Community Matrix (iii).mws: qualitative and symbolic analysis of community matrix. Revision 2 of Ecological Archives E083-022-S1, Supplement 1 to Ecology 83:1372-1385.
To remove input code while printing, select "View/Hide Content/Hide Input" from program menu.
Matrix Input: Input n number of species or system variables, and °**A**ij community matrix elements as 0**,** 1**,** or -1**.**

"31"

$$A := \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

$poly\_coef\_F0\_to\_Fn = [-1, -1, 2, 1]$

$positive\_feedback = [0, 0, 2, 1]$

$negative\_feedback = [-1, -1, 0, 0]$

$absolute\_feedback = [1, 1, 2, 1]$

$wFn = [-1., -1., 1., 1.]$

"Criterion ii"

$wD_2 = -0.33$

$ratio\_to\_model\_C = -1.$

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 0 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 1. & 1. & 1. \\ 1. & 1. & 1. \\ 1. & 1. & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. \\ 1. & 1. & 1. \\ 1. & 1. & 1. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. \\ 1. & 1. & 1. \\ 1. & 1. & 1. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & -a_{1,2} & 0 \\ -a_{2,1} & 0 & a_{2,3} \\ 0 & a_{3,2} & -a_{3,3} \end{bmatrix}$$

"system feedback"

$$F_3$$

$$a_{1,2}\, a_{2,1}\, a_{3,3}$$

$$F_2$$

$$a_{2,3}\, a_{3,2} + a_{1,2}\, a_{2,1}$$

$$F_1$$

$$-a_{3,3}$$

$$F_0$$

-1

"adjoint (-A)"

$$\begin{bmatrix} -a_{2,3}\, a_{3,2} & -a_{1,2}\, a_{3,3} & -a_{1,2}\, a_{2,3} \\ -a_{2,1}\, a_{3,3} & 0 & 0 \\ -a_{2,1}\, a_{3,2} & 0 & -a_{1,2}\, a_{2,1} \end{bmatrix}$$

"32"

$$A := \begin{bmatrix} 0 & -1 & -1 \\ 1 & 1 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

$poly\_coef\_F0\_to\_Fn = [\,-1,\ 1,\ 0,\ -2\,]$

$positive\_feedback = [\,0,\ 1,\ 1,\ 0\,]$

$negative\_feedback = [\,-1,\ 0,\ -1,\ -2\,]$

$absolute\_feedback = [\,1,\ 1,\ 2,\ 2\,]$

$wFn = [\,-1.,\ 1.,\ 0.,\ -1.\,]$

"Criterion ii"

$wD_2 = -0.50$

$ratio\_to\_model\_C = -1.5$

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 0 & 0 & 2 \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 0 & 0 & 2 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 1. & 1. & 1. \\ 1. & 1. & 1. \\ 1. & 1. & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} -2 & 0 & 0 \\ -1 & -1 & -1 \\ 0 & 0 & -2 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. \\ 1. & 1. & 1. \\ 1. & 1. & 1. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. \\ 1. & 1. & 1. \\ 1. & 1. & 1. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & -a_{1,2} & -a_{1,3} \\ a_{2,1} & a_{2,2} & -a_{2,3} \\ -a_{3,1} & 0 & 0 \end{bmatrix}$$

"system feedback"

$$F_3$$

$$-a_{1,2}\,a_{2,3}\,a_{3,1} - a_{1,3}\,a_{2,2}\,a_{3,1}$$

$$F_2$$

$$-a_{1,2}\,a_{2,1} + a_{1,3}\,a_{3,1}$$

$$F_1$$

$$a_{2,2}$$

$$F_0$$

-1

"adjoint (-A)"

$$\begin{bmatrix} 0 & 0 & a_{1,2}\,a_{2,3} + a_{1,3}\,a_{2,2} \\ a_{2,3}\,a_{3,1} & -a_{1,3}\,a_{3,1} & -a_{1,3}\,a_{2,1} \\ a_{2,2}\,a_{3,1} & a_{1,2}\,a_{3,1} & a_{1,2}\,a_{2,1} \end{bmatrix}$$

"33"

$$A := \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & -1 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

poly_coef_F0_to_Fn = [-1, -1, 0, 1]

positive_feedback = [0, 0, 0, 1]

negative_feedback = [-1, -1, 0, 0]

absolute_feedback = [1, 1, 0, 1]

wFn = [-1., -1., 1., 1.]

"Note: absence of feedback at one or more levels in system."

"Criterion ii"

$wD_2 = 1.$

ratio_to_model_C = 3.

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 1. & 1. & 1. \\ 1. & 1. & 1. \\ 1. & 1. & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. \\ 1. & 1. & 1. \\ 1. & 1. & 1. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. \\ 1. & 1. & 1. \\ 1. & 1. & 1. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & 0 & -a_{1,3} \\ a_{2,1} & 0 & 0 \\ 0 & -a_{3,2} & -a_{3,3} \end{bmatrix}$$

"system feedback"

F_3

$a_{1,3}\, a_{2,1}\, a_{3,2}$

F_2

0

F_1

$-a_{3,3}$

F_0

-1

"adjoint (-A)"

$$\begin{bmatrix} 0 & a_{1,3}\, a_{3,2} & 0 \\ a_{2,1}\, a_{3,3} & 0 & -a_{1,3}\, a_{2,1} \\ -a_{2,1}\, a_{3,2} & 0 & 0 \end{bmatrix}$$

"34"

$$A := \begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

poly_coef_F0_to_Fn = [-1, 1, 0, -1]

positive_feedback = [0, 1, 0, 0]

negative_feedback = [-1, 0, 0, -1]

absolute_feedback = [1, 1, 0, 1]

wFn = [-1., 1., 1., -1.]

"Note: absence of feedback at one or more levels in system."

"Criterion ii"

wD_2 = -1.

ratio_to_model_C = -3.

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 1. & 1. & 1. \\ 1. & 1. & 1. \\ 1. & 1. & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} -1 & 0 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. \\ 1. & 1. & 1. \\ 1. & 1. & 1. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. \\ 1. & 1. & 1. \\ 1. & 1. & 1. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & -a_{1,2} & 0 \\ 0 & a_{2,2} & -a_{2,3} \\ -a_{3,1} & 0 & 0 \end{bmatrix}$$

"system feedback"

F_3

$-a_{1,2}\, a_{2,3}\, a_{3,1}$

F_2

0

F_1

$a_{2,2}$

F_0

-1

"adjoint (-A)"

$$\begin{bmatrix} 0 & 0 & a_{1,2}\, a_{2,3} \\ a_{2,3}\, a_{3,1} & 0 & 0 \\ a_{2,2}\, a_{3,1} & a_{1,2}\, a_{3,1} & 0 \end{bmatrix}$$

"35"

$$A := \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

poly_coef_F0_to_Fn = [-1, 0, 2, 1]

positive_feedback = [0, 0, 2, 1]

negative_feedback = [-1, 0, 0, 0]

absolute_feedback = [1, 0, 2, 1]

$wFn = [\,-1.,\ 1.,\ 1.,\ 1.\,]$

"Note: absence of feedback at one or more levels in system."

"Criterion ii"

$wD_2 = 1.$

$ratio\_to\_model\_C = 3.$

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 1. & 1. & 1. \\ 1. & 1. & 1. \\ 1. & 1. & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. \\ 1. & 1. & 1. \\ 1. & 1. & 1. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. \\ 1. & 1. & 1. \\ 1. & 1. & 1. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & a_{1,2} & a_{1,3} \\ a_{2,1} & 0 & 0 \\ a_{3,1} & a_{3,2} & 0 \end{bmatrix}$$

"system feedback"

$$F_3$$

$$a_{1,3}\, a_{2,1}\, a_{3,2}$$

$$F_2$$

$$a_{1,2}\, a_{2,1} + a_{1,3}\, a_{3,1}$$

$$F_1$$

0

$$F_0$$

-1

"adjoint (-A)"

$$\begin{bmatrix} 0 & a_{1,3}\, a_{3,2} & 0 \\ 0 & -a_{1,3}\, a_{3,1} & a_{1,3}\, a_{2,1} \\ a_{2,1}\, a_{3,2} & a_{1,2}\, a_{3,1} & -a_{1,2}\, a_{2,1} \end{bmatrix}$$

"41"

$$A := \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

poly_coef_F0_to_Fn = [-1, -1, 0, -1, 1]

positive_feedback = [0, 0, 0, 0, 1]

negative_feedback = [-1, -1, 0, -1, 0]

absolute_feedback = [1, 1, 0, 1, 1]

wFn = [-1., -1., 1., -1., 1.]

"Note: absence of feedback at one or more levels in system."

"Criterion ii"

$wD_3 = 0.$

ratio_to_model_C = 0.

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & 0 & 0 & -a_{1,4} \\ 0 & 0 & -a_{2,3} & 0 \\ a_{3,1} & 0 & 0 & 0 \\ 0 & a_{4,2} & a_{4,3} & -a_{4,4} \end{bmatrix}$$

"system feedback"

$$F_4$$

$$a_{1,4}\, a_{3,1}\, a_{2,3}\, a_{4,2}$$

$$F_3$$

$$-a_{3,1}\, a_{1,4}\, a_{4,3}$$

$$F_2$$

$$0$$

$$F_1$$

$$-a_{4,4}$$

$$F_0$$

$$1$$

"adjoint (-A)"

$$\begin{bmatrix} 0 & 0 & a_{4,2}\, a_{1,4}\, a_{2,3} & 0 \\ -a_{3,1}\, a_{2,3}\, a_{4,4} & a_{3,1}\, a_{1,4}\, a_{4,3} & 0 & a_{3,1}\, a_{1,4}\, a_{2,3} \\ 0 & -a_{3,1}\, a_{1,4}\, a_{4,2} & 0 & 0 \\ -a_{3,1}\, a_{2,3}\, a_{4,2} & 0 & 0 & 0 \end{bmatrix}$$

"42"

$$A := \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

poly_coef_F0_to_Fn = [-1, 0, 0, 0, 1]

positive_feedback = [0, 0, 1, 0, 1]

negative_feedback = [-1, 0, -1, 0, 0]

absolute_feedback = [1, 0, 2, 0, 1]

wFn = [-1., 1., 0., 1., 1.]

"Note: absence of feedback at one or more levels in system."

"Criterion ii"

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Error, numeric exception: division by zero
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Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & a_{1,2} & 0 & a_{1,4} \\ 0 & 0 & -a_{2,3} & 0 \\ 0 & a_{3,2} & 0 & 0 \\ a_{4,1} & 0 & 0 & 0 \end{bmatrix}$$

"system feedback"

$$F_4$$

$$a_{2,3}\, a_{4,1}\, a_{1,4}\, a_{3,2}$$

$$F_3$$

$$0$$

$$F_2$$

$$-a_{2,3}\, a_{3,2} + a_{4,1}\, a_{1,4}$$

$$F_1$$

$$0$$

$$F_0$$

$$-1$$

"adjoint (-A)"

$$\begin{bmatrix} 0 & 0 & 0 & a_{3,2}\, a_{1,4}\, a_{2,3} \\ 0 & 0 & a_{4,1}\, a_{1,4}\, a_{2,3} & 0 \\ 0 & -a_{4,1}\, a_{1,4}\, a_{3,2} & 0 & 0 \\ a_{4,1}\, a_{2,3}\, a_{3,2} & 0 & -a_{4,1}\, a_{1,2}\, a_{2,3} & 0 \end{bmatrix}$$

"43"

$$A := \begin{bmatrix} 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

poly_coef_F0_to_Fn = [-1, 0, -1, -2, 1]

positive_feedback = [0, 0, 0, 0, 1]

negative_feedback = [-1, 0, -1, -2, 0]

absolute_feedback = [1, 0, 1, 2, 1]

wFn = [-1., 1., -1., -1., 1.]

"Note: absence of feedback at one or more levels in system."

"Criterion ii"

$wD_3 = -1.$

ratio_to_model_C = -11.

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ -2 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ -2 & -1 & -1 & 0 \\ -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & 0 & 0 & a_{1,4} \\ -a_{2,1} & 0 & a_{2,3} & -a_{2,4} \\ a_{3,1} & 0 & 0 & -a_{3,4} \\ 0 & a_{4,2} & 0 & 0 \end{bmatrix}$$

"system feedback"

$$F_4$$

$$a_{1,4}\, a_{3,1}\, a_{2,3}\, a_{4,2}$$

$$F_3$$

$$-a_{4,2}\, a_{2,3}\, a_{3,4} - a_{2,1}\, a_{1,4}\, a_{4,2}$$

$$F_2$$

$$-a_{2,4}\, a_{4,2}$$

$$F_1$$

$$0$$

$$F_0$$

$$-1$$

"adjoint (-A)"

$$\begin{bmatrix} a_{4,2}\, a_{2,3}\, a_{3,4} & 0 & a_{4,2}\, a_{1,4}\, a_{2,3} & 0 \\ 0 & 0 & 0 & a_{3,1}\, a_{1,4}\, a_{2,3} \\ a_{2,1}\, a_{3,4}\, a_{4,2} + a_{3,1}\, a_{2,4}\, a_{4,2} & a_{3,1}\, a_{1,4}\, a_{4,2} & a_{2,1}\, a_{1,4}\, a_{4,2} & 0 \\ a_{3,1}\, a_{2,3}\, a_{4,2} & 0 & 0 & 0 \end{bmatrix}$$

"44"

$$A := \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & -1 & 1 & 0 \\ -1 & 1 & -1 & 0 \\ -1 & -1 & 0 & 0 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

$poly\_coef\_F0\_to\_Fn = [\,-1, -1, 1, -3, 0\,]$

$positive\_feedback = [\,0, 1, 3, 1, 2\,]$

$negative\_feedback = [\,-1, -2, -2, -4, -2\,]$

$absolute\_feedback = [\,1, 3, 5, 5, 4\,]$

$wFn = [\,-1., -0.33, 0.20, -0.60, 0.\,]$

"Criterion ii"

$wD_3 = -0.088$

$ratio\_to\_model\_C = -0.97$

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 0 & -1 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 2 & 2 & 2 \\ 4 & 1 & 1 & 2 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 1. & 1. & 1. & 0. \\ 1. & 1. & 1. & 0. \\ 1. & 1. & 1. & 0. \\ 0. & 1. & 1. & 0. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 0. \\ 1. & 1. & 0.33 & 0. \\ 1. & 1. & 1. & 0. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 3 & 1 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 0. & 1. & 1. & 1. \\ 1. & 0.33 & 1. & 0. \\ 1. & 1. & 1. & 0. \\ 1. & 1. & 1. & 1. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} a_{1,1} & 0 & 0 & a_{1,4} \\ a_{2,1} & -a_{2,2} & a_{2,3} & 0 \\ -a_{3,1} & a_{3,2} & -a_{3,3} & 0 \\ -a_{4,1} & -a_{4,2} & 0 & 0 \end{bmatrix}$$

"system feedback"

F_4

$$-a_{2,2}\,a_{4,1}\,a_{1,4}\,a_{3,3} + a_{2,3}\,a_{4,1}\,a_{1,4}\,a_{3,2} - a_{3,3}\,a_{2,1}\,a_{1,4}\,a_{4,2} + a_{1,4}\,a_{3,1}\,a_{2,3}\,a_{4,2}$$

F_3

$$a_{1,1}\,a_{2,2}\,a_{3,3} - a_{1,1}\,a_{3,2}\,a_{2,3} - a_{2,1}\,a_{1,4}\,a_{4,2} - a_{4,1}\,a_{1,4}\,a_{3,3} - a_{4,1}\,a_{1,4}\,a_{2,2}$$

F_2

$$-a_{2,2}\,a_{3,3} + a_{2,3}\,a_{3,2} + a_{1,1}\,a_{3,3} + a_{1,1}\,a_{2,2} - a_{4,1}\,a_{1,4}$$

F_1

$$-a_{3,3} - a_{2,2} + a_{1,1}$$

F_0

-1

"adjoint (-A)"

$$\begin{bmatrix} 0, & -a_{4,2}\,a_{1,4}\,a_{3,3}, & -a_{4,2}\,a_{1,4}\,a_{2,3}, & a_{2,2}\,a_{1,4}\,a_{3,3} - a_{3,2}\,a_{1,4}\,a_{2,3} \\ 0, & a_{4,1}\,a_{1,4}\,a_{3,3}, & a_{4,1}\,a_{1,4}\,a_{2,3}, & a_{2,1}\,a_{1,4}\,a_{3,3} - a_{3,1}\,a_{1,4}\,a_{2,3} \\ 0, & a_{3,1}\,a_{1,4}\,a_{4,2} + a_{4,1}\,a_{1,4}\,a_{3,2}, & a_{2,1}\,a_{1,4}\,a_{4,2} + a_{4,1}\,a_{1,4}\,a_{2,2}, & a_{2,1}\,a_{1,4}\,a_{3,2} - a_{3,1}\,a_{1,4}\,a_{2,2} \\ -a_{2,1}\,a_{3,3}\,a_{4,2} + a_{3,1}\,a_{2,3}\,a_{4,2} - a_{4,1}\,a_{2,2}\,a_{3,3} + a_{4,1}\,a_{2,3}\,a_{3,2}, & a_{1,1}\,a_{3,3}\,a_{4,2}, & a_{1,1}\,a_{2,3}\,a_{4,2}, & -a_{1,1}\,(a_{2,2}\,a_{3,3} - a_{2,3}\,a_{3,2}) \end{bmatrix}$$

"45"

$$A := \begin{bmatrix} 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

poly_coef_F0_to_Fn = [-1, 0, 0, -1, 0]

positive_feedback = [0, 0, 1, 0, 1]

negative_feedback = [-1, 0, -1, -1, -1]

absolute_feedback = [1, 0, 2, 1, 2]

wFn = [-1., 1., 0., -1., 0.]

"Note: absence of feedback at one or more levels in system."

"Criterion ii"

$wD_3 = -1.$

ratio_to_model_C = -11.

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 1. & 1. & 0. & 1. \\ 1. & 1. & 1. & 0. \\ 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 0. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \\ 1. & 1. & 0. & 1. \\ 1. & 1. & 1. & 0. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. \\ 1. & 0. & 1. & 1. \\ 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & 0 & -a_{1,3} & -a_{1,4} \\ 0 & 0 & a_{2,3} & a_{2,4} \\ -a_{3,1} & 0 & 0 & a_{3,4} \\ 0 & -a_{4,2} & 0 & 0 \end{bmatrix}$$

"system feedback"

$$F_4$$

$$a_{1,3}\, a_{3,1}\, a_{2,4}\, a_{4,2} - a_{1,4}\, a_{3,1}\, a_{2,3}\, a_{4,2}$$

$$F_3$$

$$-a_{4,2}\, a_{2,3}\, a_{3,4}$$

$$F_2$$

$$-a_{2,4}\, a_{4,2} + a_{3,1}\, a_{1,3}$$

$$F_1$$

$$0$$

$$F_0$$

$$-1$$

"adjoint (-A)"

$$\begin{bmatrix} a_{4,2}\, a_{2,3}\, a_{3,4} & a_{4,2}\, a_{1,3}\, a_{3,4} & a_{4,2}\,(-a_{1,3}\, a_{2,4} + a_{1,4}\, a_{2,3}) & 0 \\ 0 & 0 & 0 & a_{3,1}\,(-a_{1,3}\, a_{2,4} + a_{1,4}\, a_{2,3}) \\ -a_{3,1}\, a_{2,4}\, a_{4,2} & -a_{3,1}\, a_{1,4}\, a_{4,2} & 0 & 0 \\ a_{3,1}\, a_{2,3}\, a_{4,2} & a_{3,1}\, a_{1,3}\, a_{4,2} & 0 & 0 \end{bmatrix}$$

"51"

$$A := \begin{bmatrix} 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

$poly\_coef\_F0\_to\_Fn = [\,-1,\ 0,\ 0,\ 0,\ -1,\ 1\,]$

$positive\_feedback = [\,0,\ 1,\ 1,\ 1,\ 0,\ 1\,]$

$negative\_feedback = [\,-1,\ -1,\ -1,\ -1,\ -1,\ 0\,]$

$absolute\_feedback = [\,1,\ 2,\ 2,\ 2,\ 1,\ 1\,]$

$wFn = [\,-1.,\ 0.,\ 0.,\ 0.,\ -1.,\ 1.\,]$

"Criterion ii"

$wD_4 = -0.030$

$ratio\_to\_model\_C = -3.2$

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ -1 & 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & -a_{1,2} & 0 & 0 & 0 \\ 0 & -a_{2,2} & 0 & 0 & -a_{2,5} \\ 0 & 0 & a_{3,3} & a_{3,4} & 0 \\ 0 & -a_{4,2} & -a_{4,3} & 0 & 0 \\ a_{5,1} & 0 & 0 & 0 & 0 \end{bmatrix}$$

"system feedback"

$$F_5$$

$$a_{1,2}\, a_{5,1}\, a_{4,3}\, a_{2,5}\, a_{3,4}$$

$$F_4$$

$$-a_{5,1}\, a_{1,2}\, a_{2,5}\, a_{3,3}$$

$$F_3$$

$$-a_{2,2}\, a_{3,4}\, a_{4,3} + a_{5,1}\, a_{1,2}\, a_{2,5}$$

$$F_2$$

$$-a_{3,4}\, a_{4,3} + a_{2,2}\, a_{3,3}$$

$$F_1$$

$$a_{3,3} - a_{2,2}$$

$$F_0$$

-1

"adjoint (-A)"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & a_{1,2}\, a_{4,3}\, a_{2,5}\, a_{3,4} \\ -a_{5,1}\, a_{4,3}\, a_{2,5}\, a_{3,4} & 0 & 0 & 0 & 0 \\ a_{5,1}\, a_{4,2}\, a_{2,5}\, a_{3,4} & 0 & 0 & -a_{5,1}\, a_{1,2}\, a_{2,5}\, a_{3,4} & 0 \\ -a_{5,1}\, a_{4,2}\, a_{2,5}\, a_{3,3} & 0 & a_{5,1}\, a_{1,2}\, a_{2,5}\, a_{4,3} & a_{5,1}\, a_{1,2}\, a_{2,5}\, a_{3,3} & 0 \\ a_{5,1}\, a_{2,2}\, a_{3,4}\, a_{4,3} & -a_{5,1}\, a_{1,2}\, a_{3,4}\, a_{4,3} & 0 & 0 & 0 \end{bmatrix}$$

"52"

$$A := \begin{bmatrix} 0 & 0 & -1 & 1 & -1 \\ -1 & -1 & 0 & -1 & 0 \\ -1 & -1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & -1 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

poly_coef_F0_to_Fn = [-1, -1, 3, 3, 1, 0]

positive_feedback = [0, 1, 4, 6, 6, 2]

negative_feedback = [-1, -2, -1, -3, -5, -2]

absolute_feedback = [1, 3, 5, 9, 11, 4]

wFn = [-1., -0.33, 0.60, 0.33, 0.091, 0.]

"Criterion ii"

wD_4 = -0.00024

ratio_to_model_C = -0.025

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 0 & 1 & 1 & 2 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 4 & 3 & 3 & 4 & 4 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 2 & 2 & 2 & 4 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 1. & 1. & 1. & 0. & 1. \\ 1. & 0. & 0. & 0. & 1. \\ 0. & 0.33 & 0.33 & 0. & 0. \\ 1. & 1. & 1. & 0. & 1. \\ 1. & 0. & 0. & 0. & 0. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 4 & 1 & 1 & 2 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 0. & 1. & 1. & 0. & 1. \\ 1. & 0. & 1. & 1. & 1. \\ 1. & 1. & 0. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 0. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 0. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 0. & 1. \\ 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & 0 & -a_{1,3} & a_{1,4} & -a_{1,5} \\ -a_{2,1} & -a_{2,2} & 0 & -a_{2,4} & 0 \\ -a_{3,1} & -a_{3,2} & 0 & -a_{3,4} & 0 \\ a_{4,1} & 0 & 0 & a_{4,4} & 0 \\ 0 & -a_{5,2} & 0 & 0 & -a_{5,5} \end{bmatrix}$$

"system feedback"

F_5

$$-a_{3,1}\,a_{2,2}\,a_{1,3}\,a_{4,4}\,a_{5,5} + a_{2,1}\,a_{3,2}\,a_{1,3}\,a_{4,4}\,a_{5,5} - a_{2,4}\,a_{4,1}\,a_{3,2}\,a_{1,3}\,a_{5,5} + a_{3,4}\,a_{4,1}\,a_{2,2}\,a_{1,3}\,a_{5,5}$$

F_4

$$-a_{4,1}\,a_{5,2}\,a_{1,5}\,a_{2,4} - a_{4,1}\,a_{3,2}\,a_{1,3}\,a_{2,4} + a_{4,1}\,a_{1,3}\,a_{3,4}\,a_{5,5} + a_{4,1}\,a_{2,2}\,a_{1,3}\,a_{3,4} + a_{4,1}\,a_{2,2}\,a_{1,4}\,a_{5,5} - a_{3,1}\,a_{1,3}\,a_{4,4}\,a_{5,5} + a_{3,1}\,a_{2,2}\,a_{1,3}\,a_{5,5} - a_{3,1}\,a_{2,2}\,a_{1,3}\,a_{4,4} - a_{2,1}\,a_{3,2}\,a_{1,3}\,a_{5,5} + a_{2,1}\,a_{3,2}\,a_{1,3}\,a_{4,4} + a_{2,1}\,a_{5,2}\,a_{1,5}\,a_{4,4}$$

F_3

$$a_{2,2}\,a_{4,4}\,a_{5,5} - a_{2,1}\,a_{3,2}\,a_{1,3} - a_{2,1}\,a_{5,2}\,a_{1,5} + a_{3,1}\,a_{1,3}\,a_{5,5} - a_{3,1}\,a_{1,3}\,a_{4,4} + a_{3,1}\,a_{1,3}\,a_{2,2} + a_{4,1}\,a_{1,3}\,a_{3,4} + a_{4,1}\,a_{1,4}\,a_{5,5} + a_{4,1}\,a_{2,2}\,a_{1,4}$$

F_2

$$a_{4,1}\,a_{1,4} + a_{3,1}\,a_{1,3} + a_{2,2}\,a_{4,4} + a_{4,4}\,a_{5,5} - a_{2,2}\,a_{5,5}$$

F_1

$$-a_{5,5} + a_{4,4} - a_{2,2}$$

F_0

-1

"adjoint (-A)"

$$[0\,,\ -a_{3,2}\,a_{1,3}\,a_{4,4}\,a_{5,5}\,,\ a_{2,2}\,a_{1,3}\,a_{4,4}\,a_{5,5}\,,\ a_{2,2}\,a_{1,3}\,a_{3,4}\,a_{5,5} - a_{3,2}\,a_{1,3}\,a_{2,4}\,a_{5,5}\,,\ 0\,]$$

$$[0\,,\ a_{3,1}\,a_{1,3}\,a_{4,4}\,a_{5,5} - a_{4,1}\,a_{1,3}\,a_{3,4}\,a_{5,5}\,,\ -a_{2,1}\,a_{1,3}\,a_{4,4}\,a_{5,5} + a_{4,1}\,a_{1,3}\,a_{2,4}\,a_{5,5}\,,\ -a_{2,1}\,a_{1,3}\,a_{3,4}\,a_{5,5} + a_{3,1}\,a_{1,3}\,a_{2,4}\,a_{5,5}\,,\ 0\,]$$

$$[-a_{2,1}\,a_{3,2}\,a_{4,4}\,a_{5,5} + a_{3,1}\,a_{2,2}\,a_{4,4}\,a_{5,5} - a_{4,1}\,a_{2,2}\,a_{3,4}\,a_{5,5} + a_{4,1}\,a_{3,2}\,a_{2,4}\,a_{5,5}\,,$$
$$a_{3,1}\,a_{5,2}\,a_{1,5}\,a_{4,4} + a_{4,1}\,a_{3,2}\,a_{1,4}\,a_{5,5} - a_{4,1}\,a_{5,2}\,a_{1,5}\,a_{3,4}\,,\ -a_{2,1}\,a_{5,2}\,a_{1,5}\,a_{4,4} - a_{4,1}\,a_{2,2}\,a_{1,4}\,a_{5,5} + a_{4,1}\,a_{5,2}\,a_{1,5}\,a_{2,4}\,,$$
$$a_{2,1}\,a_{3,2}\,a_{1,4}\,a_{5,5} - a_{2,1}\,a_{5,2}\,a_{1,5}\,a_{3,4} - a_{3,1}\,a_{2,2}\,a_{1,4}\,a_{5,5} + a_{3,1}\,a_{5,2}\,a_{1,5}\,a_{2,4}\,,$$
$$a_{2,1}\,a_{3,2}\,a_{1,5}\,a_{4,4} - a_{3,1}\,a_{2,2}\,a_{1,5}\,a_{4,4} + a_{4,1}\,a_{2,2}\,a_{1,5}\,a_{3,4} - a_{4,1}\,a_{3,2}\,a_{1,5}\,a_{2,4}\,]$$

$$[0\,,\ a_{4,1}\,a_{3,2}\,a_{1,3}\,a_{5,5}\,,\ -a_{4,1}\,a_{2,2}\,a_{1,3}\,a_{5,5}\,,\ a_{2,1}\,a_{3,2}\,a_{1,3}\,a_{5,5} - a_{3,1}\,a_{2,2}\,a_{1,3}\,a_{5,5}\,,\ 0\,]$$

$$[0\,,\ -a_{3,1}\,a_{5,2}\,a_{1,3}\,a_{4,4} + a_{4,1}\,a_{5,2}\,a_{1,3}\,a_{3,4}\,,\ a_{2,1}\,a_{5,2}\,a_{1,3}\,a_{4,4} - a_{4,1}\,a_{5,2}\,a_{1,3}\,a_{2,4}\,,\ a_{2,1}\,a_{5,2}\,a_{1,3}\,a_{3,4} - a_{3,1}\,a_{5,2}\,a_{1,3}\,a_{2,4}\,,$$
$$-a_{2,1}\,a_{3,2}\,a_{1,3}\,a_{4,4} + a_{3,1}\,a_{2,2}\,a_{1,3}\,a_{4,4} - a_{4,1}\,a_{2,2}\,a_{1,3}\,a_{3,4} + a_{4,1}\,a_{3,2}\,a_{1,3}\,a_{2,4}\,]$$

"53"

$$A := \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & -1 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

$poly\_coef\_F0\_to\_Fn = [\,-1, -1, -1, -1, 1, 0\,]$

$positive\_feedback = [\,0, 0, 0, 1, 2, 0\,]$

$negative\_feedback = [\,-1, -1, -1, -2, -1, 0\,]$

$absolute\_feedback = [\,1, 1, 1, 3, 3, 0\,]$

$wFn = [\,-1., -1., -1., -0.33, 0.33, 1.\,]$

"Note: absence of feedback at one or more levels in system."

"Criterion ii"

$wD_4 = -0.022$

$ratio\_to\_model\_C = -2.4$

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 2 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -2 & 0 & 2 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 2 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -2 & 0 & 2 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 2 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & 0 & 0 & a_{1,4} & 0 \\ a_{2,1} & 0 & 0 & 0 & -a_{2,5} \\ 0 & 0 & 0 & a_{3,4} & 0 \\ -a_{4,1} & a_{4,2} & 0 & 0 & 0 \\ -a_{5,1} & 0 & a_{5,3} & a_{5,4} & -a_{5,5} \end{bmatrix}$$

"system feedback"

F_5

0

F_4

$-a_{4,2}\,a_{5,3}\,a_{2,5}\,a_{3,4} + a_{2,1}\,a_{4,2}\,a_{1,4}\,a_{5,5} + a_{5,1}\,a_{4,2}\,a_{1,4}\,a_{2,5}$

F_3

$-a_{4,2}\,a_{2,5}\,a_{5,4} + a_{2,1}\,a_{4,2}\,a_{1,4} - a_{4,1}\,a_{1,4}\,a_{5,5}$

F_2

$-a_{4,1}\,a_{1,4}$

F_1

$-a_{5,5}$

F_0

-1

"adjoint (-A)"

$$\begin{bmatrix} a_{4,2}\,a_{5,3}\,a_{2,5}\,a_{3,4} & 0 & -a_{4,2}\,a_{5,3}\,a_{1,4}\,a_{2,5} & 0 & 0 \\ a_{4,1}\,a_{5,3}\,a_{2,5}\,a_{3,4} & 0 & -a_{4,1}\,a_{5,3}\,a_{1,4}\,a_{2,5} & 0 & 0 \\ a_{2,1}\,a_{4,2}\,a_{3,4}\,a_{5,5} + a_{5,1}\,a_{4,2}\,a_{2,5}\,a_{3,4} & 0 & -a_{2,1}\,a_{4,2}\,a_{1,4}\,a_{5,5} - a_{5,1}\,a_{4,2}\,a_{1,4}\,a_{2,5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ a_{2,1}\,a_{4,2}\,a_{3,4}\,a_{5,3} & 0 & -a_{2,1}\,a_{4,2}\,a_{1,4}\,a_{5,3} & 0 & 0 \end{bmatrix}$$

"54"

$$A := \begin{bmatrix} -1 & -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

poly_coef_F0_to_Fn = [-1, 0, -1, 1, 1, 1]

positive_feedback = [0, 1, 1, 2, 2, 2]

negative_feedback = [-1, -1, -2, -1, -1, -1]

absolute_feedback = [1, 2, 3, 3, 3, 3]

wFn = [-1., 0., -0.33, 0.33, 0.33, 0.33]

"Criterion ii"

$wD_4 = 0.0041$

ratio_to_model_C = 0.44

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} -1 & 0 & 1 & 0 & -1 \\ -1 & -1 & 0 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 2 & 0 & 2 \\ 1 & 1 & 1 & 0 & 2 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 3 & 0 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 1. & 0. & 1. & 1. & 1. \\ 1. & 1. & 0. & 1. & 0. \\ 1. & 1. & 1. & 1. & 0. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 0.33 & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 2 & 1 & -1 & 0 & 1 \\ 1 & 1 & -1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & 1 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 2 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 0. & 1. & 0. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 & 1 \\ 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 1 & 1 & -1 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 3 & 0 \\ 1 & 1 & 1 & 0 & 2 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. \\ 1. & 0. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 0. \\ 1. & 1. & 1. & 0.33 & 1. \\ 1. & 1. & 1. & 1. & 0. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} -a_{1,1} & -a_{1,2} & 0 & a_{1,4} & 0 \\ a_{2,1} & 0 & 0 & -a_{2,4} & 0 \\ 0 & -a_{3,2} & a_{3,3} & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{4,5} \\ 0 & 0 & a_{5,3} & -a_{5,4} & 0 \end{bmatrix}$$

"system feedback"

$$F_5$$

$$a_{1,1}\,a_{3,2}\,a_{5,3}\,a_{2,4}\,a_{4,5} + a_{1,2}\,a_{2,1}\,a_{3,3}\,a_{4,5}\,a_{5,4} - a_{1,4}\,a_{2,1}\,a_{3,2}\,a_{4,5}\,a_{5,3}$$

$$F_4$$

$$a_{3,2}\,a_{5,3}\,a_{2,4}\,a_{4,5} + a_{1,1}\,a_{3,3}\,a_{4,5}\,a_{5,4} - a_{2,1}\,a_{1,2}\,a_{4,5}\,a_{5,4}$$

$$F_3$$

$$a_{3,3}\,a_{4,5}\,a_{5,4} - a_{1,1}\,a_{4,5}\,a_{5,4} + a_{2,1}\,a_{1,2}\,a_{3,3}$$

$$F_2$$

$$-a_{4,5}\,a_{5,4} + a_{1,1}\,a_{3,3} - a_{2,1}\,a_{1,2}$$

$$F_1$$

$$a_{3,3} - a_{1,1}$$

$$F_0$$

-1

"adjoint (-A)"

$$\begin{bmatrix} -a_{3,2}\,a_{5,3}\,a_{2,4}\,a_{4,5}, & a_{1,2}\,a_{3,3}\,a_{4,5}\,a_{5,4} - a_{3,2}\,a_{5,3}\,a_{1,4}\,a_{4,5}, & a_{1,2}\,a_{5,3}\,a_{2,4}\,a_{4,5}, & 0, & -a_{1,2}\,a_{3,3}\,a_{2,4}\,a_{4,5} \\ -a_{2,1}\,a_{3,3}\,a_{4,5}\,a_{5,4}, & -a_{1,1}\,a_{3,3}\,a_{4,5}\,a_{5,4}, & -a_{1,1}\,a_{5,3}\,a_{2,4}\,a_{4,5} + a_{2,1}\,a_{5,3}\,a_{1,4}\,a_{4,5}, & 0, & a_{1,1}\,a_{3,3}\,a_{2,4}\,a_{4,5} - a_{2,1}\,a_{3,3}\,a_{1,4}\,a_{4,5} \\ -a_{2,1}\,a_{3,2}\,a_{4,5}\,a_{5,4}, & -a_{1,1}\,a_{3,2}\,a_{4,5}\,a_{5,4}, & a_{2,1}\,a_{1,2}\,a_{4,5}\,a_{5,4}, & 0, & a_{1,1}\,a_{3,2}\,a_{2,4}\,a_{4,5} - a_{2,1}\,a_{3,2}\,a_{1,4}\,a_{4,5} \\ -a_{2,1}\,a_{3,2}\,a_{4,5}\,a_{5,3}, & -a_{1,1}\,a_{3,2}\,a_{4,5}\,a_{5,3}, & a_{2,1}\,a_{1,2}\,a_{4,5}\,a_{5,3}, & 0, & -a_{2,1}\,a_{1,2}\,a_{3,3}\,a_{4,5} \\ 0, & 0, & 0, & a_{1,1}\,a_{3,2}\,a_{2,4}\,a_{5,3} + a_{2,1}\,a_{1,2}\,a_{3,3}\,a_{5,4} - a_{2,1}\,a_{3,2}\,a_{1,4}\,a_{5,3}, & 0 \end{bmatrix}$$

"55"

$$A := \begin{bmatrix} 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & -1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

poly_coef_F0_to_Fn = [-1, 1, 1, 1, 0, 0]

positive_feedback = [0, 1, 1, 2, 1, 0]

negative_feedback = [-1, 0, 0, -1, -1, 0]

absolute_feedback = [1, 1, 1, 3, 2, 0]

wFn = [-1., 1., 1., 0.33, 0., 1.]

"Note: absence of feedback at one or more levels in system."

"Criterion ii"

$wD_4 = 0.$

ratio_to_model_C = 0.

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 & 0 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 0. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 0. & 1. & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & a_{1,2} & 0 & -a_{1,4} & 0 \\ 0 & a_{2,2} & 0 & -a_{2,4} & 0 \\ 0 & 0 & 0 & -a_{3,4} & 0 \\ 0 & 0 & -a_{4,3} & 0 & -a_{4,5} \\ a_{5,1} & a_{5,2} & 0 & 0 & 0 \end{bmatrix}$$

"system feedback"

F_5

0

F_4

$a_{5,1}\,a_{1,2}\,a_{2,4}\,a_{4,5} - a_{5,1}\,a_{1,4}\,a_{4,5}\,a_{2,2}$

F_3

$-a_{2,2}\,a_{3,4}\,a_{4,3} + a_{5,2}\,a_{2,4}\,a_{4,5} + a_{5,1}\,a_{1,4}\,a_{4,5}$

F_2

$a_{3,4}\,a_{4,3}$

F_1

$a_{2,2}$

F_0

-1

"adjoint (-A)"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -a_{5,1}\,a_{2,2}\,a_{3,4}\,a_{4,5} & a_{5,1}\,a_{1,2}\,a_{3,4}\,a_{4,5} & a_{5,1}\,(-a_{1,2}\,a_{2,4}\,a_{4,5} + a_{2,2}\,a_{1,4}\,a_{4,5}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ a_{5,1}\,a_{2,2}\,a_{3,4}\,a_{4,3} & -a_{5,1}\,a_{1,2}\,a_{3,4}\,a_{4,3} & a_{5,1}\,(a_{1,2}\,a_{2,4}\,a_{4,3} - a_{2,2}\,a_{1,4}\,a_{4,3}) & 0 & 0 \end{bmatrix}$$

"61"

$$A := \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & -1 & 0 \\ -1 & 1 & 1 & 0 & -1 & 0 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

$poly\_coef\_F0\_to\_Fn = [\,-1, 1, -1, 1, 5, -5, 0\,]$

$positive\_feedback = [\,0, 2, 3, 7, 13, 10, 7\,]$

$negative\_feedback = [\,-1, -1, -4, -6, -8, -15, -7\,]$

$absolute\_feedback = [\,1, 3, 7, 13, 21, 25, 14\,]$

$wFn = [\,-1., 0.33, -0.14, 0.077, 0.24, -0.20, 0.\,]$

"Criterion ii"

$wD_5 = 0.$

$ratio\_to\_model\_C = 0.$

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 0 & 2 & 0 & -2 & 0 & -2 \\ 0 & 3 & 0 & -3 & 0 & -3 \\ 0 & 2 & 0 & -2 & 0 & -2 \\ 0 & -1 & 0 & 1 & 0 & 1 \\ 0 & 3 & 0 & -3 & 0 & -3 \\ 0 & -1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 2 & 4 & 2 & 6 & 4 & 4 \\ 2 & 9 & 2 & 3 & 4 & 3 \\ 2 & 4 & 2 & 8 & 4 & 4 \\ 4 & 5 & 4 & 5 & 6 & 3 \\ 2 & 5 & 2 & 3 & 4 & 3 \\ 10 & 5 & 4 & 5 & 6 & 3 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 0. & 0.50 & 0. & 0.33 & 0. & 0.50 \\ 0. & 0.33 & 0. & 1. & 0. & 1. \\ 0. & 0.50 & 0. & 0.25 & 0. & 0.50 \\ 0. & 0.20 & 0. & 0.20 & 0. & 0.33 \\ 0. & 0.60 & 0. & 1. & 0. & 1. \\ 0. & 0.20 & 0. & 0.20 & 0. & 0.33 \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & -3 & 0 & 3 & 0 & 3 \\ 0 & -3 & 0 & 3 & 0 & 3 \\ 0 & -2 & 0 & 2 & 0 & 2 \\ 0 & -2 & 0 & 2 & 0 & 2 \\ 0 & -5 & 0 & 5 & 0 & 5 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 4 & 5 & 4 & 5 & 6 & 3 \\ 2 & 5 & 2 & 3 & 4 & 3 \\ 2 & 9 & 2 & 3 & 4 & 3 \\ 2 & 4 & 2 & 8 & 4 & 4 \\ 2 & 4 & 2 & 6 & 10 & 4 \\ 2 & 7 & 2 & 7 & 4 & 7 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 0. & 0.20 & 0. & 0.20 & 0. & 0.33 \\ 0. & 0.60 & 0. & 1. & 0. & 1. \\ 0. & 0.33 & 0. & 1. & 0. & 1. \\ 0. & 0.50 & 0. & 0.25 & 0. & 0.50 \\ 0. & 0.50 & 0. & 0.33 & 0. & 0.50 \\ 0. & 0.71 & 0. & 0.71 & 0. & 0.71 \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & -3 & 0 & 3 & 0 & 3 \\ 0 & -3 & 0 & 3 & 0 & 3 \\ 0 & -2 & 0 & 2 & 0 & 2 \\ 0 & -2 & 0 & 2 & 0 & 2 \\ 0 & -5 & 0 & 5 & 0 & 5 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 10 & 5 & 4 & 5 & 6 & 3 \\ 2 & 9 & 2 & 3 & 4 & 3 \\ 2 & 9 & 12 & 3 & 4 & 3 \\ 2 & 4 & 2 & 6 & 4 & 4 \\ 2 & 4 & 2 & 6 & 4 & 4 \\ 2 & 7 & 2 & 7 & 4 & 7 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 0. & 0.20 & 0. & 0.20 & 0. & 0.33 \\ 0. & 0.33 & 0. & 1. & 0. & 1. \\ 0. & 0.33 & 0. & 1. & 0. & 1. \\ 0. & 0.50 & 0. & 0.33 & 0. & 0.50 \\ 0. & 0.50 & 0. & 0.33 & 0. & 0.50 \\ 0. & 0.71 & 0. & 0.71 & 0. & 0.71 \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & 0 & 0 & -a_{1,4} & 0 & a_{1,6} \\ 0 & a_{2,2} & 0 & 0 & -a_{2,5} & 0 \\ 0 & -a_{3,2} & a_{3,3} & a_{3,4} & a_{3,5} & a_{3,6} \\ a_{4,1} & 0 & -a_{4,3} & 0 & 0 & 0 \\ a_{5,1} & 0 & 0 & -a_{5,4} & -a_{5,5} & 0 \\ -a_{6,1} & a_{6,2} & a_{6,3} & 0 & -a_{6,5} & 0 \end{bmatrix}$$

"system feedback"

F_6

$$a_{5,4}\,a_{4,1}\,a_{2,2}\,a_{3,3}\,a_{1,6}\,a_{6,5} + a_{5,4}\,a_{4,1}\,a_{2,2}\,a_{6,3}\,a_{1,6}\,a_{3,5} - a_{5,4}\,a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{1,6}\,a_{2,5} - a_{5,4}\,a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{1,6}\,a_{2,5}$$
$$- a_{5,4}\,a_{6,1}\,a_{2,2}\,a_{4,3}\,a_{1,6}\,a_{3,5} + a_{5,4}\,a_{6,1}\,a_{3,2}\,a_{4,3}\,a_{1,6}\,a_{2,5} - a_{5,5}\,a_{4,1}\,a_{2,2}\,a_{6,3}\,a_{1,4}\,a_{3,6} - a_{5,5}\,a_{4,1}\,a_{2,2}\,a_{6,3}\,a_{1,6}\,a_{3,4}$$
$$+ a_{5,5}\,a_{6,1}\,a_{2,2}\,a_{4,3}\,a_{1,4}\,a_{3,6} + a_{5,5}\,a_{6,1}\,a_{2,2}\,a_{4,3}\,a_{1,6}\,a_{3,4} + a_{6,5}\,a_{5,1}\,a_{2,2}\,a_{4,3}\,a_{1,4}\,a_{3,6} + a_{6,5}\,a_{5,1}\,a_{2,2}\,a_{4,3}\,a_{1,6}\,a_{3,4}$$
$$- a_{3,4}\,a_{5,1}\,a_{6,2}\,a_{4,3}\,a_{1,6}\,a_{2,5} - a_{3,6}\,a_{5,1}\,a_{6,2}\,a_{4,3}\,a_{1,4}\,a_{2,5}$$

F_5

$$-a_{6,2}\,a_{4,3}\,a_{5,4}\,a_{2,5}\,a_{3,6} + a_{2,2}\,a_{4,3}\,a_{5,4}\,a_{3,6}\,a_{6,5} - a_{4,1}\,a_{3,3}\,a_{5,4}\,a_{1,6}\,a_{6,5} + a_{4,1}\,a_{6,3}\,a_{1,4}\,a_{3,6}\,a_{5,5} + a_{4,1}\,a_{6,3}\,a_{3,4}\,a_{1,6}\,a_{5,5}$$
$$- a_{4,1}\,a_{6,3}\,a_{5,4}\,a_{1,6}\,a_{3,5} - a_{5,1}\,a_{4,3}\,a_{1,4}\,a_{3,6}\,a_{6,5} - a_{5,1}\,a_{4,3}\,a_{3,4}\,a_{1,6}\,a_{6,5} - a_{6,1}\,a_{4,3}\,a_{1,4}\,a_{3,6}\,a_{5,5} - a_{6,1}\,a_{4,3}\,a_{3,4}\,a_{1,6}\,a_{5,5}$$
$$+ a_{6,1}\,a_{4,3}\,a_{5,4}\,a_{1,6}\,a_{3,5} - a_{4,1}\,a_{2,2}\,a_{6,3}\,a_{1,4}\,a_{3,6} - a_{4,1}\,a_{2,2}\,a_{6,3}\,a_{1,6}\,a_{3,4} + a_{6,1}\,a_{2,2}\,a_{4,3}\,a_{1,4}\,a_{3,6} + a_{6,1}\,a_{2,2}\,a_{4,3}\,a_{1,6}\,a_{3,4}$$
$$- a_{5,1}\,a_{2,2}\,a_{6,3}\,a_{1,6}\,a_{3,5} + a_{5,1}\,a_{3,2}\,a_{6,3}\,a_{1,6}\,a_{2,5} + a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{1,6}\,a_{2,5} - a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{1,6}\,a_{6,5} - a_{6,1}\,a_{2,2}\,a_{3,3}\,a_{1,6}\,a_{5,5}$$
$$- a_{4,1}\,a_{2,2}\,a_{3,3}\,a_{1,4}\,a_{5,5} - a_{5,1}\,a_{2,2}\,a_{4,3}\,a_{1,4}\,a_{3,5} + a_{5,1}\,a_{3,2}\,a_{4,3}\,a_{1,4}\,a_{2,5} + a_{6,2}\,a_{2,5}\,a_{4,1}\,a_{5,4}\,a_{1,6} - a_{4,1}\,a_{5,4}\,a_{1,6}\,a_{6,5}\,a_{2,2}$$

F_4

$$-a_{4,3}\,a_{5,4}\,a_{3,6}\,a_{6,5} - a_{2,2}\,a_{6,3}\,a_{3,6}\,a_{5,5} - a_{2,2}\,a_{4,3}\,a_{3,5}\,a_{5,4} + a_{2,2}\,a_{4,3}\,a_{3,4}\,a_{5,5} + a_{3,2}\,a_{4,3}\,a_{2,5}\,a_{5,4} + a_{4,1}\,a_{6,3}\,a_{1,4}\,a_{3,6}$$
$$+ a_{4,1}\,a_{6,3}\,a_{3,4}\,a_{1,6} - a_{6,1}\,a_{4,3}\,a_{1,4}\,a_{3,6} - a_{6,1}\,a_{4,3}\,a_{3,4}\,a_{1,6} + a_{4,1}\,a_{5,4}\,a_{1,6}\,a_{6,5} + a_{5,5}\,a_{4,1}\,a_{1,4}\,a_{3,3} + a_{5,5}\,a_{4,1}\,a_{1,4}\,a_{2,2}$$
$$+ a_{5,1}\,a_{4,3}\,a_{1,4}\,a_{3,5} - a_{3,3}\,a_{2,2}\,a_{4,1}\,a_{1,4} - a_{6,2}\,a_{2,5}\,a_{5,1}\,a_{1,6} + a_{6,3}\,a_{3,5}\,a_{5,1}\,a_{1,6} + a_{6,5}\,a_{5,1}\,a_{1,6}\,a_{3,3} + a_{6,5}\,a_{5,1}\,a_{1,6}\,a_{2,2}$$
$$+ a_{6,1}\,a_{1,6}\,a_{5,5}\,a_{3,3} + a_{6,1}\,a_{1,6}\,a_{5,5}\,a_{2,2} - a_{6,1}\,a_{1,6}\,a_{3,3}\,a_{2,2}$$

F_3

$$-a_{6,5}\,a_{5,1}\,a_{1,6} - a_{6,1}\,a_{1,6}\,a_{5,5} + a_{6,1}\,a_{1,6}\,a_{3,3} + a_{6,1}\,a_{1,6}\,a_{2,2} + a_{6,3}\,a_{3,6}\,a_{5,5} - a_{6,3}\,a_{3,6}\,a_{2,2} + a_{5,4}\,a_{4,3}\,a_{3,5} - a_{5,5}\,a_{4,1}\,a_{1,4}$$
$$- a_{5,5}\,a_{3,4}\,a_{4,3} - a_{5,5}\,a_{3,3}\,a_{2,2} + a_{4,1}\,a_{1,4}\,a_{3,3} + a_{4,1}\,a_{1,4}\,a_{2,2} + a_{2,2}\,a_{3,4}\,a_{4,3}$$

F_2

$$-a_{6,1}\,a_{1,6} + a_{6,3}\,a_{3,6} + a_{5,5}\,a_{3,3} + a_{5,5}\,a_{2,2} - a_{4,1}\,a_{1,4} - a_{3,4}\,a_{4,3} - a_{3,3}\,a_{2,2}$$

F_1

$$-a_{5,5} + a_{3,3} + a_{2,2}$$

F_0

-1

"adjoint (-A)"

$$[-a_{2,2}\,a_{4,3}\,a_{5,4}\,a_{3,6}\,a_{6,5} + a_{6,2}\,a_{4,3}\,a_{5,4}\,a_{2,5}\,a_{3,6},$$
$$a_{3,2}\,a_{4,3}\,a_{5,4}\,a_{1,6}\,a_{6,5} + a_{6,2}\,a_{4,3}\,a_{1,4}\,a_{3,6}\,a_{5,5} + a_{6,2}\,a_{4,3}\,a_{3,4}\,a_{1,6}\,a_{5,5} - a_{6,2}\,a_{4,3}\,a_{5,4}\,a_{1,6}\,a_{3,5},$$

$$a_{2,2}\,a_{4,3}\,a_{5,4}\,a_{1,6}\,a_{6,5} - a_{6,2}\,a_{4,3}\,a_{5,4}\,a_{1,6}\,a_{2,5},\ a_{2,2}\,a_{3,3}\,a_{5,4}\,a_{1,6}\,a_{6,5} - a_{2,2}\,a_{6,3}\,a_{1,4}\,a_{3,6}\,a_{5,5} - a_{2,2}\,a_{6,3}\,a_{3,4}\,a_{1,6}\,a_{5,5}$$
$$+ a_{2,2}\,a_{6,3}\,a_{5,4}\,a_{1,6}\,a_{3,5} - a_{3,2}\,a_{6,3}\,a_{5,4}\,a_{1,6}\,a_{2,5} - a_{6,2}\,a_{3,3}\,a_{5,4}\,a_{1,6}\,a_{2,5},$$
$$a_{2,2}\,a_{4,3}\,a_{1,4}\,a_{3,6}\,a_{6,5} + a_{2,2}\,a_{4,3}\,a_{3,4}\,a_{1,6}\,a_{6,5} - a_{6,2}\,a_{4,3}\,a_{1,4}\,a_{2,5}\,a_{3,6} - a_{6,2}\,a_{4,3}\,a_{3,4}\,a_{1,6}\,a_{2,5},$$
$$-a_{2,2}\,a_{4,3}\,a_{1,4}\,a_{3,6}\,a_{5,5} - a_{2,2}\,a_{4,3}\,a_{3,4}\,a_{1,6}\,a_{5,5} + a_{2,2}\,a_{4,3}\,a_{5,4}\,a_{1,6}\,a_{3,5} - a_{3,2}\,a_{4,3}\,a_{5,4}\,a_{1,6}\,a_{2,5}]$$
$$[-a_{4,1}\,a_{6,3}\,a_{5,4}\,a_{2,5}\,a_{3,6} + a_{6,1}\,a_{4,3}\,a_{5,4}\,a_{2,5}\,a_{3,6},\ a_{4,1}\,a_{3,3}\,a_{5,4}\,a_{1,6}\,a_{6,5} - a_{4,1}\,a_{6,3}\,a_{1,4}\,a_{3,6}\,a_{5,5} - a_{4,1}\,a_{6,3}\,a_{3,4}\,a_{1,6}\,a_{5,5}$$
$$+ a_{4,1}\,a_{6,3}\,a_{5,4}\,a_{1,6}\,a_{3,5} + a_{5,1}\,a_{4,3}\,a_{1,4}\,a_{3,6}\,a_{6,5} + a_{5,1}\,a_{4,3}\,a_{3,4}\,a_{1,6}\,a_{6,5} + a_{6,1}\,a_{4,3}\,a_{1,4}\,a_{3,6}\,a_{5,5} + a_{6,1}\,a_{4,3}\,a_{3,4}\,a_{1,6}\,a_{5,5}$$
$$- a_{6,1}\,a_{4,3}\,a_{5,4}\,a_{1,6}\,a_{3,5},\ a_{4,1}\,a_{6,3}\,a_{5,4}\,a_{1,6}\,a_{2,5} - a_{6,1}\,a_{4,3}\,a_{5,4}\,a_{1,6}\,a_{2,5},$$
$$-a_{5,1}\,a_{6,3}\,a_{1,4}\,a_{2,5}\,a_{3,6} - a_{5,1}\,a_{6,3}\,a_{3,4}\,a_{1,6}\,a_{2,5} - a_{6,1}\,a_{3,3}\,a_{5,4}\,a_{1,6}\,a_{2,5},$$
$$a_{4,1}\,a_{6,3}\,a_{1,4}\,a_{2,5}\,a_{3,6} + a_{4,1}\,a_{6,3}\,a_{3,4}\,a_{1,6}\,a_{2,5} - a_{6,1}\,a_{4,3}\,a_{1,4}\,a_{2,5}\,a_{3,6} - a_{6,1}\,a_{4,3}\,a_{3,4}\,a_{1,6}\,a_{2,5},$$
$$-a_{4,1}\,a_{3,3}\,a_{5,4}\,a_{1,6}\,a_{2,5} - a_{5,1}\,a_{4,3}\,a_{1,4}\,a_{2,5}\,a_{3,6} - a_{5,1}\,a_{4,3}\,a_{3,4}\,a_{1,6}\,a_{2,5}]$$
$$[-a_{4,1}\,(a_{2,2}\,a_{5,4}\,a_{3,6}\,a_{6,5} - a_{6,2}\,a_{5,4}\,a_{2,5}\,a_{3,6}),$$
$$a_{4,1}\,(a_{3,2}\,a_{5,4}\,a_{1,6}\,a_{6,5} + a_{6,2}\,a_{1,4}\,a_{3,6}\,a_{5,5} + a_{6,2}\,a_{3,4}\,a_{1,6}\,a_{5,5} - a_{6,2}\,a_{5,4}\,a_{1,6}\,a_{3,5}),$$
$$-a_{4,1}\,(-a_{2,2}\,a_{5,4}\,a_{1,6}\,a_{6,5} + a_{6,2}\,a_{5,4}\,a_{1,6}\,a_{2,5}),\ -a_{5,1}\,a_{2,2}\,a_{1,4}\,a_{3,6}\,a_{6,5} - a_{5,1}\,a_{2,2}\,a_{3,4}\,a_{1,6}\,a_{6,5} + a_{5,1}\,a_{6,2}\,a_{1,4}\,a_{2,5}\,a_{3,6}$$
$$+ a_{5,1}\,a_{6,2}\,a_{3,4}\,a_{1,6}\,a_{2,5} - a_{6,1}\,a_{2,2}\,a_{1,4}\,a_{3,6}\,a_{5,5} - a_{6,1}\,a_{2,2}\,a_{3,4}\,a_{1,6}\,a_{5,5} + a_{6,1}\,a_{2,2}\,a_{5,4}\,a_{1,6}\,a_{3,5} - a_{6,1}\,a_{3,2}\,a_{5,4}\,a_{1,6}\,a_{2,5},$$
$$a_{4,1}\,(a_{2,2}\,a_{1,4}\,a_{3,6}\,a_{6,5} + a_{2,2}\,a_{3,4}\,a_{1,6}\,a_{6,5} - a_{6,2}\,a_{1,4}\,a_{2,5}\,a_{3,6} - a_{6,2}\,a_{3,4}\,a_{1,6}\,a_{2,5}),$$
$$-a_{4,1}\,(a_{2,2}\,a_{1,4}\,a_{3,6}\,a_{5,5} + a_{2,2}\,a_{3,4}\,a_{1,6}\,a_{5,5} - a_{2,2}\,a_{5,4}\,a_{1,6}\,a_{3,5} + a_{3,2}\,a_{5,4}\,a_{1,6}\,a_{2,5})]$$
$$[a_{4,1}\,a_{2,2}\,a_{6,3}\,a_{3,6}\,a_{5,5} - a_{5,1}\,a_{2,2}\,a_{4,3}\,a_{3,6}\,a_{6,5} + a_{5,1}\,a_{6,2}\,a_{4,3}\,a_{2,5}\,a_{3,6} - a_{6,1}\,a_{2,2}\,a_{4,3}\,a_{3,6}\,a_{5,5},$$
$$-a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{1,6}\,a_{5,5} - a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{1,6}\,a_{5,5} + a_{5,1}\,a_{3,2}\,a_{4,3}\,a_{1,6}\,a_{6,5} - a_{5,1}\,a_{6,2}\,a_{4,3}\,a_{1,6}\,a_{3,5} + a_{6,1}\,a_{3,2}\,a_{4,3}\,a_{1,6}\,a_{5,5},$$
$$-a_{4,1}\,a_{2,2}\,a_{6,3}\,a_{1,6}\,a_{5,5} + a_{5,1}\,a_{2,2}\,a_{4,3}\,a_{1,6}\,a_{6,5} - a_{5,1}\,a_{6,2}\,a_{4,3}\,a_{1,6}\,a_{2,5} + a_{6,1}\,a_{2,2}\,a_{4,3}\,a_{1,6}\,a_{5,5},$$
$$a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{1,6}\,a_{6,5} + a_{5,1}\,a_{2,2}\,a_{6,3}\,a_{1,6}\,a_{3,5} - a_{5,1}\,a_{3,2}\,a_{6,3}\,a_{1,6}\,a_{2,5} - a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{1,6}\,a_{2,5} + a_{6,1}\,a_{2,2}\,a_{3,3}\,a_{1,6}\,a_{5,5},$$
$$-a_{4,1}\,a_{2,2}\,a_{3,3}\,a_{1,6}\,a_{6,5} - a_{4,1}\,a_{2,2}\,a_{6,3}\,a_{1,6}\,a_{3,5} + a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{1,6}\,a_{2,5} + a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{1,6}\,a_{2,5} + a_{6,1}\,a_{2,2}\,a_{4,3}\,a_{1,6}\,a_{3,5}$$
$$- a_{6,1}\,a_{3,2}\,a_{4,3}\,a_{1,6}\,a_{2,5},\ a_{4,1}\,a_{2,2}\,a_{3,3}\,a_{1,6}\,a_{5,5} + a_{5,1}\,a_{2,2}\,a_{4,3}\,a_{1,6}\,a_{3,5} - a_{5,1}\,a_{3,2}\,a_{4,3}\,a_{1,6}\,a_{2,5}]$$
$$[-a_{4,1}\,a_{2,2}\,a_{6,3}\,a_{3,6}\,a_{5,4} + a_{6,1}\,a_{2,2}\,a_{4,3}\,a_{3,6}\,a_{5,4},$$
$$a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{1,6}\,a_{5,4} + a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{1,6}\,a_{5,4} + a_{5,1}\,a_{6,2}\,a_{4,3}\,a_{1,4}\,a_{3,6} + a_{5,1}\,a_{6,2}\,a_{4,3}\,a_{1,6}\,a_{3,4} - a_{6,1}\,a_{3,2}\,a_{4,3}\,a_{1,6}\,a_{5,4},$$
$$a_{4,1}\,a_{2,2}\,a_{6,3}\,a_{1,6}\,a_{5,4} - a_{6,1}\,a_{2,2}\,a_{4,3}\,a_{1,6}\,a_{5,4},\ -a_{5,1}\,a_{2,2}\,a_{6,3}\,a_{1,4}\,a_{3,6} - a_{5,1}\,a_{2,2}\,a_{6,3}\,a_{1,6}\,a_{3,4} - a_{6,1}\,a_{2,2}\,a_{3,3}\,a_{1,6}\,a_{5,4},$$
$$a_{4,1}\,a_{2,2}\,a_{6,3}\,a_{1,4}\,a_{3,6} + a_{4,1}\,a_{2,2}\,a_{6,3}\,a_{1,6}\,a_{3,4} - a_{6,1}\,a_{2,2}\,a_{4,3}\,a_{1,4}\,a_{3,6} - a_{6,1}\,a_{2,2}\,a_{4,3}\,a_{1,6}\,a_{3,4},$$
$$-a_{4,1}\,a_{2,2}\,a_{3,3}\,a_{1,6}\,a_{5,4} - a_{5,1}\,a_{2,2}\,a_{4,3}\,a_{1,4}\,a_{3,6} - a_{5,1}\,a_{2,2}\,a_{4,3}\,a_{1,6}\,a_{3,4}]$$
$$[a_{4,1}\,a_{2,2}\,a_{3,3}\,a_{5,4}\,a_{6,5} - a_{4,1}\,a_{2,2}\,a_{6,3}\,a_{3,4}\,a_{5,5} + a_{4,1}\,a_{2,2}\,a_{6,3}\,a_{3,5}\,a_{5,4} - a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{2,5}\,a_{5,4} - a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{2,5}\,a_{5,4}$$
$$+ a_{5,1}\,a_{2,2}\,a_{4,3}\,a_{3,4}\,a_{6,5} - a_{5,1}\,a_{6,2}\,a_{4,3}\,a_{2,5}\,a_{3,4} + a_{6,1}\,a_{2,2}\,a_{4,3}\,a_{3,4}\,a_{5,5} - a_{6,1}\,a_{2,2}\,a_{4,3}\,a_{3,5}\,a_{5,4} + a_{6,1}\,a_{3,2}\,a_{4,3}\,a_{2,5}\,a_{5,4},$$
$$-a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{1,4}\,a_{5,5} - a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{1,4}\,a_{5,5} + a_{5,1}\,a_{3,2}\,a_{4,3}\,a_{1,4}\,a_{6,5} - a_{5,1}\,a_{6,2}\,a_{4,3}\,a_{1,4}\,a_{3,5} + a_{6,1}\,a_{3,2}\,a_{4,3}\,a_{1,4}\,a_{5,5},$$
$$-a_{4,1}\,a_{2,2}\,a_{6,3}\,a_{1,4}\,a_{5,5} + a_{5,1}\,a_{2,2}\,a_{4,3}\,a_{1,4}\,a_{6,5} - a_{5,1}\,a_{6,2}\,a_{4,3}\,a_{1,4}\,a_{2,5} + a_{6,1}\,a_{2,2}\,a_{4,3}\,a_{1,4}\,a_{5,5},$$
$$a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{1,4}\,a_{6,5} + a_{5,1}\,a_{2,2}\,a_{6,3}\,a_{1,4}\,a_{3,5} - a_{5,1}\,a_{3,2}\,a_{6,3}\,a_{1,4}\,a_{2,5} - a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{1,4}\,a_{2,5} + a_{6,1}\,a_{2,2}\,a_{3,3}\,a_{1,4}\,a_{5,5},$$
$$-a_{4,1}\,a_{2,2}\,a_{3,3}\,a_{1,4}\,a_{6,5} - a_{4,1}\,a_{2,2}\,a_{6,3}\,a_{1,4}\,a_{3,5} + a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{1,4}\,a_{2,5} + a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{1,4}\,a_{2,5} + a_{6,1}\,a_{2,2}\,a_{4,3}\,a_{1,4}\,a_{3,5}$$
$$- a_{6,1}\,a_{3,2}\,a_{4,3}\,a_{1,4}\,a_{2,5},\ a_{4,1}\,a_{2,2}\,a_{3,3}\,a_{1,4}\,a_{5,5} + a_{5,1}\,a_{2,2}\,a_{4,3}\,a_{1,4}\,a_{3,5} - a_{5,1}\,a_{3,2}\,a_{4,3}\,a_{1,4}\,a_{2,5}]$$

"62"

$$A := \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & -1 \\ -1 & 0 & 0 & -1 & 1 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

poly_coef_F0_to_Fn = [-1, 2, -2, 3, -9, 10, -3]

positive_feedback = [0, 2, 2, 7, 2, 12, 1]

negative_feedback = [-1, 0, -4, -4, -11, -2, -4]

absolute_feedback = [1, 2, 6, 11, 13, 14, 5]

wFn = [-1., 1., -0.33, 0.27, -0.69, 0.71, -0.60]

"Criterion ii"

$wD_5 = 0.0034$

$ratio\_to\_model\_C = 5.1$

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 0 & 0 & 3 & 0 & 0 & 0 \\ -1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -4 & 1 & -1 & -2 \\ 1 & 1 & 2 & -2 & -1 & 1 \\ -1 & -1 & 7 & -4 & -2 & 2 \\ 2 & -1 & -2 & 2 & 1 & -1 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 0 & 0 & 5 & 0 & 0 & 0 \\ 1 & 1 & 5 & 3 & 1 & 1 \\ 1 & 1 & 8 & 3 & 1 & 4 \\ 1 & 1 & 4 & 2 & 1 & 1 \\ 3 & 3 & 9 & 4 & 2 & 2 \\ 4 & 1 & 4 & 2 & 1 & 1 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 1. & 1. & 0.60 & 1. & 1. & 1. \\ 1. & 1. & 0.20 & 0.33 & 1. & 1. \\ 1. & 1. & 0.50 & 0.33 & 1. & 0.50 \\ 1. & 1. & 0.50 & 1. & 1. & 1. \\ 0.33 & 0.33 & 0.78 & 1. & 1. & 1. \\ 0.50 & 1. & 0.50 & 1. & 1. & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} -3 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & -5 & 2 & 1 & -1 \\ 0 & 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & -3 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 5 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 7 & 2 & 1 & 1 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 2 & 2 & 7 & 3 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 0.60 & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 0.71 & 1. & 1. & 1. \\ 1. & 1. & 0.60 & 1. & 1. & 1. \\ 1. & 1. & 0.60 & 1. & 1. & 1. \\ 0. & 0. & 0.43 & 1. & 0. & 0. \\ 1. & 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -5 & 2 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & 3 & 0 & 0 \\ 0 & 0 & -3 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 4 & 7 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 5 & 0 & 0 \\ 2 & 2 & 7 & 3 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 0.50 & 0.71 & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 0.60 & 0.60 & 1. & 1. \\ 0. & 0. & 0.43 & 1. & 1. & 0. \\ 1. & 1. & 1. & 1. & 1. & 0.60 \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & 0 & 0 & -a_{1,4} & 0 & -a_{1,6} \\ -a_{2,1} & 0 & 0 & -a_{2,4} & a_{2,5} & a_{2,6} \\ -a_{3,1} & 0 & 0 & 0 & 0 & 0 \\ -a_{4,1} & a_{4,2} & 0 & a_{4,4} & 0 & 0 \\ 0 & -a_{5,2} & a_{5,3} & -a_{5,4} & a_{5,5} & 0 \\ a_{6,1} & a_{6,2} & a_{6,3} & 0 & 0 & 0 \end{bmatrix}$$

"system feedback"

$$F_6$$

$$-a_{1,4}\,a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{2,6}\,a_{5,5} - a_{2,4}\,a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{1,6}\,a_{5,5} + a_{5,4}\,a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{1,6}\,a_{2,5} - a_{5,2}\,a_{3,1}\,a_{6,3}\,a_{4,4}\,a_{1,6}\,a_{2,5} - a_{1,6}\,a_{3,1}\,a_{6,2}\,a_{5,3}\,a_{2,5}\,a_{4,4}$$

$$F_5$$

$$a_{6,3}\,a_{3,1}\,a_{1,4}\,a_{4,2}\,a_{2,6} + a_{6,2}\,a_{2,5}\,a_{5,3}\,a_{1,6}\,a_{3,1} + a_{6,3}\,a_{1,6}\,a_{3,1}\,a_{4,2}\,a_{2,4} + a_{6,3}\,a_{1,6}\,a_{3,1}\,a_{5,2}\,a_{2,5} + a_{3,1}\,a_{6,3}\,a_{4,4}\,a_{1,6}\,a_{5,5} + a_{4,1}\,a_{6,2}\,a_{2,4}\,a_{1,6}\,a_{5,5} + a_{6,1}\,a_{4,2}\,a_{1,4}\,a_{2,6}\,a_{5,5} + a_{6,1}\,a_{4,2}\,a_{2,4}\,a_{1,6}\,a_{5,5} - a_{6,1}\,a_{4,2}\,a_{5,4}\,a_{1,6}\,a_{2,5} + a_{6,1}\,a_{5,2}\,a_{4,4}\,a_{1,6}\,a_{2,5} + a_{4,1}\,a_{6,2}\,a_{1,4}\,a_{2,6}\,a_{5,5} + a_{2,1}\,a_{6,2}\,a_{4,4}\,a_{1,6}\,a_{5,5} + a_{3,1}\,a_{4,2}\,a_{5,3}\,a_{1,4}\,a_{2,5} - a_{6,2}\,a_{2,5}\,a_{4,1}\,a_{5,4}\,a_{1,6}$$

$$F_4$$

$$-a_{6,1}\,a_{1,4}\,a_{4,2}\,a_{2,6} - a_{6,2}\,a_{2,4}\,a_{4,1}\,a_{1,6} - a_{6,2}\,a_{2,1}\,a_{1,6}\,a_{5,5} - a_{6,2}\,a_{2,1}\,a_{1,6}\,a_{4,4} - a_{6,3}\,a_{1,6}\,a_{3,1}\,a_{5,5} - a_{6,3}\,a_{1,6}\,a_{3,1}\,a_{4,4} - a_{6,1}\,a_{1,6}\,a_{5,2}\,a_{2,5} - a_{6,1}\,a_{1,6}\,a_{5,5}\,a_{4,4} - a_{6,1}\,a_{1,6}\,a_{4,2}\,a_{2,4} + a_{6,2}\,a_{4,4}\,a_{2,6}\,a_{5,5} - a_{6,2}\,a_{2,6}\,a_{4,1}\,a_{1,4} + a_{5,2}\,a_{2,5}\,a_{4,1}\,a_{1,4} - a_{5,5}\,a_{4,2}\,a_{2,1}\,a_{1,4}$$

$$F_3$$

$$a_{6,2}\,a_{2,1}\,a_{1,6} + a_{6,3}\,a_{1,6}\,a_{3,1} + a_{6,1}\,a_{1,6}\,a_{5,5} + a_{6,1}\,a_{1,6}\,a_{4,4} - a_{6,2}\,a_{2,6}\,a_{5,5} - a_{6,2}\,a_{2,6}\,a_{4,4} - a_{5,4}\,a_{4,2}\,a_{2,5} + a_{5,2}\,a_{2,5}\,a_{4,4} - a_{5,5}\,a_{4,1}\,a_{1,4} + a_{5,5}\,a_{4,2}\,a_{2,4} + a_{4,2}\,a_{2,1}\,a_{1,4}$$

$$F_2$$

$$-a_{6,1}\,a_{1,6} + a_{6,2}\,a_{2,6} - a_{5,2}\,a_{2,5} - a_{5,5}\,a_{4,4} + a_{4,1}\,a_{1,4} - a_{4,2}\,a_{2,4}$$

$$F_1$$

$$a_{5,5} + a_{4,4}$$

$$F_0$$

-1

"adjoint (-A)"

$$[0\,,\,0\,,\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{2,6}\,a_{5,5} + a_{4,2}\,a_{6,3}\,a_{2,4}\,a_{1,6}\,a_{5,5} - a_{4,2}\,a_{6,3}\,a_{5,4}\,a_{1,6}\,a_{2,5} + a_{5,2}\,a_{6,3}\,a_{4,4}\,a_{1,6}\,a_{2,5} + a_{6,2}\,a_{5,3}\,a_{4,4}\,a_{1,6}\,a_{2,5}\,,$$

$$0\ ,\ 0\ ,\ 0\ ]$$

$$[-a_{3,1}\,a_{6,3}\,a_{4,4}\,a_{2,6}\,a_{5,5}\ ,\ -a_{3,1}\,a_{6,3}\,a_{4,4}\,a_{1,6}\,a_{5,5}\ ,$$

$$a_{2,1}\,a_{6,3}\,a_{4,4}\,a_{1,6}\,a_{5,5}+a_{4,1}\,a_{6,3}\,a_{1,4}\,a_{2,6}\,a_{5,5}+a_{4,1}\,a_{6,3}\,a_{2,4}\,a_{1,6}\,a_{5,5}-a_{4,1}\,a_{6,3}\,a_{5,4}\,a_{1,6}\,a_{2,5}-a_{6,1}\,a_{5,3}\,a_{4,4}\,a_{1,6}\,a_{2,5}\ ,$$

$$a_{3,1}\,a_{6,3}\,(-a_{1,4}\,a_{2,6}\,a_{5,5}-a_{2,4}\,a_{1,6}\,a_{5,5}+a_{5,4}\,a_{1,6}\,a_{2,5})\ ,\ a_{3,1}\,a_{6,3}\,a_{4,4}\,a_{1,6}\,a_{2,5}\ ,\ -a_{3,1}\,a_{5,3}\,a_{4,4}\,a_{1,6}\,a_{2,5}\ ]$$

$$[a_{3,1}\,a_{6,2}\,a_{4,4}\,a_{2,6}\,a_{5,5}\ ,\ a_{3,1}\,a_{6,2}\,a_{4,4}\,a_{1,6}\,a_{5,5}\ ,\ -a_{2,1}\,a_{6,2}\,a_{4,4}\,a_{1,6}\,a_{5,5}-a_{4,1}\,a_{6,2}\,a_{1,4}\,a_{2,6}\,a_{5,5}-a_{4,1}\,a_{6,2}\,a_{2,4}\,a_{1,6}\,a_{5,5}$$

$$+a_{6,2}\,a_{2,5}\,a_{4,1}\,a_{5,4}\,a_{1,6}-a_{6,1}\,a_{4,2}\,a_{1,4}\,a_{2,6}\,a_{5,5}-a_{6,1}\,a_{4,2}\,a_{2,4}\,a_{1,6}\,a_{5,5}+a_{6,1}\,a_{4,2}\,a_{5,4}\,a_{1,6}\,a_{2,5}-a_{6,1}\,a_{5,2}\,a_{4,4}\,a_{1,6}\,a_{2,5}\ ,$$

$$-a_{3,1}\,a_{6,2}\,(-a_{1,4}\,a_{2,6}\,a_{5,5}-a_{2,4}\,a_{1,6}\,a_{5,5}+a_{5,4}\,a_{1,6}\,a_{2,5})\ ,\ -a_{3,1}\,a_{6,2}\,a_{4,4}\,a_{1,6}\,a_{2,5}\ ,$$

$$-a_{3,1}\,(a_{4,2}\,a_{1,4}\,a_{2,6}\,a_{5,5}+a_{4,2}\,a_{2,4}\,a_{1,6}\,a_{5,5}-a_{4,2}\,a_{5,4}\,a_{1,6}\,a_{2,5}+a_{5,2}\,a_{4,4}\,a_{1,6}\,a_{2,5})]$$

$$[a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{2,6}\,a_{5,5}\ ,\ a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{1,6}\,a_{5,5}\ ,$$

$$-a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{1,6}\,a_{5,5}+a_{4,1}\,a_{5,2}\,a_{6,3}\,a_{1,6}\,a_{2,5}+a_{4,1}\,a_{6,2}\,a_{5,3}\,a_{1,6}\,a_{2,5}+a_{6,1}\,a_{4,2}\,a_{5,3}\,a_{1,6}\,a_{2,5}\ ,$$

$$a_{3,1}\,(-a_{5,2}\,a_{6,3}\,a_{1,6}\,a_{2,5}-a_{6,2}\,a_{5,3}\,a_{1,6}\,a_{2,5})\ ,\ -a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{1,6}\,a_{2,5}\ ,\ a_{3,1}\,a_{4,2}\,a_{5,3}\,a_{1,6}\,a_{2,5}\ ]$$

$$[-a_{3,1}\,(-a_{4,2}\,a_{6,3}\,a_{2,6}\,a_{5,4}+a_{5,2}\,a_{6,3}\,a_{2,6}\,a_{4,4}+a_{6,2}\,a_{5,3}\,a_{2,6}\,a_{4,4})\ ,$$

$$a_{3,1}\,(a_{4,2}\,a_{6,3}\,a_{1,6}\,a_{5,4}-a_{5,2}\,a_{6,3}\,a_{1,6}\,a_{4,4}-a_{6,2}\,a_{5,3}\,a_{1,6}\,a_{4,4})\ ,\ -a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{1,6}\,a_{5,4}+a_{2,1}\,a_{5,2}\,a_{6,3}\,a_{1,6}\,a_{4,4}$$

$$+a_{2,1}\,a_{6,2}\,a_{5,3}\,a_{1,6}\,a_{4,4}+a_{4,1}\,a_{5,2}\,a_{6,3}\,a_{1,4}\,a_{2,6}+a_{4,1}\,a_{5,2}\,a_{6,3}\,a_{1,6}\,a_{2,4}+a_{4,1}\,a_{6,2}\,a_{5,3}\,a_{1,4}\,a_{2,6}+a_{4,1}\,a_{6,2}\,a_{5,3}\,a_{1,6}\,a_{2,4}$$

$$+a_{6,1}\,a_{4,2}\,a_{5,3}\,a_{1,4}\,a_{2,6}+a_{6,1}\,a_{4,2}\,a_{5,3}\,a_{1,6}\,a_{2,4}\ ,$$

$$-a_{3,1}\,(a_{5,2}\,a_{6,3}\,a_{1,4}\,a_{2,6}+a_{5,2}\,a_{6,3}\,a_{1,6}\,a_{2,4}+a_{6,2}\,a_{5,3}\,a_{1,4}\,a_{2,6}+a_{6,2}\,a_{5,3}\,a_{1,6}\,a_{2,4})\ ,\ a_{3,1}\,a_{4,2}\,a_{6,3}\,(-a_{1,4}\,a_{2,6}-a_{1,6}\,a_{2,4})\ ,$$

$$-a_{3,1}\,a_{4,2}\,a_{5,3}\,(-a_{1,4}\,a_{2,6}-a_{1,6}\,a_{2,4})]$$

$$[a_{3,1}\,(a_{4,2}\,a_{6,3}\,a_{2,4}\,a_{5,5}-a_{4,2}\,a_{6,3}\,a_{2,5}\,a_{5,4}+a_{5,2}\,a_{6,3}\,a_{2,5}\,a_{4,4}+a_{6,2}\,a_{5,3}\,a_{2,5}\,a_{4,4})\ ,\ -a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{5,5}\ ,$$

$$a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{5,5}-a_{4,1}\,a_{5,2}\,a_{6,3}\,a_{1,4}\,a_{2,5}-a_{4,1}\,a_{6,2}\,a_{5,3}\,a_{1,4}\,a_{2,5}-a_{6,1}\,a_{4,2}\,a_{5,3}\,a_{1,4}\,a_{2,5}\ ,$$

$$a_{3,1}\,(a_{5,2}\,a_{6,3}\,a_{1,4}\,a_{2,5}+a_{6,2}\,a_{5,3}\,a_{1,4}\,a_{2,5})\ ,\ a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{2,5}\ ,\ -a_{3,1}\,a_{4,2}\,a_{5,3}\,a_{1,4}\,a_{2,5}\ ]$$

"63"

$$A := \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & -1 & -1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

$poly\_coef\_F0\_to\_Fn = [\,-1,\ 2,\ -1,\ -1,\ -1,\ 5,\ 1\,]$

$positive\_feedback = [\,0,\ 3,\ 3,\ 3,\ 3,\ 8,\ 5\,]$

$negative\_feedback = [\,-1,\ -1,\ -4,\ -4,\ -4,\ -3,\ -4\,]$

$absolute\_feedback = [\,1,\ 4,\ 7,\ 7,\ 7,\ 11,\ 9\,]$

$wFn = [\,-1.,\ 0.50,\ -0.14,\ -0.14,\ -0.14,\ 0.45,\ 0.11\,]$

"Criterion ii"

$wD_5 = 0.0017$

$ratio\_to\_model\_C = 2.6$

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 & -2 & 2 \\ -2 & 1 & 0 & -1 & 4 & -4 \\ 2 & -1 & 0 & 1 & -4 & 5 \\ 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 4 & 2 & 0 & 2 & 1 & 1 \\ 5 & 2 & 0 & 2 & 1 & 1 \\ 5 & 5 & 0 & 4 & 2 & 2 \\ 2 & 1 & 0 & 1 & 4 & 4 \\ 2 & 1 & 0 & 1 & 4 & 5 \\ 0 & 0 & 9 & 0 & 0 & 0 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 0. & 0. & 1. & 0. & 1. & 1. \\ 0.20 & 0. & 1. & 0. & 1. & 1. \\ 0.20 & 0.20 & 1. & 0. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 0.11 & 1. & 1. & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 7 & 0 & 2 & 1 & 1 \\ 0 & 0 & 9 & 0 & 0 & 0 \\ 4 & 2 & 0 & 7 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. & 1. \\ 0.20 & 0.14 & 1. & 0. & 1. & 1. \\ 1. & 1. & 0.11 & 1. & 1. & 1. \\ 0. & 0. & 1. & 0.14 & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 9 & 0 & 0 & 0 & 0 & 0 \\ 5 & 2 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 0.11 & 1. & 1. & 1. & 1. & 1. \\ 0.20 & 0. & 1. & 0. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. \\ 0. & 0. & 1. & 0. & 1. & 1. \\ 1. & 1. & 1. & 1. & 0.11 & 1. \\ 1. & 1. & 1. & 1. & 1. & 0.11 \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} a_{1,1} & a_{1,2} & 0 & 0 & 0 & 0 \\ -a_{2,1} & a_{2,2} & -a_{2,3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -a_{3,6} \\ a_{4,1} & -a_{4,2} & -a_{4,3} & -a_{4,4} & 0 & 0 \\ a_{5,1} & 0 & 0 & a_{5,4} & a_{5,5} & 0 \\ 0 & 0 & 0 & a_{6,4} & a_{6,5} & 0 \end{bmatrix}$$

"system feedback"

F_6

$$a_{4,4}\,a_{5,1}\,a_{1,2}\,a_{2,3}\,a_{3,6}\,a_{6,5} + a_{4,3}\,a_{1,1}\,a_{2,2}\,a_{5,4}\,a_{3,6}\,a_{6,5} + a_{2,3}\,a_{1,1}\,a_{4,2}\,a_{5,4}\,a_{3,6}\,a_{6,5} + a_{2,1}\,a_{1,2}\,a_{4,3}\,a_{5,4}\,a_{3,6}\,a_{6,5} + a_{2,3}\,a_{4,1}\,a_{1,2}\,a_{5,4}\,a_{3,6}\,a_{6,5} - a_{4,3}\,a_{1,1}\,a_{2,2}\,a_{6,4}\,a_{3,6}\,a_{5,5} - a_{2,3}\,a_{1,1}\,a_{4,2}\,a_{6,4}\,a_{3,6}\,a_{5,5} - a_{2,1}\,a_{1,2}\,a_{4,3}\,a_{6,4}\,a_{3,6}\,a_{5,5} - a_{2,3}\,a_{4,1}\,a_{1,2}\,a_{6,4}\,a_{3,6}\,a_{5,5}$$

F_5

$$-a_{2,2}\,a_{4,3}\,a_{5,4}\,a_{3,6}\,a_{6,5} + a_{2,2}\,a_{4,3}\,a_{6,4}\,a_{3,6}\,a_{5,5} - a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{3,6}\,a_{6,5} + a_{4,2}\,a_{2,3}\,a_{6,4}\,a_{3,6}\,a_{5,5} + a_{2,1}\,a_{1,2}\,a_{4,3}\,a_{3,6}\,a_{6,4} + a_{4,1}\,a_{1,2}\,a_{2,3}\,a_{3,6}\,a_{6,4} + a_{1,1}\,a_{2,2}\,a_{4,3}\,a_{3,6}\,a_{6,4} + a_{1,1}\,a_{4,2}\,a_{2,3}\,a_{3,6}\,a_{6,4} + a_{5,1}\,a_{1,2}\,a_{2,3}\,a_{3,6}\,a_{6,5} + a_{4,3}\,a_{3,6}\,a_{6,4}\,a_{5,5}\,a_{1,1} - a_{6,5}\,a_{4,3}\,a_{3,6}\,a_{5,4}\,a_{1,1}$$

F_4

$$-a_{4,2}\,a_{2,3}\,a_{3,6}\,a_{6,4} + a_{6,5}\,a_{4,3}\,a_{3,6}\,a_{5,4} - a_{4,3}\,a_{3,6}\,a_{6,4}\,a_{5,5} - a_{2,2}\,a_{4,3}\,a_{3,6}\,a_{6,4} - a_{4,3}\,a_{3,6}\,a_{6,4}\,a_{1,1} + a_{5,5}\,a_{4,4}\,a_{2,1}\,a_{1,2} + a_{5,5}\,a_{4,4}\,a_{2,2}\,a_{1,1}$$

F_3

$$a_{4,3}\,a_{3,6}\,a_{6,4} - a_{5,5}\,a_{4,4}\,a_{2,2} - a_{5,5}\,a_{4,4}\,a_{1,1} + a_{5,5}\,a_{2,1}\,a_{1,2} + a_{5,5}\,a_{2,2}\,a_{1,1} - a_{4,4}\,a_{2,1}\,a_{1,2} - a_{4,4}\,a_{2,2}\,a_{1,1}$$

F_2

$$a_{5,5}\,a_{4,4} - a_{5,5}\,a_{2,2} - a_{5,5}\,a_{1,1} + a_{4,4}\,a_{2,2} + a_{4,4}\,a_{1,1} - a_{2,1}\,a_{1,2} - a_{2,2}\,a_{1,1}$$

F_1

$$a_{5,5} - a_{4,4} + a_{2,2} + a_{1,1}$$

F_0

-1

"adjoint (-A)"

$$[a_{2,2}\,a_{4,3}\,a_{5,4}\,a_{3,6}\,a_{6,5} - a_{2,2}\,a_{4,3}\,a_{6,4}\,a_{3,6}\,a_{5,5} + a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{3,6}\,a_{6,5} - a_{4,2}\,a_{2,3}\,a_{6,4}\,a_{3,6}\,a_{5,5},\ -a_{1,2}\,a_{4,3}\,(a_{5,4}\,a_{3,6}\,a_{6,5} - a_{6,4}\,a_{3,6}\,a_{5,5}),\ 0,\ a_{1,2}\,a_{2,3}\,(a_{5,4}\,a_{3,6}\,a_{6,5} - a_{6,4}\,a_{3,6}\,a_{5,5}),\ a_{1,2}\,a_{2,3}\,a_{4,4}\,a_{3,6}\,a_{6,5},\ -a_{1,2}\,a_{2,3}\,a_{4,4}\,a_{3,6}\,a_{5,5}]$$

$$[a_{2,1}\,a_{4,3}\,a_{5,4}\,a_{3,6}\,a_{6,5} - a_{2,1}\,a_{4,3}\,a_{6,4}\,a_{3,6}\,a_{5,5} + a_{4,1}\,a_{2,3}\,a_{5,4}\,a_{3,6}\,a_{6,5} - a_{4,1}\,a_{2,3}\,a_{6,4}\,a_{3,6}\,a_{5,5} + a_{5,1}\,a_{2,3}\,a_{4,4}\,a_{3,6}\,a_{6,5},\ a_{1,1}\,a_{4,3}\,(a_{5,4}\,a_{3,6}\,a_{6,5} - a_{6,4}\,a_{3,6}\,a_{5,5}),\ 0,\ -a_{1,1}\,a_{2,3}\,(a_{5,4}\,a_{3,6}\,a_{6,5} - a_{6,4}\,a_{3,6}\,a_{5,5}),\ -a_{1,1}\,a_{2,3}\,a_{4,4}\,a_{3,6}\,a_{6,5},\ a_{1,1}\,a_{2,3}\,a_{4,4}\,a_{3,6}\,a_{5,5}]$$

$$[-a_{2,1}\,a_{4,2}\,a_{5,4}\,a_{3,6}\,a_{6,5} + a_{2,1}\,a_{4,2}\,a_{6,4}\,a_{3,6}\,a_{5,5} + a_{4,1}\,a_{2,2}\,a_{5,4}\,a_{3,6}\,a_{6,5} - a_{4,1}\,a_{2,2}\,a_{6,4}\,a_{3,6}\,a_{5,5} + a_{5,1}\,a_{2,2}\,a_{4,4}\,a_{3,6}\,a_{6,5},\ -a_{1,1}\,a_{4,2}\,a_{5,4}\,a_{3,6}\,a_{6,5} + a_{1,1}\,a_{4,2}\,a_{6,4}\,a_{3,6}\,a_{5,5} - a_{4,1}\,a_{1,2}\,a_{5,4}\,a_{3,6}\,a_{6,5} + a_{4,1}\,a_{1,2}\,a_{6,4}\,a_{3,6}\,a_{5,5} - a_{5,1}\,a_{1,2}\,a_{4,4}\,a_{3,6}\,a_{6,5},\ 0,\ -a_{1,1}\,a_{2,2}\,a_{5,4}\,a_{3,6}\,a_{6,5} + a_{1,1}\,a_{2,2}\,a_{6,4}\,a_{3,6}\,a_{5,5} - a_{2,1}\,a_{1,2}\,a_{5,4}\,a_{3,6}\,a_{6,5} + a_{2,1}\,a_{1,2}\,a_{6,4}\,a_{3,6}\,a_{5,5},\ -a_{1,1}\,a_{2,2}\,a_{4,4}\,a_{3,6}\,a_{6,5} - a_{2,1}\,a_{1,2}\,a_{4,4}\,a_{3,6}\,a_{6,5},\ a_{1,1}\,a_{2,2}\,a_{4,4}\,a_{3,6}\,a_{5,5} + a_{2,1}\,a_{1,2}\,a_{4,4}\,a_{3,6}\,a_{5,5}]$$

$$[-a_{5,1}\,(a_{2,2}\,a_{4,3}\,a_{3,6}\,a_{6,5} + a_{4,2}\,a_{2,3}\,a_{3,6}\,a_{6,5}),\ a_{5,1}\,a_{1,2}\,a_{4,3}\,a_{3,6}\,a_{6,5},\ 0,\ -a_{5,1}\,a_{1,2}\,a_{2,3}\,a_{3,6}\,a_{6,5},\ a_{1,1}\,a_{2,2}\,a_{4,3}\,a_{3,6}\,a_{6,5} + a_{1,1}\,a_{4,2}\,a_{2,3}\,a_{3,6}\,a_{6,5} + a_{2,1}\,a_{1,2}\,a_{4,3}\,a_{3,6}\,a_{6,5} + a_{4,1}\,a_{1,2}\,a_{2,3}\,a_{3,6}\,a_{6,5},\ -a_{1,1}\,a_{2,2}\,a_{4,3}\,a_{3,6}\,a_{5,5} - a_{1,1}\,a_{4,2}\,a_{2,3}\,a_{3,6}\,a_{5,5} - a_{2,1}\,a_{1,2}\,a_{4,3}\,a_{3,6}\,a_{5,5} - a_{4,1}\,a_{1,2}\,a_{2,3}\,a_{3,6}\,a_{5,5}]$$

$$[a_{5,1}(a_{2,2}a_{4,3}a_{3,6}a_{6,4}+a_{4,2}a_{2,3}a_{3,6}a_{6,4}),-a_{5,1}a_{1,2}a_{4,3}a_{3,6}a_{6,4},0,a_{5,1}a_{1,2}a_{2,3}a_{3,6}a_{6,4},$$
$$-a_{1,1}a_{2,2}a_{4,3}a_{3,6}a_{6,4}-a_{1,1}a_{4,2}a_{2,3}a_{3,6}a_{6,4}-a_{2,1}a_{1,2}a_{4,3}a_{3,6}a_{6,4}-a_{4,1}a_{1,2}a_{2,3}a_{3,6}a_{6,4},$$
$$a_{1,1}a_{2,2}a_{4,3}a_{3,6}a_{5,4}+a_{1,1}a_{4,2}a_{2,3}a_{3,6}a_{5,4}+a_{2,1}a_{1,2}a_{4,3}a_{3,6}a_{5,4}+a_{4,1}a_{1,2}a_{2,3}a_{3,6}a_{5,4}+a_{5,1}a_{1,2}a_{2,3}a_{3,6}a_{4,4}]$$
$$[0,0,-a_{1,1}a_{2,2}a_{4,3}a_{5,4}a_{6,5}+a_{1,1}a_{2,2}a_{4,3}a_{5,5}a_{6,4}-a_{1,1}a_{4,2}a_{2,3}a_{5,4}a_{6,5}+a_{1,1}a_{4,2}a_{2,3}a_{5,5}a_{6,4}-a_{2,1}a_{1,2}a_{4,3}a_{5,4}a_{6,5}$$
$$+a_{2,1}a_{1,2}a_{4,3}a_{5,5}a_{6,4}-a_{4,1}a_{1,2}a_{2,3}a_{5,4}a_{6,5}+a_{4,1}a_{1,2}a_{2,3}a_{5,5}a_{6,4}-a_{5,1}a_{1,2}a_{2,3}a_{4,4}a_{6,5},0,0,0]$$

"64"

$$A := \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ -1 & -1 & -1 & 1 & 1 & 0 \\ 0 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

poly_coef_F0_to_Fn = [-1, -2, -3, -2, -1, -4, 1]

positive_feedback = [0, 0, 1, 3, 5, 2, 2]

negative_feedback = [-1, -2, -4, -5, -6, -6, -1]

absolute_feedback = [1, 2, 5, 8, 11, 8, 3]

wFn = [-1., -1., -0.60, -0.25, -0.091, -0.50, 0.33]

"Criterion ii"

wD_5 = -0.0069

ratio_to_model_C = -11.

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} -1 & 2 & -1 & 1 & -1 & 0 \\ -1 & 3 & -1 & 1 & -2 & -1 \\ 1 & -4 & 1 & -2 & 3 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 1 & 4 & 1 & 1 & 5 & 2 \\ 1 & 3 & 1 & 1 & 4 & 1 \\ 1 & 4 & 1 & 2 & 5 & 1 \\ 0 & 3 & 0 & 0 & 3 & 0 \\ 3 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 1. & 0.50 & 1. & 1. & 0.20 & 0. \\ 1. & 1. & 1. & 1. & 0.50 & 1. \\ 1. & 1. & 1. & 1. & 0.60 & 1. \\ 1. & 0.33 & 1. & 1. & 0.33 & 1. \\ 0.33 & 1. & 1. & 1. & 0.33 & 1. \\ 1. & 1. & 1. & 1. & 0.33 & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -3 & 1 & -1 & 2 & 1 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 3 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 3 & 3 & 0 & 6 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 3 & 1 & 1 & 4 & 1 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 0.33 & 1. & 1. & 1. & 0.33 & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. \\ 0.33 & 0.33 & 0.33 & 1. & 0. & 1. \\ 1. & 1. & 1. & 0.33 & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 0.50 & 1. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & -3 & 1 & -1 & 2 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 3 & 3 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 1 & 3 & 1 & 1 & 4 & 2 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 0.33 & 1. \\ 1. & 0.33 & 1. & 1. & 1. & 1. \\ 0.33 & 0.33 & 1. & 1. & 0. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 0.33 & 1. \\ 1. & 1. & 1. & 1. & 0.50 & 0. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & -a_{1,5} & a_{1,6} \\ 0 & 0 & 0 & a_{2,4} & 0 & a_{2,6} \\ -a_{3,1} & -a_{3,2} & -a_{3,3} & a_{3,4} & a_{3,5} & 0 \\ 0 & -a_{4,2} & -a_{4,3} & -a_{4,4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{5,6} \\ a_{6,1} & -a_{6,2} & 0 & a_{6,4} & 0 & 0 \end{bmatrix}$$

"system feedback"

$$F_6$$

$$a_{3,1}\,a_{6,2}\,a_{4,3}\,a_{2,4}\,a_{1,5}\,a_{5,6} + a_{3,2}\,a_{6,1}\,a_{4,3}\,a_{2,4}\,a_{1,5}\,a_{5,6} - a_{3,3}\,a_{6,1}\,a_{4,2}\,a_{2,4}\,a_{1,5}\,a_{5,6}$$

$$F_5$$

$$a_{6,2}\,a_{4,3}\,a_{2,4}\,a_{3,5}\,a_{5,6} - a_{6,1}\,a_{4,2}\,a_{2,4}\,a_{1,5}\,a_{5,6} - a_{3,1}\,a_{4,3}\,a_{6,4}\,a_{1,5}\,a_{5,6} - a_{6,1}\,a_{3,3}\,a_{4,4}\,a_{1,5}\,a_{5,6} - a_{6,1}\,a_{4,3}\,a_{3,4}\,a_{1,5}\,a_{5,6}$$
$$- a_{6,1}\,a_{3,2}\,a_{4,3}\,a_{1,6}\,a_{2,4} + a_{6,1}\,a_{4,2}\,a_{3,3}\,a_{1,6}\,a_{2,4} - a_{3,1}\,a_{6,2}\,a_{4,3}\,a_{1,6}\,a_{2,4}$$

$$F_4$$

$$a_{6,4}\,a_{4,3}\,a_{3,1}\,a_{1,6} + a_{6,4}\,a_{4,3}\,a_{3,2}\,a_{2,6} - a_{6,4}\,a_{4,3}\,a_{3,5}\,a_{5,6} - a_{6,1}\,a_{1,5}\,a_{5,6}\,a_{4,4} - a_{6,1}\,a_{1,5}\,a_{5,6}\,a_{3,3} - a_{6,4}\,a_{4,2}\,a_{2,6}\,a_{3,3} + a_{6,1}\,a_{1,6}\,a_{4,2}\,a_{2,4} + a_{6,1}\,a_{1,6}\,a_{4,3}\,a_{3,4} + a_{6,1}\,a_{1,6}\,a_{4,4}\,a_{3,3} - a_{6,2}\,a_{2,6}\,a_{4,3}\,a_{3,4} - a_{6,2}\,a_{2,6}\,a_{4,4}\,a_{3,3}$$

$$F_3$$

$$-a_{6,1}\,a_{1,5}\,a_{5,6} - a_{6,4}\,a_{4,2}\,a_{2,6} + a_{6,1}\,a_{1,6}\,a_{4,4} + a_{6,1}\,a_{1,6}\,a_{3,3} - a_{6,2}\,a_{2,6}\,a_{4,4} - a_{6,2}\,a_{2,6}\,a_{3,3} + a_{4,3}\,a_{3,2}\,a_{2,4} - a_{3,3}\,a_{4,2}\,a_{2,4}$$

$$F_2$$

$$a_{6,1}\,a_{1,6} - a_{6,2}\,a_{2,6} - a_{4,2}\,a_{2,4} - a_{4,3}\,a_{3,4} - a_{4,4}\,a_{3,3}$$

$$F_1$$

$$-a_{4,4} - a_{3,3}$$

$$F_0$$

$$-1$$

"adjoint (-A)"

$$\left[-a_{6,2}\,a_{4,3}\,a_{2,4}\,a_{3,5}\,a_{5,6},\ -a_{3,2}\,a_{4,3}\,a_{6,4}\,a_{1,5}\,a_{5,6} + a_{4,2}\,a_{3,3}\,a_{6,4}\,a_{1,5}\,a_{5,6} + a_{6,2}\,a_{3,3}\,a_{4,4}\,a_{1,5}\,a_{5,6} + a_{6,2}\,a_{4,3}\,a_{3,4}\,a_{1,5}\,a_{5,6},\ -a_{6,2}\,a_{4,3}\,a_{2,4}\,a_{1,5}\,a_{5,6},\ a_{6,2}\,a_{3,3}\,a_{2,4}\,a_{1,5}\,a_{5,6},\ a_{3,2}\,a_{4,3}\,a_{6,4}\,a_{1,5}\,a_{2,6} - a_{4,2}\,a_{3,3}\,a_{6,4}\,a_{1,5}\,a_{2,6} - a_{6,2}\,a_{3,3}\,a_{4,4}\,a_{1,5}\,a_{2,6} + a_{6,2}\,a_{4,3}\,a_{2,4}\,a_{1,6}\,a_{3,5} - a_{6,2}\,a_{4,3}\,a_{3,4}\,a_{1,5}\,a_{2,6},\ a_{3,2}\,a_{4,3}\,a_{2,4}\,a_{1,5}\,a_{5,6} - a_{4,2}\,a_{3,3}\,a_{2,4}\,a_{1,5}\,a_{5,6}\right]$$

$$\left[-a_{6,1}\,a_{4,3}\,a_{2,4}\,a_{3,5}\,a_{5,6},\ a_{3,1}\,a_{4,3}\,a_{6,4}\,a_{1,5}\,a_{5,6} + a_{6,1}\,a_{3,3}\,a_{4,4}\,a_{1,5}\,a_{5,6} + a_{6,1}\,a_{4,3}\,a_{3,4}\,a_{1,5}\,a_{5,6},\ -a_{6,1}\,a_{4,3}\,a_{2,4}\,a_{1,5}\,a_{5,6},\ a_{6,1}\,a_{3,3}\,a_{2,4}\,a_{1,5}\,a_{5,6},\ -a_{3,1}\,a_{4,3}\,a_{6,4}\,a_{1,5}\,a_{2,6} - a_{6,1}\,a_{3,3}\,a_{4,4}\,a_{1,5}\,a_{2,6} + a_{6,1}\,a_{4,3}\,a_{2,4}\,a_{1,6}\,a_{3,5} - a_{6,1}\,a_{4,3}\,a_{3,4}\,a_{1,5}\,a_{2,6},\ -a_{3,1}\,a_{4,3}\,a_{2,4}\,a_{1,5}\,a_{5,6}\right]$$

$$\left[a_{6,1}\,a_{4,2}\,a_{2,4}\,a_{3,5}\,a_{5,6},\ -a_{3,1}\,a_{4,2}\,a_{6,4}\,a_{1,5}\,a_{5,6} - a_{3,1}\,a_{6,2}\,a_{4,4}\,a_{1,5}\,a_{5,6} - a_{6,1}\,a_{3,2}\,a_{4,4}\,a_{1,5}\,a_{5,6} - a_{6,1}\,a_{4,2}\,a_{3,4}\,a_{1,5}\,a_{5,6},\ a_{6,1}\,a_{4,2}\,a_{2,4}\,a_{1,5}\,a_{5,6},\ -a_{3,1}\,a_{6,2}\,a_{2,4}\,a_{1,5}\,a_{5,6} - a_{6,1}\,a_{3,2}\,a_{2,4}\,a_{1,5}\,a_{5,6},\ a_{3,1}\,a_{4,2}\,a_{6,4}\,a_{1,5}\,a_{2,6} + a_{3,1}\,a_{6,2}\,a_{4,4}\,a_{1,5}\,a_{2,6} + a_{6,1}\,a_{3,2}\,a_{4,4}\,a_{1,5}\,a_{2,6} - a_{6,1}\,a_{4,2}\,a_{2,4}\,a_{1,6}\,a_{3,5} + a_{6,1}\,a_{4,2}\,a_{3,4}\,a_{1,5}\,a_{2,6},\ a_{3,1}\,a_{4,2}\,a_{2,4}\,a_{1,5}\,a_{5,6}\right]$$

$$\left[0,\ a_{3,1}\,a_{6,2}\,a_{4,3}\,a_{1,5}\,a_{5,6} + a_{6,1}\,a_{3,2}\,a_{4,3}\,a_{1,5}\,a_{5,6} - a_{6,1}\,a_{4,2}\,a_{3,3}\,a_{1,5}\,a_{5,6},\ 0,\ 0,\ -a_{3,1}\,a_{6,2}\,a_{4,3}\,a_{1,5}\,a_{2,6} - a_{6,1}\,a_{3,2}\,a_{4,3}\,a_{1,5}\,a_{2,6} + a_{6,1}\,a_{4,2}\,a_{3,3}\,a_{1,5}\,a_{2,6},\ 0\right]$$

$$\left[-a_{3,1}\,a_{6,2}\,a_{4,3}\,a_{2,4}\,a_{5,6} - a_{6,1}\,a_{3,2}\,a_{4,3}\,a_{2,4}\,a_{5,6} + a_{6,1}\,a_{4,2}\,a_{3,3}\,a_{2,4}\,a_{5,6},\ 0,\ 0,\ 0,\ a_{3,1}\,a_{6,2}\,a_{4,3}\,a_{1,6}\,a_{2,4} + a_{6,1}\,a_{3,2}\,a_{4,3}\,a_{1,6}\,a_{2,4} - a_{6,1}\,a_{4,2}\,a_{3,3}\,a_{1,6}\,a_{2,4},\ 0\right]$$

$$\left[0,\ 0,\ 0,\ 0,\ a_{3,1}\,a_{6,2}\,a_{4,3}\,a_{1,5}\,a_{2,4} + a_{6,1}\,a_{3,2}\,a_{4,3}\,a_{1,5}\,a_{2,4} - a_{6,1}\,a_{4,2}\,a_{3,3}\,a_{1,5}\,a_{2,4},\ 0\right]$$

"65"

$$A := \begin{bmatrix} 0 & 1 & 0 & -1 & -1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

$poly\_coef\_F0\_to\_Fn = [\,-1, -1, 2, 1, -2, -2, 0\,]$

$positive\_feedback = [\,0, 1, 4, 5, 5, 7, 6\,]$

$negative\_feedback = [\,-1, -2, -2, -4, -7, -9, -6\,]$

$absolute\_feedback = [\,1, 3, 6, 9, 12, 16, 12\,]$

$wFn = [\,-1., -0.33, 0.33, 0.11, -0.17, -0.12, 0.\,]$

"Criterion ii"

$wD_5 = -0.000019$

$ratio\_to\_model\_C = -0.029$

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -4 & 0 & 0 \\ 0 & 0 & 4 & -4 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 0 & 12 & 4 & 4 & 0 & 4 \\ 0 & 0 & 4 & 4 & 0 & 4 \\ 0 & 0 & 4 & 4 & 0 & 4 \\ 6 & 0 & 2 & 2 & 6 & 2 \\ 6 & 0 & 2 & 2 & 6 & 2 \\ 0 & 0 & 4 & 4 & 0 & 4 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 1. & 0. & 0. & 0. & 1. & 0. \\ 1. & 1. & 1. & 1. & 1. & 0. \\ 1. & 1. & 1. & 1. & 1. & 0. \\ 0. & 1. & 1. & 1. & 0. & 0. \\ 0. & 1. & 1. & 1. & 0. & 0. \\ 1. & 1. & 0. & 0. & 1. & 0. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & -4 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4 & 4 & 0 & 0 \\ 0 & 0 & -4 & 4 & 0 & 0 \\ 0 & 0 & -2 & 2 & 0 & 0 \\ 0 & 0 & -4 & 4 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 12 & 0 & 4 & 4 & 0 & 4 \\ 0 & 12 & 4 & 4 & 0 & 4 \\ 0 & 0 & 8 & 4 & 0 & 4 \\ 0 & 0 & 4 & 8 & 0 & 4 \\ 6 & 0 & 2 & 2 & 6 & 2 \\ 0 & 0 & 4 & 4 & 0 & 8 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 0. & 1. & 1. & 1. & 1. & 0. \\ 1. & 0. & 0. & 0. & 1. & 0. \\ 1. & 1. & 0.50 & 1. & 1. & 0. \\ 1. & 1. & 1. & 0.50 & 1. & 0. \\ 0. & 1. & 1. & 1. & 0. & 0. \\ 1. & 1. & 1. & 1. & 1. & 0. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & -4 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4 & 4 & 0 & 0 \\ 0 & 0 & -4 & 4 & 0 & 0 \\ 0 & 0 & -2 & 2 & 0 & 0 \\ 0 & 0 & -4 & 4 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 4 & 4 & 0 & 4 \\ 0 & 0 & 4 & 4 & 0 & 4 \\ 0 & 0 & 4 & 4 & 0 & 4 \\ 0 & 0 & 4 & 4 & 0 & 4 \\ 6 & 0 & 2 & 2 & 6 & 2 \\ 0 & 0 & 4 & 4 & 0 & 4 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. & 0. \\ 1. & 1. & 0. & 0. & 1. & 0. \\ 1. & 1. & 1. & 1. & 1. & 0. \\ 1. & 1. & 1. & 1. & 1. & 0. \\ 0. & 1. & 1. & 1. & 0. & 0. \\ 1. & 1. & 1. & 1. & 1. & 0. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & a_{1,2} & 0 & -a_{1,4} & -a_{1,5} & 0 \\ -a_{2,1} & 0 & 0 & 0 & 0 & a_{2,6} \\ 0 & -a_{3,2} & a_{3,3} & 0 & 0 & -a_{3,6} \\ 0 & -a_{4,2} & a_{4,3} & 0 & 0 & -a_{4,6} \\ 0 & 0 & 0 & a_{5,4} & -a_{5,5} & 0 \\ 0 & a_{6,2} & -a_{6,3} & 0 & 0 & -a_{6,6} \end{bmatrix}$$

"system feedback"

F_6

$$-a_{3,2}\,a_{2,1}\,a_{4,3}\,a_{1,4}\,a_{5,5}\,a_{6,6} - a_{3,2}\,a_{2,1}\,a_{6,3}\,a_{1,4}\,a_{4,6}\,a_{5,5} + a_{1,4}\,a_{2,1}\,a_{4,2}\,a_{3,3}\,a_{5,5}\,a_{6,6} + a_{3,6}\,a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{5,5} + a_{1,4}\,a_{2,1}\,a_{6,2}\,a_{3,3}\,a_{4,6}\,a_{5,5} - a_{3,6}\,a_{2,1}\,a_{6,2}\,a_{4,3}\,a_{1,4}\,a_{5,5} - a_{3,2}\,a_{2,1}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{6,6} - a_{3,2}\,a_{2,1}\,a_{6,3}\,a_{5,4}\,a_{1,5}\,a_{4,6} + a_{1,5}\,a_{2,1}\,a_{4,2}\,a_{3,3}\,a_{5,4}\,a_{6,6} + a_{3,6}\,a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{1,5}\,a_{5,4} + a_{1,5}\,a_{2,1}\,a_{6,2}\,a_{3,3}\,a_{4,6}\,a_{5,4} - a_{3,6}\,a_{2,1}\,a_{6,2}\,a_{4,3}\,a_{1,5}\,a_{5,4}$$

F_5

$$-a_{6,6}\,a_{5,4}\,a_{4,2}\,a_{2,1}\,a_{1,5} - a_{6,6}\,a_{5,5}\,a_{4,2}\,a_{2,1}\,a_{1,4} + a_{6,6}\,a_{5,5}\,a_{3,3}\,a_{2,1}\,a_{1,2} - a_{6,6}\,a_{4,3}\,a_{3,2}\,a_{2,1}\,a_{1,4} + a_{6,3}\,a_{3,6}\,a_{5,5}\,a_{2,1}\,a_{1,2} + a_{6,3}\,a_{3,6}\,a_{4,2}\,a_{2,1}\,a_{1,4} + a_{6,6}\,a_{4,2}\,a_{2,1}\,a_{1,4}\,a_{3,3} - a_{6,2}\,a_{2,1}\,a_{1,4}\,a_{4,6}\,a_{5,5} + a_{6,2}\,a_{2,1}\,a_{1,4}\,a_{4,6}\,a_{3,3} - a_{5,4}\,a_{4,3}\,a_{3,2}\,a_{2,1}\,a_{1,5} + a_{5,4}\,a_{4,2}\,a_{2,1}\,a_{1,5}\,a_{3,3} - a_{5,5}\,a_{4,3}\,a_{3,2}\,a_{2,1}\,a_{1,4} + a_{5,5}\,a_{4,2}\,a_{2,1}\,a_{1,4}\,a_{3,3} - a_{6,3}\,a_{3,2}\,a_{2,1}\,a_{1,4}\,a_{4,6} - a_{6,2}\,a_{2,1}\,a_{5,4}\,a_{1,5}\,a_{4,6} - a_{6,2}\,a_{2,1}\,a_{4,3}\,a_{1,4}\,a_{3,6}$$

F_4

$$-a_{6,2}\,a_{2,1}\,a_{1,4}\,a_{4,6} + a_{6,3}\,a_{3,2}\,a_{2,6}\,a_{5,5} - a_{6,2}\,a_{2,6}\,a_{5,5}\,a_{3,3} + a_{6,3}\,a_{3,6}\,a_{2,1}\,a_{1,2} - a_{6,6}\,a_{5,5}\,a_{2,1}\,a_{1,2} - a_{6,6}\,a_{4,2}\,a_{2,1}\,a_{1,4} + a_{6,6}\,a_{3,3}\,a_{2,1}\,a_{1,2} - a_{5,4}\,a_{4,2}\,a_{2,1}\,a_{1,5} - a_{5,5}\,a_{4,2}\,a_{2,1}\,a_{1,4} + a_{5,5}\,a_{3,3}\,a_{2,1}\,a_{1,2} - a_{4,3}\,a_{3,2}\,a_{2,1}\,a_{1,4} + a_{4,2}\,a_{2,1}\,a_{1,4}\,a_{3,3}$$

F_3

$$a_{6,3}\,a_{3,2}\,a_{2,6} + a_{6,2}\,a_{2,6}\,a_{5,5} - a_{6,2}\,a_{2,6}\,a_{3,3} + a_{6,3}\,a_{3,6}\,a_{5,5} + a_{3,3}\,a_{5,5}\,a_{6,6} - a_{6,6}\,a_{2,1}\,a_{1,2} - a_{5,5}\,a_{2,1}\,a_{1,2} - a_{4,2}\,a_{2,1}\,a_{1,4} + a_{3,3}\,a_{2,1}\,a_{1,2}$$

F_2

$$a_{6,2}\,a_{2,6} + a_{6,3}\,a_{3,6} - a_{5,5}\,a_{6,6} + a_{3,3}\,a_{6,6} + a_{5,5}\,a_{3,3} - a_{2,1}\,a_{1,2}$$

F_1

$$-a_{6,6} - a_{5,5} + a_{3,3}$$

F_0

-1

"adjoint (-A)"

$$\begin{bmatrix} 0, & a_{3,2}\,a_{4,3}\,a_{1,4}\,a_{5,5}\,a_{6,6} + a_{3,2}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{6,6} + a_{3,2}\,a_{6,3}\,a_{1,4}\,a_{4,6}\,a_{5,5} + a_{3,2}\,a_{6,3}\,a_{5,4}\,a_{1,5}\,a_{4,6} - a_{4,2}\,a_{3,3}\,a_{1,4}\,a_{5,5}\,a_{6,6} - a_{4,2}\,a_{3,3}\,a_{5,4}\,a_{1,5}\,a_{6,6} - a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{3,6}\,a_{5,5} - a_{4,2}\,a_{6,3}\,a_{5,4}\,a_{1,5}\,a_{3,6} - a_{6,2}\,a_{3,3}\,a_{1,4}\,a_{4,6}\,a_{5,5} - a_{6,2}\,a_{3,3}\,a_{5,4}\,a_{1,5}\,a_{4,6} + a_{6,2}\,a_{4,3}\,a_{1,4}\,a_{3,6}\,a_{5,5} + a_{6,2}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{3,6}, \\ -a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{2,6}\,a_{5,5} - a_{4,2}\,a_{6,3}\,a_{5,4}\,a_{1,5}\,a_{2,6} + a_{6,2}\,a_{4,3}\,a_{1,4}\,a_{2,6}\,a_{5,5} + a_{6,2}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{2,6}, \\ a_{3,2}\,a_{6,3}\,a_{1,4}\,a_{2,6}\,a_{5,5} + a_{3,2}\,a_{6,3}\,a_{5,4}\,a_{1,5}\,a_{2,6} - a_{6,2}\,a_{3,3}\,a_{1,4}\,a_{2,6}\,a_{5,5} - a_{6,2}\,a_{3,3}\,a_{5,4}\,a_{1,5}\,a_{2,6}, 0, \\ a_{3,2}\,a_{4,3}\,a_{1,4}\,a_{2,6}\,a_{5,5} + a_{3,2}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{2,6} - a_{4,2}\,a_{3,3}\,a_{1,4}\,a_{2,6}\,a_{5,5} - a_{4,2}\,a_{3,3}\,a_{5,4}\,a_{1,5}\,a_{2,6} \end{bmatrix}$$

$$[0, 0, a_{2,1}(a_{4,3}a_{1,4}a_{5,5}a_{6,6} + a_{4,3}a_{5,4}a_{1,5}a_{6,6} + a_{6,3}a_{1,4}a_{4,6}a_{5,5} + a_{6,3}a_{5,4}a_{1,5}a_{4,6}),$$
$$-a_{2,1}(a_{3,3}a_{1,4}a_{5,5}a_{6,6} + a_{3,3}a_{5,4}a_{1,5}a_{6,6} + a_{6,3}a_{1,4}a_{3,6}a_{5,5} + a_{6,3}a_{5,4}a_{1,5}a_{3,6}), 0,$$
$$-a_{2,1}(-a_{3,3}a_{1,4}a_{4,6}a_{5,5} - a_{3,3}a_{5,4}a_{1,5}a_{4,6} + a_{4,3}a_{1,4}a_{3,6}a_{5,5} + a_{4,3}a_{5,4}a_{1,5}a_{3,6})]$$
$$[0, 0, -a_{2,1}(-a_{4,2}a_{1,4}a_{5,5}a_{6,6} - a_{4,2}a_{5,4}a_{1,5}a_{6,6} - a_{6,2}a_{1,4}a_{4,6}a_{5,5} - a_{6,2}a_{5,4}a_{1,5}a_{4,6}),$$
$$a_{2,1}(-a_{3,2}a_{1,4}a_{5,5}a_{6,6} - a_{3,2}a_{5,4}a_{1,5}a_{6,6} - a_{6,2}a_{1,4}a_{3,6}a_{5,5} - a_{6,2}a_{5,4}a_{1,5}a_{3,6}), 0,$$
$$a_{2,1}(a_{3,2}a_{1,4}a_{4,6}a_{5,5} + a_{3,2}a_{5,4}a_{1,5}a_{4,6} - a_{4,2}a_{1,4}a_{3,6}a_{5,5} - a_{4,2}a_{5,4}a_{1,5}a_{3,6})]$$
$$[-a_{2,1}(-a_{3,2}a_{4,3}a_{5,5}a_{6,6} - a_{3,2}a_{6,3}a_{4,6}a_{5,5} + a_{4,2}a_{3,3}a_{5,5}a_{6,6} + a_{4,2}a_{6,3}a_{3,6}a_{5,5} + a_{6,2}a_{3,3}a_{4,6}a_{5,5} - a_{6,2}a_{4,3}a_{3,6}a_{5,5}),$$
$$0, -a_{2,1}a_{1,2}(-a_{4,3}a_{5,5}a_{6,6} - a_{6,3}a_{4,6}a_{5,5}), a_{2,1}a_{1,2}(-a_{3,3}a_{5,5}a_{6,6} - a_{6,3}a_{3,6}a_{5,5}),$$
$$a_{2,1}(-a_{3,2}a_{4,3}a_{1,5}a_{6,6} - a_{3,2}a_{6,3}a_{1,5}a_{4,6} + a_{4,2}a_{3,3}a_{1,5}a_{6,6} + a_{4,2}a_{6,3}a_{1,5}a_{3,6} + a_{6,2}a_{3,3}a_{1,5}a_{4,6} - a_{6,2}a_{4,3}a_{1,5}a_{3,6}),$$
$$a_{2,1}a_{1,2}(a_{3,3}a_{4,6}a_{5,5} - a_{4,3}a_{3,6}a_{5,5})]$$
$$[a_{2,1}(a_{3,2}a_{4,3}a_{5,4}a_{6,6} + a_{3,2}a_{6,3}a_{4,6}a_{5,4} - a_{4,2}a_{3,3}a_{5,4}a_{6,6} - a_{4,2}a_{6,3}a_{3,6}a_{5,4} - a_{6,2}a_{3,3}a_{4,6}a_{5,4} + a_{6,2}a_{4,3}a_{3,6}a_{5,4}), 0$$
$$, a_{2,1}a_{1,2}(a_{4,3}a_{5,4}a_{6,6} + a_{6,3}a_{4,6}a_{5,4}), -a_{2,1}a_{1,2}(a_{3,3}a_{5,4}a_{6,6} + a_{6,3}a_{3,6}a_{5,4}),$$
$$-a_{2,1}(-a_{3,2}a_{4,3}a_{1,4}a_{6,6} - a_{3,2}a_{6,3}a_{1,4}a_{4,6} + a_{4,2}a_{3,3}a_{1,4}a_{6,6} + a_{4,2}a_{6,3}a_{1,4}a_{3,6} + a_{6,2}a_{3,3}a_{1,4}a_{4,6} - a_{6,2}a_{4,3}a_{1,4}a_{3,6}),$$
$$-a_{2,1}a_{1,2}(-a_{3,3}a_{4,6}a_{5,4} + a_{4,3}a_{3,6}a_{5,4})]$$
$$[0, 0, a_{2,1}(-a_{4,2}a_{6,3}a_{1,4}a_{5,5} - a_{4,2}a_{6,3}a_{1,5}a_{5,4} + a_{6,2}a_{4,3}a_{1,4}a_{5,5} + a_{6,2}a_{4,3}a_{1,5}a_{5,4}),$$
$$-a_{2,1}(-a_{3,2}a_{6,3}a_{1,4}a_{5,5} - a_{3,2}a_{6,3}a_{1,5}a_{5,4} + a_{6,2}a_{3,3}a_{1,4}a_{5,5} + a_{6,2}a_{3,3}a_{1,5}a_{5,4}), 0,$$
$$-a_{2,1}(-a_{3,2}a_{4,3}a_{1,4}a_{5,5} - a_{3,2}a_{4,3}a_{1,5}a_{5,4} + a_{4,2}a_{3,3}a_{1,4}a_{5,5} + a_{4,2}a_{3,3}a_{1,5}a_{5,4})]$$

"71"

$$A := \begin{bmatrix} 0 & 0 & 0 & -1 & -1 & 0 & -1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ -1 & -1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 1 \\ -1 & 0 & -1 & -1 & 1 & 0 & 1 \\ 0 & -1 & -1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

$poly\_coef\_F0\_to\_Fn = [-1, 3, -1, -3, 6, 0, 1, -2]$

$positive\_feedback = [0, 3, 4, 9, 23, 30, 29, 13]$

$negative\_feedback = [-1, 0, -5, -12, -17, -30, -28, -15]$

$absolute\_feedback = [1, 3, 9, 21, 40, 60, 57, 28]$

$wFn = [-1., 1., -0.11, -0.14, 0.15, 0., 0.018, -0.071]$

"Criterion ii"

$wD_6 = -0.39\ 10^{-5}$

$ratio\_to\_model\_C = -0.21$

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} -1 & 0 & -1 & 0 & 0 & 1 & -1 \\ 0 & 2 & -2 & -2 & 0 & -2 & 2 \\ 1 & -4 & 3 & 4 & 0 & 3 & -1 \\ 1 & 2 & -1 & -2 & 0 & -1 & 1 \\ 2 & 0 & 2 & 2 & 0 & 0 & 0 \\ 0 & 4 & -2 & -4 & -2 & -2 & 2 \\ -1 & -2 & -1 & 0 & 0 & 1 & -1 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 5 & 16 & 5 & 4 & 0 & 3 & 3 \\ 10 & 16 & 12 & 10 & 0 & 6 & 6 \\ 15 & 24 & 13 & 12 & 0 & 13 & 15 \\ 15 & 14 & 7 & 8 & 0 & 5 & 5 \\ 8 & 10 & 6 & 6 & 0 & 4 & 4 \\ 28 & 36 & 18 & 18 & 28 & 12 & 12 \\ 5 & 12 & 5 & 4 & 0 & 3 & 3 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 0.20 & 0. & 0.20 & 0. & 1. & 0.33 & 0.33 \\ 0. & 0.12 & 0.17 & 0.20 & 1. & 0.33 & 0.33 \\ 0.067 & 0.17 & 0.23 & 0.33 & 1. & 0.23 & 0.067 \\ 0.067 & 0.14 & 0.14 & 0.25 & 1. & 0.20 & 0.20 \\ 0.25 & 0. & 0.33 & 0.33 & 1. & 0. & 0. \\ 0. & 0.11 & 0.11 & 0.22 & 0.071 & 0.17 & 0.17 \\ 0.20 & 0.17 & 0.20 & 0. & 1. & 0.33 & 0.33 \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -2 & 1 & 0 & 0 & 1 & -1 \\ -1 & -2 & 1 & 2 & 0 & 1 & -1 \\ -1 & 2 & -1 & -2 & 0 & -3 & 1 \\ -1 & 2 & -1 & -2 & 0 & -1 & -1 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 28 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 16 & 5 & 4 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 15 & 14 & 7 & 20 & 0 & 5 & 5 \\ 15 & 14 & 7 & 8 & 0 & 5 & 5 \\ 11 & 18 & 9 & 8 & 0 & 21 & 7 \\ 11 & 18 & 9 & 8 & 0 & 7 & 21 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 0.071 & 1. & 1. & 1. & 1. & 1. & 1. \\ 0.20 & 0. & 0.20 & 0. & 1. & 0.33 & 0.33 \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 0.067 & 0.14 & 0.14 & 0. & 1. & 0.20 & 0.20 \\ 0.067 & 0.14 & 0.14 & 0.25 & 1. & 0.20 & 0.20 \\ 0.091 & 0.11 & 0.11 & 0.25 & 1. & 0.14 & 0.14 \\ 0.091 & 0.11 & 0.11 & 0.25 & 1. & 0.14 & 0.048 \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ -1 & -2 & 1 & 2 & 0 & 1 & -1 \\ -1 & -2 & 1 & 2 & 2 & 1 & -1 \\ -1 & 2 & -1 & -2 & 0 & -1 & 1 \\ -1 & 2 & -1 & -2 & 0 & -1 & 1 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 12 & 5 & 4 & 0 & 3 & 3 \\ 0 & 0 & 28 & 0 & 0 & 0 & 0 \\ 15 & 14 & 7 & 8 & 0 & 5 & 5 \\ 15 & 14 & 7 & 8 & 28 & 5 & 5 \\ 11 & 18 & 9 & 8 & 0 & 7 & 7 \\ 11 & 18 & 9 & 8 & 0 & 7 & 7 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 0.20 & 0.17 & 0.20 & 0. & 1. & 0.33 & 0.33 \\ 1. & 1. & 0.071 & 1. & 1. & 1. & 1. \\ 0.067 & 0.14 & 0.14 & 0.25 & 1. & 0.20 & 0.20 \\ 0.067 & 0.14 & 0.14 & 0.25 & 0.071 & 0.20 & 0.20 \\ 0.091 & 0.11 & 0.11 & 0.25 & 1. & 0.14 & 0.14 \\ 0.091 & 0.11 & 0.11 & 0.25 & 1. & 0.14 & 0.14 \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & 0 & 0 & -a_{1,4} & -a_{1,5} & 0 & -a_{1,7} \\ -a_{2,1} & 0 & 0 & 0 & 0 & 0 & a_{2,7} \\ a_{3,1} & a_{3,2} & 0 & 0 & a_{3,5} & 0 & a_{3,7} \\ -a_{4,1} & -a_{4,2} & 0 & a_{4,4} & -a_{4,5} & 0 & 0 \\ 0 & 0 & 0 & -a_{5,4} & a_{5,5} & a_{5,6} & a_{5,7} \\ -a_{6,1} & 0 & -a_{6,3} & -a_{6,4} & a_{6,5} & 0 & a_{6,7} \\ 0 & -a_{7,2} & -a_{7,3} & 0 & a_{7,5} & 0 & a_{7,7} \end{bmatrix}$$

"system feedback"

$$F_7$$

$$\begin{aligned} &-a_{7,2}\,a_{3,1}\,a_{6,3}\,a_{4,4}\,a_{1,5}\,a_{2,7}\,a_{5,6} - a_{1,4}\,a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{7,5}\,a_{3,7}\,a_{5,6} + a_{1,4}\,a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{3,5}\,a_{5,6}\,a_{7,7} \\ &\quad - a_{1,4}\,a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{4,5}\,a_{5,6}\,a_{7,7} + a_{1,4}\,a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{7,5}\,a_{2,7}\,a_{5,6} - a_{1,4}\,a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{7,5}\,a_{2,7}\,a_{5,6} \\ &\quad + a_{1,7}\,a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{4,4}\,a_{5,6}\,a_{7,5} - a_{1,5}\,a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{4,4}\,a_{5,6}\,a_{7,7} - a_{7,3}\,a_{6,1}\,a_{3,2}\,a_{1,4}\,a_{4,5}\,a_{2,7}\,a_{5,6} \\ &\quad - a_{7,3}\,a_{6,1}\,a_{3,2}\,a_{4,4}\,a_{1,5}\,a_{2,7}\,a_{5,6} + a_{7,3}\,a_{6,1}\,a_{4,2}\,a_{1,4}\,a_{3,5}\,a_{2,7}\,a_{5,6} + a_{7,3}\,a_{2,1}\,a_{3,2}\,a_{4,4}\,a_{1,5}\,a_{5,6}\,a_{6,7} \\ &\quad - a_{7,3}\,a_{2,1}\,a_{3,2}\,a_{4,4}\,a_{6,5}\,a_{1,7}\,a_{5,6} + a_{7,3}\,a_{2,1}\,a_{3,2}\,a_{6,4}\,a_{4,5}\,a_{1,7}\,a_{5,6} - a_{7,3}\,a_{2,1}\,a_{4,2}\,a_{1,4}\,a_{3,5}\,a_{5,6}\,a_{6,7} \\ &\quad + a_{7,3}\,a_{2,1}\,a_{4,2}\,a_{1,4}\,a_{6,5}\,a_{3,7}\,a_{5,6} + a_{7,3}\,a_{2,1}\,a_{4,2}\,a_{6,4}\,a_{1,5}\,a_{3,7}\,a_{5,6} - a_{7,3}\,a_{2,1}\,a_{4,2}\,a_{6,4}\,a_{3,5}\,a_{1,7}\,a_{5,6} \\ &\quad + a_{7,3}\,a_{3,1}\,a_{4,2}\,a_{1,4}\,a_{6,5}\,a_{2,7}\,a_{5,6} + a_{7,3}\,a_{3,1}\,a_{4,2}\,a_{6,4}\,a_{1,5}\,a_{2,7}\,a_{5,6} - a_{7,3}\,a_{4,1}\,a_{3,2}\,a_{1,4}\,a_{6,5}\,a_{2,7}\,a_{5,6} \\ &\quad + a_{7,3}\,a_{2,1}\,a_{3,2}\,a_{1,4}\,a_{4,5}\,a_{5,6}\,a_{6,7} - a_{7,3}\,a_{4,1}\,a_{3,2}\,a_{6,4}\,a_{1,5}\,a_{2,7}\,a_{5,6} - a_{7,2}\,a_{2,1}\,a_{6,3}\,a_{1,4}\,a_{4,5}\,a_{3,7}\,a_{5,6} \\ &\quad - a_{7,2}\,a_{2,1}\,a_{6,3}\,a_{4,4}\,a_{1,5}\,a_{3,7}\,a_{5,6} + a_{7,2}\,a_{2,1}\,a_{6,3}\,a_{4,4}\,a_{3,5}\,a_{1,7}\,a_{5,6} - a_{7,2}\,a_{3,1}\,a_{6,3}\,a_{1,4}\,a_{4,5}\,a_{2,7}\,a_{5,6} \\ &\quad + a_{7,2}\,a_{4,1}\,a_{6,3}\,a_{1,4}\,a_{3,5}\,a_{2,7}\,a_{5,6} \end{aligned}$$

$$F_6$$

$$\begin{aligned} &a_{6,3}\,a_{3,2}\,a_{2,1}\,a_{5,6}\,a_{1,4}\,a_{4,5} - a_{6,3}\,a_{3,5}\,a_{5,6}\,a_{4,2}\,a_{2,1}\,a_{1,4} + a_{6,3}\,a_{3,2}\,a_{2,1}\,a_{1,5}\,a_{5,6}\,a_{4,4} + a_{7,2}\,a_{6,3}\,a_{4,4}\,a_{3,5}\,a_{2,7}\,a_{5,6} \\ &\quad - a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{3,5}\,a_{2,7}\,a_{5,6} + a_{3,2}\,a_{6,3}\,a_{4,4}\,a_{7,5}\,a_{2,7}\,a_{5,6} - a_{3,2}\,a_{7,3}\,a_{4,4}\,a_{6,5}\,a_{2,7}\,a_{5,6} + a_{3,2}\,a_{7,3}\,a_{6,4}\,a_{4,5}\,a_{2,7}\,a_{5,6} \\ &\quad + a_{3,1}\,a_{6,3}\,a_{1,4}\,a_{4,5}\,a_{5,6}\,a_{7,7} + a_{3,1}\,a_{6,3}\,a_{4,4}\,a_{1,5}\,a_{5,6}\,a_{7,7} - a_{3,1}\,a_{6,3}\,a_{4,4}\,a_{7,5}\,a_{1,7}\,a_{5,6} - a_{3,1}\,a_{7,3}\,a_{1,4}\,a_{4,5}\,a_{5,6}\,a_{6,7} \\ &\quad - a_{3,1}\,a_{7,3}\,a_{4,4}\,a_{1,5}\,a_{5,6}\,a_{6,7} + a_{3,1}\,a_{7,3}\,a_{4,4}\,a_{6,5}\,a_{1,7}\,a_{5,6} - a_{3,1}\,a_{7,3}\,a_{6,4}\,a_{4,5}\,a_{1,7}\,a_{5,6} - a_{4,1}\,a_{6,3}\,a_{1,4}\,a_{3,5}\,a_{5,6}\,a_{7,7} \\ &\quad + a_{4,1}\,a_{6,3}\,a_{1,4}\,a_{7,5}\,a_{3,7}\,a_{5,6} + a_{4,1}\,a_{7,3}\,a_{1,4}\,a_{3,5}\,a_{5,6}\,a_{6,7} - a_{4,1}\,a_{7,3}\,a_{1,4}\,a_{6,5}\,a_{3,7}\,a_{5,6} - a_{4,1}\,a_{7,3}\,a_{6,4}\,a_{1,5}\,a_{3,7}\,a_{5,6} \\ &\quad + a_{4,1}\,a_{7,3}\,a_{6,4}\,a_{3,5}\,a_{1,7}\,a_{5,6} - a_{6,1}\,a_{7,3}\,a_{1,4}\,a_{4,5}\,a_{3,7}\,a_{5,6} - a_{6,1}\,a_{7,3}\,a_{4,4}\,a_{1,5}\,a_{3,7}\,a_{5,6} + a_{6,1}\,a_{7,3}\,a_{4,4}\,a_{3,5}\,a_{1,7}\,a_{5,6} \\ &\quad - a_{2,1}\,a_{4,2}\,a_{1,4}\,a_{6,5}\,a_{5,6}\,a_{7,7} + a_{2,1}\,a_{4,2}\,a_{1,4}\,a_{7,5}\,a_{5,6}\,a_{6,7} - a_{2,1}\,a_{4,2}\,a_{6,4}\,a_{1,5}\,a_{5,6}\,a_{7,7} + a_{2,1}\,a_{4,2}\,a_{6,4}\,a_{7,5}\,a_{1,7}\,a_{5,6} \\ &\quad + a_{2,1}\,a_{7,2}\,a_{1,4}\,a_{4,5}\,a_{5,6}\,a_{6,7} + a_{2,1}\,a_{7,2}\,a_{4,4}\,a_{1,5}\,a_{5,6}\,a_{6,7} - a_{2,1}\,a_{7,2}\,a_{4,4}\,a_{6,5}\,a_{1,7}\,a_{5,6} + a_{2,1}\,a_{7,2}\,a_{6,4}\,a_{4,5}\,a_{1,7}\,a_{5,6} \\ &\quad - a_{4,1}\,a_{7,2}\,a_{1,4}\,a_{6,5}\,a_{2,7}\,a_{5,6} - a_{4,1}\,a_{7,2}\,a_{6,4}\,a_{1,5}\,a_{2,7}\,a_{5,6} - a_{6,1}\,a_{4,2}\,a_{1,4}\,a_{7,5}\,a_{2,7}\,a_{5,6} - a_{6,1}\,a_{7,2}\,a_{1,4}\,a_{4,5}\,a_{2,7}\,a_{5,6} \\ &\quad - a_{6,1}\,a_{7,2}\,a_{4,4}\,a_{1,5}\,a_{2,7}\,a_{5,6} + a_{6,1}\,a_{3,2}\,a_{7,3}\,a_{1,5}\,a_{2,7}\,a_{5,6} + a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{1,5}\,a_{5,6}\,a_{7,7} - a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{7,5}\,a_{1,7}\,a_{5,6} \\ &\quad - a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{1,5}\,a_{5,6}\,a_{6,7} + a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{6,5}\,a_{1,7}\,a_{5,6} + a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{1,5}\,a_{3,7}\,a_{5,6} - a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{3,5}\,a_{1,7}\,a_{5,6} \\ &\quad + a_{3,1}\,a_{7,2}\,a_{6,3}\,a_{1,5}\,a_{2,7}\,a_{5,6} + a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{5,4}\,a_{1,7}\,a_{4,5} - a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{3,5}\,a_{5,7} + a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{3,7}\,a_{5,5} \\ &\quad + a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{1,5}\,a_{3,7} - a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{1,7}\,a_{3,5} + a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{2,7}\,a_{5,5} + a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{1,5}\,a_{2,7} \end{aligned}$$

$$- a_{4,1}\, a_{3,2}\, a_{7,3}\, a_{1,4}\, a_{2,7}\, a_{5,5} - a_{4,1}\, a_{3,2}\, a_{7,3}\, a_{5,4}\, a_{1,5}\, a_{2,7} + a_{2,1}\, a_{3,2}\, a_{7,3}\, a_{1,4}\, a_{4,5}\, a_{5,7} + a_{2,1}\, a_{3,2}\, a_{7,3}\, a_{4,4}\, a_{1,5}\, a_{5,7}$$
$$- a_{2,1}\, a_{3,2}\, a_{7,3}\, a_{4,4}\, a_{1,7}\, a_{5,5}$$

F_5

$$a_{6,4}\, a_{4,2}\, a_{2,1}\, a_{1,5}\, a_{5,6} - a_{6,3}\, a_{3,1}\, a_{1,4}\, a_{4,5}\, a_{5,6} - a_{6,3}\, a_{3,2}\, a_{2,1}\, a_{1,5}\, a_{5,6} + a_{6,3}\, a_{3,5}\, a_{5,6}\, a_{4,1}\, a_{1,4} + a_{5,6}\, a_{6,5}\, a_{4,2}\, a_{2,1}\, a_{1,4}$$
$$- a_{6,3}\, a_{3,1}\, a_{1,5}\, a_{5,6}\, a_{4,4} + a_{7,5}\, a_{5,6}\, a_{6,3}\, a_{3,1}\, a_{1,7} - a_{7,5}\, a_{5,6}\, a_{6,4}\, a_{4,1}\, a_{1,7} - a_{7,3}\, a_{3,1}\, a_{1,4}\, a_{4,2}\, a_{2,7} - a_{7,3}\, a_{3,1}\, a_{1,4}\, a_{4,5}\, a_{5,7}$$
$$+ a_{7,3}\, a_{3,1}\, a_{1,5}\, a_{5,6}\, a_{6,7} - a_{7,3}\, a_{3,2}\, a_{2,1}\, a_{1,5}\, a_{5,7} + a_{7,3}\, a_{3,5}\, a_{5,4}\, a_{4,1}\, a_{1,7} - a_{7,3}\, a_{3,5}\, a_{5,6}\, a_{6,1}\, a_{1,7} + a_{7,5}\, a_{5,4}\, a_{4,2}\, a_{1,7}\, a_{2,1}$$
$$+ a_{7,2}\, a_{2,1}\, a_{1,4}\, a_{4,5}\, a_{5,7} - a_{7,2}\, a_{2,1}\, a_{1,5}\, a_{5,6}\, a_{6,7} - a_{7,2}\, a_{1,7}\, a_{2,1}\, a_{5,5}\, a_{4,4} - a_{7,3}\, a_{3,1}\, a_{1,7}\, a_{5,6}\, a_{6,5} - a_{7,3}\, a_{3,1}\, a_{1,7}\, a_{5,4}\, a_{4,5}$$
$$+ a_{7,3}\, a_{3,1}\, a_{1,7}\, a_{5,5}\, a_{4,4} + a_{7,3}\, a_{3,2}\, a_{2,7}\, a_{4,1}\, a_{1,4} + a_{7,3}\, a_{3,5}\, a_{5,7}\, a_{4,1}\, a_{1,4} - a_{7,5}\, a_{5,6}\, a_{6,7}\, a_{4,1}\, a_{1,4} + a_{7,2}\, a_{1,7}\, a_{2,1}\, a_{5,4}\, a_{4,5}$$
$$+ a_{7,3}\, a_{3,7}\, a_{6,1}\, a_{1,5}\, a_{5,6} - a_{7,3}\, a_{3,7}\, a_{5,4}\, a_{4,1}\, a_{1,5} - a_{7,3}\, a_{3,7}\, a_{5,5}\, a_{4,1}\, a_{1,4} - a_{7,3}\, a_{3,7}\, a_{4,2}\, a_{2,1}\, a_{1,4} + a_{5,7}\, a_{7,5}\, a_{4,2}\, a_{2,1}\, a_{1,4}$$
$$+ a_{7,7}\, a_{6,1}\, a_{1,4}\, a_{4,5}\, a_{5,6} - a_{7,7}\, a_{6,3}\, a_{3,1}\, a_{1,5}\, a_{5,6} + a_{7,7}\, a_{6,4}\, a_{4,1}\, a_{1,5}\, a_{5,6} + a_{7,7}\, a_{6,1}\, a_{1,5}\, a_{5,6}\, a_{4,4} + a_{7,7}\, a_{5,6}\, a_{6,5}\, a_{4,1}\, a_{1,4}$$
$$- a_{7,7}\, a_{5,4}\, a_{4,2}\, a_{2,1}\, a_{1,5} - a_{7,7}\, a_{5,5}\, a_{4,2}\, a_{2,1}\, a_{1,4} + a_{7,2}\, a_{1,7}\, a_{2,1}\, a_{5,6}\, a_{6,5} + a_{7,2}\, a_{2,1}\, a_{1,5}\, a_{5,7}\, a_{4,4} - a_{7,3}\, a_{3,1}\, a_{1,5}\, a_{5,7}\, a_{4,4}$$
$$+ a_{7,3}\, a_{3,2}\, a_{1,7}\, a_{2,1}\, a_{5,5} + a_{7,3}\, a_{3,2}\, a_{1,7}\, a_{2,1}\, a_{4,4} - a_{7,5}\, a_{5,6}\, a_{6,1}\, a_{1,7}\, a_{4,4} + a_{7,2}\, a_{2,7}\, a_{6,1}\, a_{1,5}\, a_{5,6} - a_{7,2}\, a_{2,7}\, a_{5,4}\, a_{4,1}\, a_{1,5}$$
$$- a_{7,2}\, a_{2,7}\, a_{5,5}\, a_{4,1}\, a_{1,4} - a_{7,2}\, a_{6,3}\, a_{3,5}\, a_{2,7}\, a_{5,6} - a_{3,2}\, a_{6,3}\, a_{7,5}\, a_{2,7}\, a_{5,6} + a_{3,2}\, a_{7,3}\, a_{6,5}\, a_{2,7}\, a_{5,6} + a_{4,2}\, a_{6,4}\, a_{7,5}\, a_{2,7}\, a_{5,6}$$
$$- a_{7,2}\, a_{4,4}\, a_{6,5}\, a_{2,7}\, a_{5,6} + a_{7,2}\, a_{6,4}\, a_{4,5}\, a_{2,7}\, a_{5,6} - a_{7,3}\, a_{4,4}\, a_{6,5}\, a_{3,7}\, a_{5,6} + a_{7,3}\, a_{6,4}\, a_{4,5}\, a_{3,7}\, a_{5,6} - a_{6,3}\, a_{4,4}\, a_{3,5}\, a_{5,6}\, a_{7,7}$$
$$+ a_{6,3}\, a_{4,4}\, a_{7,5}\, a_{3,7}\, a_{5,6} + a_{7,3}\, a_{4,4}\, a_{3,5}\, a_{5,6}\, a_{6,7} - a_{4,2}\, a_{7,3}\, a_{5,4}\, a_{2,7}\, a_{3,5} - a_{3,2}\, a_{7,3}\, a_{4,4}\, a_{2,7}\, a_{5,5} + a_{3,2}\, a_{7,3}\, a_{5,4}\, a_{2,7}\, a_{4,5}$$

F_4

$$a_{5,4}\, a_{4,2}\, a_{2,1}\, a_{1,5} + a_{5,5}\, a_{4,2}\, a_{2,1}\, a_{1,4} - a_{4,4}\, a_{7,5}\, a_{5,6}\, a_{6,7} + a_{4,4}\, a_{6,5}\, a_{5,6}\, a_{7,7} + a_{7,3}\, a_{6,5}\, a_{3,7}\, a_{5,6} - a_{7,3}\, a_{3,5}\, a_{5,6}\, a_{6,7}$$
$$- a_{6,3}\, a_{7,5}\, a_{3,7}\, a_{5,6} + a_{6,3}\, a_{3,5}\, a_{5,6}\, a_{7,7} - a_{6,1}\, a_{1,4}\, a_{4,5}\, a_{5,6} + a_{6,3}\, a_{3,1}\, a_{1,5}\, a_{5,6} - a_{6,4}\, a_{4,1}\, a_{1,5}\, a_{5,6} + a_{6,3}\, a_{4,4}\, a_{3,5}\, a_{5,6}$$
$$- a_{6,1}\, a_{1,5}\, a_{5,6}\, a_{4,4} - a_{5,6}\, a_{6,5}\, a_{4,1}\, a_{1,4} - a_{7,2}\, a_{2,1}\, a_{1,5}\, a_{5,7} + a_{7,3}\, a_{3,1}\, a_{1,5}\, a_{5,7} - a_{7,3}\, a_{3,2}\, a_{1,7}\, a_{2,1} - a_{7,5}\, a_{5,4}\, a_{4,1}\, a_{1,7}$$
$$+ a_{7,5}\, a_{5,4}\, a_{4,2}\, a_{2,7} + a_{7,5}\, a_{5,6}\, a_{6,1}\, a_{1,7} + a_{7,7}\, a_{5,4}\, a_{4,1}\, a_{1,5} + a_{7,7}\, a_{5,5}\, a_{4,1}\, a_{1,4} + a_{7,7}\, a_{4,2}\, a_{2,1}\, a_{1,4} - a_{5,7}\, a_{7,5}\, a_{4,1}\, a_{1,4}$$
$$- a_{7,7}\, a_{6,1}\, a_{1,5}\, a_{5,6} + a_{7,2}\, a_{1,7}\, a_{2,1}\, a_{5,5} + a_{7,2}\, a_{1,7}\, a_{2,1}\, a_{4,4} - a_{7,3}\, a_{3,1}\, a_{1,7}\, a_{5,5} - a_{7,3}\, a_{3,1}\, a_{1,7}\, a_{4,4} + a_{7,3}\, a_{3,2}\, a_{2,7}\, a_{5,5}$$
$$+ a_{7,3}\, a_{3,2}\, a_{2,7}\, a_{4,4} + a_{7,3}\, a_{3,5}\, a_{5,7}\, a_{4,4} + a_{7,2}\, a_{2,7}\, a_{5,6}\, a_{6,5} + a_{7,2}\, a_{2,7}\, a_{5,4}\, a_{4,5} - a_{7,2}\, a_{2,7}\, a_{5,5}\, a_{4,4} + a_{7,2}\, a_{2,7}\, a_{4,1}\, a_{1,4}$$
$$+ a_{7,3}\, a_{3,7}\, a_{5,4}\, a_{4,5} - a_{7,3}\, a_{3,7}\, a_{5,5}\, a_{4,4} + a_{7,3}\, a_{3,7}\, a_{4,1}\, a_{1,4} - a_{6,4}\, a_{4,5}\, a_{5,6}\, a_{7,7}$$

F_3

$$-a_{7,2}\, a_{1,7}\, a_{2,1} + a_{7,3}\, a_{3,1}\, a_{1,7} - a_{7,3}\, a_{3,2}\, a_{2,7} - a_{7,3}\, a_{3,5}\, a_{5,7} + a_{7,5}\, a_{5,6}\, a_{6,7} + a_{7,2}\, a_{2,7}\, a_{5,5} + a_{7,2}\, a_{2,7}\, a_{4,4} + a_{7,3}\, a_{3,7}\, a_{5,5}$$
$$+ a_{7,3}\, a_{3,7}\, a_{4,4} - a_{4,4}\, a_{5,7}\, a_{7,5} - a_{6,5}\, a_{5,6}\, a_{7,7} - a_{5,4}\, a_{4,5}\, a_{7,7} + a_{4,4}\, a_{5,5}\, a_{7,7} - a_{7,7}\, a_{4,1}\, a_{1,4} + a_{6,1}\, a_{1,5}\, a_{5,6} - a_{6,3}\, a_{3,5}\, a_{5,6}$$
$$+ a_{6,4}\, a_{4,5}\, a_{5,6} - a_{4,4}\, a_{5,6}\, a_{6,5} - a_{5,4}\, a_{4,1}\, a_{1,5} - a_{5,5}\, a_{4,1}\, a_{1,4} - a_{4,2}\, a_{2,1}\, a_{1,4}$$

F_2

$$-a_{7,2}\, a_{2,7} - a_{7,3}\, a_{3,7} + a_{5,7}\, a_{7,5} - a_{5,5}\, a_{7,7} - a_{7,7}\, a_{4,4} + a_{5,6}\, a_{6,5} + a_{5,4}\, a_{4,5} - a_{5,5}\, a_{4,4} + a_{4,1}\, a_{1,4}$$

F_1

$$a_{7,7} + a_{5,5} + a_{4,4}$$

F_0

-1

"adjoint (-A)"

$$\Big[-a_{3,2}\, a_{6,3}\, a_{4,4}\, a_{7,5}\, a_{2,7}\, a_{5,6} + a_{3,2}\, a_{7,3}\, a_{4,4}\, a_{6,5}\, a_{2,7}\, a_{5,6} - a_{3,2}\, a_{7,3}\, a_{6,4}\, a_{4,5}\, a_{2,7}\, a_{5,6} + a_{4,2}\, a_{7,3}\, a_{6,4}\, a_{3,5}\, a_{2,7}\, a_{5,6}$$
$$- a_{7,2}\, a_{6,3}\, a_{4,4}\, a_{3,5}\, a_{2,7}\, a_{5,6},\ a_{3,2}\, a_{6,3}\, a_{1,4}\, a_{4,5}\, a_{5,6}\, a_{7,7} + a_{3,2}\, a_{6,3}\, a_{4,4}\, a_{1,5}\, a_{5,6}\, a_{7,7} - a_{3,2}\, a_{6,3}\, a_{4,4}\, a_{7,5}\, a_{1,7}\, a_{5,6}$$
$$- a_{3,2}\, a_{7,3}\, a_{1,4}\, a_{4,5}\, a_{5,6}\, a_{6,7} - a_{3,2}\, a_{7,3}\, a_{4,4}\, a_{1,5}\, a_{5,6}\, a_{6,7} + a_{3,2}\, a_{7,3}\, a_{4,4}\, a_{6,5}\, a_{1,7}\, a_{5,6} - a_{3,2}\, a_{7,3}\, a_{6,4}\, a_{4,5}\, a_{1,7}\, a_{5,6}$$
$$- a_{4,2}\, a_{6,3}\, a_{1,4}\, a_{3,5}\, a_{5,6}\, a_{7,7} + a_{4,2}\, a_{6,3}\, a_{1,4}\, a_{7,5}\, a_{3,7}\, a_{5,6} + a_{4,2}\, a_{7,3}\, a_{1,4}\, a_{3,5}\, a_{5,6}\, a_{6,7} - a_{4,2}\, a_{7,3}\, a_{1,4}\, a_{6,5}\, a_{3,7}\, a_{5,6}$$
$$- a_{4,2}\, a_{7,3}\, a_{6,4}\, a_{1,5}\, a_{3,7}\, a_{5,6} + a_{4,2}\, a_{7,3}\, a_{6,4}\, a_{3,5}\, a_{1,7}\, a_{5,6} + a_{7,2}\, a_{6,3}\, a_{1,4}\, a_{4,5}\, a_{3,7}\, a_{5,6} + a_{7,2}\, a_{6,3}\, a_{4,4}\, a_{1,5}\, a_{3,7}\, a_{5,6}$$
$$- a_{7,2}\, a_{6,3}\, a_{4,4}\, a_{3,5}\, a_{1,7}\, a_{5,6},\ -a_{4,2}\, a_{6,3}\, a_{1,4}\, a_{7,5}\, a_{2,7}\, a_{5,6} + a_{4,2}\, a_{7,3}\, a_{1,4}\, a_{6,5}\, a_{2,7}\, a_{5,6} + a_{4,2}\, a_{7,3}\, a_{6,4}\, a_{1,5}\, a_{2,7}\, a_{5,6}$$
$$- a_{7,2}\, a_{6,3}\, a_{1,4}\, a_{4,5}\, a_{2,7}\, a_{5,6} - a_{7,2}\, a_{6,3}\, a_{4,4}\, a_{1,5}\, a_{2,7}\, a_{5,6},$$
$$-a_{3,2}\, a_{6,3}\, a_{1,4}\, a_{7,5}\, a_{2,7}\, a_{5,6} + a_{3,2}\, a_{7,3}\, a_{1,4}\, a_{6,5}\, a_{2,7}\, a_{5,6} + a_{3,2}\, a_{7,3}\, a_{6,4}\, a_{1,5}\, a_{2,7}\, a_{5,6} - a_{7,2}\, a_{6,3}\, a_{1,4}\, a_{3,5}\, a_{2,7}\, a_{5,6},\ 0,$$
$$a_{3,2}\, a_{7,3}\, a_{1,4}\, a_{4,5}\, a_{2,7}\, a_{5,6} + a_{3,2}\, a_{7,3}\, a_{4,4}\, a_{1,5}\, a_{2,7}\, a_{5,6} - a_{4,2}\, a_{7,3}\, a_{1,4}\, a_{3,5}\, a_{2,7}\, a_{5,6},$$
$$-a_{3,2}\, a_{6,3}\, a_{1,4}\, a_{4,5}\, a_{2,7}\, a_{5,6} - a_{3,2}\, a_{6,3}\, a_{4,4}\, a_{1,5}\, a_{2,7}\, a_{5,6} + a_{4,2}\, a_{6,3}\, a_{1,4}\, a_{3,5}\, a_{2,7}\, a_{5,6}\Big]$$
$$\Big[-a_{2,1}\, a_{6,3}\, a_{4,4}\, a_{3,5}\, a_{5,6}\, a_{7,7} + a_{2,1}\, a_{6,3}\, a_{4,4}\, a_{7,5}\, a_{3,7}\, a_{5,6} + a_{2,1}\, a_{7,3}\, a_{4,4}\, a_{3,5}\, a_{5,6}\, a_{6,7} - a_{2,1}\, a_{7,3}\, a_{4,4}\, a_{6,5}\, a_{3,7}\, a_{5,6}$$
$$+ a_{2,1}\, a_{7,3}\, a_{6,4}\, a_{4,5}\, a_{3,7}\, a_{5,6} + a_{3,1}\, a_{6,3}\, a_{4,4}\, a_{7,5}\, a_{2,7}\, a_{5,6} - a_{3,1}\, a_{7,3}\, a_{4,4}\, a_{6,5}\, a_{2,7}\, a_{5,6} + a_{3,1}\, a_{7,3}\, a_{6,4}\, a_{4,5}\, a_{2,7}\, a_{5,6}$$

$$
\begin{aligned}
&- a_{4,1}\,a_{7,3}\,a_{6,4}\,a_{3,5}\,a_{2,7}\,a_{5,6} - a_{6,1}\,a_{7,3}\,a_{4,4}\,a_{3,5}\,a_{2,7}\,a_{5,6}\,,\; -a_{3,1}\,a_{6,3}\,a_{1,4}\,a_{4,5}\,a_{5,6}\,a_{7,7} - a_{3,1}\,a_{6,3}\,a_{4,4}\,a_{1,5}\,a_{5,6}\,a_{7,7}\\
&+ a_{3,1}\,a_{6,3}\,a_{4,4}\,a_{7,5}\,a_{1,7}\,a_{5,6} + a_{3,1}\,a_{7,3}\,a_{1,4}\,a_{4,5}\,a_{5,6}\,a_{6,7} + a_{3,1}\,a_{7,3}\,a_{4,4}\,a_{1,5}\,a_{5,6}\,a_{6,7} - a_{3,1}\,a_{7,3}\,a_{4,4}\,a_{6,5}\,a_{1,7}\,a_{5,6}\\
&+ a_{3,1}\,a_{7,3}\,a_{6,4}\,a_{4,5}\,a_{1,7}\,a_{5,6} + a_{4,1}\,a_{6,3}\,a_{1,4}\,a_{3,5}\,a_{5,6}\,a_{7,7} - a_{4,1}\,a_{6,3}\,a_{1,4}\,a_{7,5}\,a_{3,7}\,a_{5,6} - a_{4,1}\,a_{7,3}\,a_{1,4}\,a_{3,5}\,a_{5,6}\,a_{6,7}\\
&+ a_{4,1}\,a_{7,3}\,a_{1,4}\,a_{6,5}\,a_{3,7}\,a_{5,6} + a_{4,1}\,a_{7,3}\,a_{6,4}\,a_{1,5}\,a_{3,7}\,a_{5,6} - a_{4,1}\,a_{7,3}\,a_{6,4}\,a_{3,5}\,a_{1,7}\,a_{5,6} + a_{6,1}\,a_{7,3}\,a_{1,4}\,a_{4,5}\,a_{3,7}\,a_{5,6}\\
&+ a_{6,1}\,a_{7,3}\,a_{4,4}\,a_{1,5}\,a_{3,7}\,a_{5,6} - a_{6,1}\,a_{7,3}\,a_{4,4}\,a_{3,5}\,a_{1,7}\,a_{5,6}\,,\; -a_{2,1}\,a_{6,3}\,a_{1,4}\,a_{4,5}\,a_{5,6}\,a_{7,7} - a_{2,1}\,a_{6,3}\,a_{4,4}\,a_{1,5}\,a_{5,6}\,a_{7,7}\\
&+ a_{2,1}\,a_{6,3}\,a_{4,4}\,a_{7,5}\,a_{1,7}\,a_{5,6} + a_{2,1}\,a_{7,3}\,a_{1,4}\,a_{4,5}\,a_{5,6}\,a_{6,7} + a_{2,1}\,a_{7,3}\,a_{4,4}\,a_{1,5}\,a_{5,6}\,a_{6,7} - a_{2,1}\,a_{7,3}\,a_{4,4}\,a_{6,5}\,a_{1,7}\,a_{5,6}\\
&+ a_{2,1}\,a_{7,3}\,a_{6,4}\,a_{4,5}\,a_{1,7}\,a_{5,6} + a_{4,1}\,a_{6,3}\,a_{1,4}\,a_{7,5}\,a_{2,7}\,a_{5,6} - a_{4,1}\,a_{7,3}\,a_{1,4}\,a_{6,5}\,a_{2,7}\,a_{5,6} - a_{4,1}\,a_{7,3}\,a_{6,4}\,a_{1,5}\,a_{2,7}\,a_{5,6}\\
&- a_{6,1}\,a_{7,3}\,a_{1,4}\,a_{4,5}\,a_{2,7}\,a_{5,6} - a_{6,1}\,a_{7,3}\,a_{4,4}\,a_{1,5}\,a_{2,7}\,a_{5,6}\,,\; -a_{2,1}\,a_{6,3}\,a_{1,4}\,a_{3,5}\,a_{5,6}\,a_{7,7} + a_{2,1}\,a_{6,3}\,a_{1,4}\,a_{7,5}\,a_{3,7}\,a_{5,6}\\
&+ a_{2,1}\,a_{7,3}\,a_{1,4}\,a_{3,5}\,a_{5,6}\,a_{6,7} - a_{2,1}\,a_{7,3}\,a_{1,4}\,a_{6,5}\,a_{3,7}\,a_{5,6} - a_{2,1}\,a_{7,3}\,a_{6,4}\,a_{1,5}\,a_{3,7}\,a_{5,6} + a_{2,1}\,a_{7,3}\,a_{6,4}\,a_{3,5}\,a_{1,7}\,a_{5,6}\\
&+ a_{3,1}\,a_{6,3}\,a_{1,4}\,a_{7,5}\,a_{2,7}\,a_{5,6} - a_{3,1}\,a_{7,3}\,a_{1,4}\,a_{6,5}\,a_{2,7}\,a_{5,6} - a_{3,1}\,a_{7,3}\,a_{6,4}\,a_{1,5}\,a_{2,7}\,a_{5,6} - a_{6,1}\,a_{7,3}\,a_{1,4}\,a_{3,5}\,a_{2,7}\,a_{5,6}\,,\; 0\,,\\
&-a_{2,1}\,a_{7,3}\,a_{1,4}\,a_{4,5}\,a_{3,7}\,a_{5,6} - a_{2,1}\,a_{7,3}\,a_{4,4}\,a_{1,5}\,a_{3,7}\,a_{5,6} + a_{2,1}\,a_{7,3}\,a_{4,4}\,a_{3,5}\,a_{1,7}\,a_{5,6} - a_{3,1}\,a_{7,3}\,a_{1,4}\,a_{4,5}\,a_{2,7}\,a_{5,6}\\
&- a_{3,1}\,a_{7,3}\,a_{4,4}\,a_{1,5}\,a_{2,7}\,a_{5,6} + a_{4,1}\,a_{7,3}\,a_{1,4}\,a_{3,5}\,a_{2,7}\,a_{5,6}\,,\; a_{2,1}\,a_{6,3}\,a_{1,4}\,a_{4,5}\,a_{3,7}\,a_{5,6} + a_{2,1}\,a_{6,3}\,a_{4,4}\,a_{1,5}\,a_{3,7}\,a_{5,6}\\
&- a_{2,1}\,a_{6,3}\,a_{4,4}\,a_{3,5}\,a_{1,7}\,a_{5,6} + a_{3,1}\,a_{6,3}\,a_{1,4}\,a_{4,5}\,a_{2,7}\,a_{5,6} + a_{3,1}\,a_{6,3}\,a_{4,4}\,a_{1,5}\,a_{2,7}\,a_{5,6} - a_{4,1}\,a_{6,3}\,a_{1,4}\,a_{3,5}\,a_{2,7}\,a_{5,6}\,]\\
&[\,a_{2,1}\,a_{3,2}\,a_{4,4}\,a_{6,5}\,a_{5,6}\,a_{7,7} - a_{2,1}\,a_{3,2}\,a_{4,4}\,a_{7,5}\,a_{5,6}\,a_{6,7} - a_{2,1}\,a_{3,2}\,a_{6,4}\,a_{4,5}\,a_{5,6}\,a_{7,7} + a_{2,1}\,a_{4,2}\,a_{6,4}\,a_{3,5}\,a_{5,6}\,a_{7,7}\\
&- a_{2,1}\,a_{4,2}\,a_{6,4}\,a_{7,5}\,a_{3,7}\,a_{5,6} - a_{2,1}\,a_{7,2}\,a_{4,4}\,a_{3,5}\,a_{5,6}\,a_{6,7} + a_{2,1}\,a_{7,2}\,a_{4,4}\,a_{6,5}\,a_{3,7}\,a_{5,6} - a_{2,1}\,a_{7,2}\,a_{6,4}\,a_{4,5}\,a_{3,7}\,a_{5,6}\\
&- a_{3,1}\,a_{4,2}\,a_{6,4}\,a_{7,5}\,a_{2,7}\,a_{5,6} + a_{3,1}\,a_{7,2}\,a_{4,4}\,a_{6,5}\,a_{2,7}\,a_{5,6} - a_{3,1}\,a_{7,2}\,a_{6,4}\,a_{4,5}\,a_{2,7}\,a_{5,6} + a_{4,1}\,a_{3,2}\,a_{6,4}\,a_{7,5}\,a_{2,7}\,a_{5,6}\\
&+ a_{4,1}\,a_{7,2}\,a_{6,4}\,a_{3,5}\,a_{2,7}\,a_{5,6} + a_{6,1}\,a_{3,2}\,a_{4,4}\,a_{7,5}\,a_{2,7}\,a_{5,6} + a_{6,1}\,a_{7,2}\,a_{4,4}\,a_{3,5}\,a_{2,7}\,a_{5,6}\,,\; a_{3,1}\,a_{4,2}\,a_{1,4}\,a_{6,5}\,a_{5,6}\,a_{7,7}\\
&- a_{3,1}\,a_{4,2}\,a_{1,4}\,a_{7,5}\,a_{5,6}\,a_{6,7} + a_{3,1}\,a_{4,2}\,a_{6,4}\,a_{1,5}\,a_{5,6}\,a_{7,7} - a_{3,1}\,a_{4,2}\,a_{6,4}\,a_{7,5}\,a_{1,7}\,a_{5,6} - a_{3,1}\,a_{7,2}\,a_{1,4}\,a_{4,5}\,a_{5,6}\,a_{6,7}\\
&- a_{3,1}\,a_{7,2}\,a_{4,4}\,a_{1,5}\,a_{5,6}\,a_{6,7} + a_{3,1}\,a_{7,2}\,a_{4,4}\,a_{6,5}\,a_{1,7}\,a_{5,6} - a_{3,1}\,a_{7,2}\,a_{6,4}\,a_{4,5}\,a_{1,7}\,a_{5,6} - a_{4,1}\,a_{3,2}\,a_{1,4}\,a_{6,5}\,a_{5,6}\,a_{7,7}\\
&+ a_{4,1}\,a_{3,2}\,a_{1,4}\,a_{7,5}\,a_{5,6}\,a_{6,7} - a_{4,1}\,a_{3,2}\,a_{6,4}\,a_{1,5}\,a_{5,6}\,a_{7,7} + a_{4,1}\,a_{3,2}\,a_{6,4}\,a_{7,5}\,a_{1,7}\,a_{5,6} + a_{4,1}\,a_{7,2}\,a_{1,4}\,a_{3,5}\,a_{5,6}\,a_{6,7}\\
&- a_{4,1}\,a_{7,2}\,a_{1,4}\,a_{6,5}\,a_{3,7}\,a_{5,6} - a_{4,1}\,a_{7,2}\,a_{6,4}\,a_{1,5}\,a_{3,7}\,a_{5,6} + a_{4,1}\,a_{7,2}\,a_{6,4}\,a_{3,5}\,a_{1,7}\,a_{5,6} - a_{6,1}\,a_{3,2}\,a_{1,4}\,a_{4,5}\,a_{5,6}\,a_{7,7}\\
&- a_{6,1}\,a_{3,2}\,a_{4,4}\,a_{1,5}\,a_{5,6}\,a_{7,7} + a_{6,1}\,a_{3,2}\,a_{4,4}\,a_{7,5}\,a_{1,7}\,a_{5,6} + a_{6,1}\,a_{4,2}\,a_{1,4}\,a_{3,5}\,a_{5,6}\,a_{7,7} - a_{6,1}\,a_{4,2}\,a_{1,4}\,a_{7,5}\,a_{3,7}\,a_{5,6}\\
&- a_{6,1}\,a_{7,2}\,a_{1,4}\,a_{4,5}\,a_{3,7}\,a_{5,6} - a_{6,1}\,a_{7,2}\,a_{4,4}\,a_{1,5}\,a_{3,7}\,a_{5,6} + a_{6,1}\,a_{7,2}\,a_{4,4}\,a_{3,5}\,a_{1,7}\,a_{5,6}\,,\; a_{2,1}\,a_{4,2}\,a_{1,4}\,a_{6,5}\,a_{5,6}\,a_{7,7}\\
&- a_{2,1}\,a_{4,2}\,a_{1,4}\,a_{7,5}\,a_{5,6}\,a_{6,7} + a_{2,1}\,a_{4,2}\,a_{6,4}\,a_{1,5}\,a_{5,6}\,a_{7,7} - a_{2,1}\,a_{4,2}\,a_{6,4}\,a_{7,5}\,a_{1,7}\,a_{5,6} - a_{2,1}\,a_{7,2}\,a_{1,4}\,a_{4,5}\,a_{5,6}\,a_{6,7}\\
&- a_{2,1}\,a_{7,2}\,a_{4,4}\,a_{1,5}\,a_{5,6}\,a_{6,7} + a_{2,1}\,a_{7,2}\,a_{4,4}\,a_{6,5}\,a_{1,7}\,a_{5,6} - a_{2,1}\,a_{7,2}\,a_{6,4}\,a_{4,5}\,a_{1,7}\,a_{5,6} + a_{4,1}\,a_{7,2}\,a_{1,4}\,a_{6,5}\,a_{2,7}\,a_{5,6}\\
&+ a_{4,1}\,a_{7,2}\,a_{6,4}\,a_{1,5}\,a_{2,7}\,a_{5,6} + a_{6,1}\,a_{4,2}\,a_{1,4}\,a_{7,5}\,a_{2,7}\,a_{5,6} + a_{6,1}\,a_{7,2}\,a_{1,4}\,a_{4,5}\,a_{2,7}\,a_{5,6} + a_{6,1}\,a_{7,2}\,a_{4,4}\,a_{1,5}\,a_{2,7}\,a_{5,6}\,,\\
&a_{2,1}\,a_{3,2}\,a_{1,4}\,a_{6,5}\,a_{5,6}\,a_{7,7} - a_{2,1}\,a_{3,2}\,a_{1,4}\,a_{7,5}\,a_{5,6}\,a_{6,7} + a_{2,1}\,a_{3,2}\,a_{6,4}\,a_{1,5}\,a_{5,6}\,a_{7,7} - a_{2,1}\,a_{3,2}\,a_{6,4}\,a_{7,5}\,a_{1,7}\,a_{5,6}\\
&- a_{2,1}\,a_{7,2}\,a_{1,4}\,a_{3,5}\,a_{5,6}\,a_{6,7} + a_{2,1}\,a_{7,2}\,a_{1,4}\,a_{6,5}\,a_{3,7}\,a_{5,6} + a_{2,1}\,a_{7,2}\,a_{6,4}\,a_{1,5}\,a_{3,7}\,a_{5,6} - a_{2,1}\,a_{7,2}\,a_{6,4}\,a_{3,5}\,a_{1,7}\,a_{5,6}\\
&+ a_{3,1}\,a_{7,2}\,a_{1,4}\,a_{6,5}\,a_{2,7}\,a_{5,6} + a_{3,1}\,a_{7,2}\,a_{6,4}\,a_{1,5}\,a_{2,7}\,a_{5,6} + a_{6,1}\,a_{3,2}\,a_{1,4}\,a_{7,5}\,a_{2,7}\,a_{5,6} + a_{6,1}\,a_{7,2}\,a_{1,4}\,a_{3,5}\,a_{2,7}\,a_{5,6}\,,\; 0\,,\\
&a_{2,1}\,a_{3,2}\,a_{1,4}\,a_{4,5}\,a_{5,6}\,a_{7,7} + a_{2,1}\,a_{3,2}\,a_{4,4}\,a_{1,5}\,a_{5,6}\,a_{7,7} - a_{2,1}\,a_{3,2}\,a_{4,4}\,a_{7,5}\,a_{1,7}\,a_{5,6} - a_{2,1}\,a_{4,2}\,a_{1,4}\,a_{3,5}\,a_{5,6}\,a_{7,7}\\
&+ a_{2,1}\,a_{4,2}\,a_{1,4}\,a_{7,5}\,a_{3,7}\,a_{5,6} + a_{2,1}\,a_{7,2}\,a_{1,4}\,a_{4,5}\,a_{3,7}\,a_{5,6} + a_{2,1}\,a_{7,2}\,a_{4,4}\,a_{1,5}\,a_{3,7}\,a_{5,6} - a_{2,1}\,a_{7,2}\,a_{4,4}\,a_{3,5}\,a_{1,7}\,a_{5,6}\\
&+ a_{3,1}\,a_{4,2}\,a_{1,4}\,a_{7,5}\,a_{2,7}\,a_{5,6} + a_{3,1}\,a_{7,2}\,a_{1,4}\,a_{4,5}\,a_{2,7}\,a_{5,6} + a_{3,1}\,a_{7,2}\,a_{4,4}\,a_{1,5}\,a_{2,7}\,a_{5,6} - a_{4,1}\,a_{3,2}\,a_{1,4}\,a_{7,5}\,a_{2,7}\,a_{5,6}\\
&- a_{4,1}\,a_{7,2}\,a_{1,4}\,a_{3,5}\,a_{2,7}\,a_{5,6}\,,\; -a_{2,1}\,a_{3,2}\,a_{1,4}\,a_{4,5}\,a_{5,6}\,a_{6,7} - a_{2,1}\,a_{3,2}\,a_{4,4}\,a_{1,5}\,a_{5,6}\,a_{6,7} + a_{2,1}\,a_{3,2}\,a_{4,4}\,a_{6,5}\,a_{1,7}\,a_{5,6}\\
&- a_{2,1}\,a_{3,2}\,a_{6,4}\,a_{4,5}\,a_{1,7}\,a_{5,6} + a_{2,1}\,a_{4,2}\,a_{6,4}\,a_{1,5}\,a_{3,7}\,a_{5,6} - a_{2,1}\,a_{4,2}\,a_{6,4}\,a_{3,5}\,a_{1,7}\,a_{5,6} + a_{3,1}\,a_{4,2}\,a_{6,4}\,a_{1,5}\,a_{2,7}\,a_{5,6}\\
&+ a_{3,1}\,a_{4,2}\,a_{6,4}\,a_{7,5}\,a_{1,7}\,a_{5,6} + a_{6,1}\,a_{3,2}\,a_{4,4}\,a_{1,5}\,a_{2,7}\,a_{5,6} + a_{6,1}\,a_{3,2}\,a_{4,4}\,a_{7,5}\,a_{1,7}\,a_{5,6} - a_{6,1}\,a_{4,2}\,a_{1,4}\,a_{3,5}\,a_{2,7}\,a_{5,6}\,]\\
&[\,a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{4,5}\,a_{5,6}\,a_{7,7} - a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{4,5}\,a_{5,6}\,a_{6,7} - a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{3,5}\,a_{5,6}\,a_{7,7} + a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{7,5}\,a_{3,7}\,a_{5,6}\\
&+ a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{3,5}\,a_{5,6}\,a_{6,7} - a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{6,5}\,a_{3,7}\,a_{5,6} + a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{4,5}\,a_{3,7}\,a_{5,6} + a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{7,5}\,a_{2,7}\,a_{5,6}\\
&- a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{6,5}\,a_{2,7}\,a_{5,6} + a_{3,1}\,a_{7,2}\,a_{6,3}\,a_{4,5}\,a_{2,7}\,a_{5,6} - a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{7,5}\,a_{2,7}\,a_{5,6} + a_{4,1}\,a_{3,2}\,a_{7,3}\,a_{6,5}\,a_{2,7}\,a_{5,6}\\
&- a_{4,1}\,a_{7,2}\,a_{6,3}\,a_{3,5}\,a_{2,7}\,a_{5,6} + a_{6,1}\,a_{3,2}\,a_{7,3}\,a_{4,5}\,a_{2,7}\,a_{5,6} - a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{3,5}\,a_{2,7}\,a_{5,6}\,,\; -a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{1,5}\,a_{5,6}\,a_{7,7}\\
&+ a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{7,5}\,a_{1,7}\,a_{5,6} + a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{1,5}\,a_{5,6}\,a_{6,7} - a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{6,5}\,a_{1,7}\,a_{5,6} + a_{3,1}\,a_{7,2}\,a_{6,3}\,a_{4,5}\,a_{1,7}\,a_{5,6}\\
&+ a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{1,5}\,a_{5,6}\,a_{7,7} - a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{7,5}\,a_{1,7}\,a_{5,6} - a_{4,1}\,a_{3,2}\,a_{7,3}\,a_{1,5}\,a_{5,6}\,a_{6,7} + a_{4,1}\,a_{3,2}\,a_{7,3}\,a_{6,5}\,a_{1,7}\,a_{5,6}\\
&+ a_{4,1}\,a_{7,2}\,a_{6,3}\,a_{1,5}\,a_{3,7}\,a_{5,6} - a_{4,1}\,a_{7,2}\,a_{6,3}\,a_{3,5}\,a_{1,7}\,a_{5,6} + a_{6,1}\,a_{3,2}\,a_{7,3}\,a_{4,5}\,a_{1,7}\,a_{5,6} + a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{1,5}\,a_{3,7}\,a_{5,6}\\
&- a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{3,5}\,a_{1,7}\,a_{5,6}\,,\; -a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{1,5}\,a_{5,6}\,a_{7,7} + a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{7,5}\,a_{1,7}\,a_{5,6} + a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{1,5}\,a_{5,6}\,a_{6,7}\\
&- a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{6,5}\,a_{1,7}\,a_{5,6} + a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{4,5}\,a_{1,7}\,a_{5,6} - a_{4,1}\,a_{7,2}\,a_{6,3}\,a_{1,5}\,a_{2,7}\,a_{5,6} - a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{1,5}\,a_{2,7}\,a_{5,6}\,,
\end{aligned}
$$

$$-a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{1,5}\,a_{5,6}\,a_{7,7}+a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{7,5}\,a_{1,7}\,a_{5,6}+a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{1,5}\,a_{5,6}\,a_{6,7}-a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{6,5}\,a_{1,7}\,a_{5,6}$$
$$-a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{1,5}\,a_{3,7}\,a_{5,6}+a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{3,5}\,a_{1,7}\,a_{5,6}-a_{3,1}\,a_{7,2}\,a_{6,3}\,a_{1,5}\,a_{2,7}\,a_{5,6}-a_{6,1}\,a_{3,2}\,a_{7,3}\,a_{1,5}\,a_{2,7}\,a_{5,6}\,,\,0\,,$$
$$-a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{4,5}\,a_{1,7}\,a_{5,6}-a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{1,5}\,a_{3,7}\,a_{5,6}+a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{3,5}\,a_{1,7}\,a_{5,6}-a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{1,5}\,a_{2,7}\,a_{5,6}$$
$$+a_{4,1}\,a_{3,2}\,a_{7,3}\,a_{1,5}\,a_{2,7}\,a_{5,6}\,,\,a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{4,5}\,a_{1,7}\,a_{5,6}+a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{1,5}\,a_{3,7}\,a_{5,6}-a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{3,5}\,a_{1,7}\,a_{5,6}$$
$$+a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{1,5}\,a_{2,7}\,a_{5,6}-a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{1,5}\,a_{2,7}\,a_{5,6}\,]$$
$$[\,a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{4,4}\,a_{5,6}\,a_{7,7}-a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{4,4}\,a_{5,6}\,a_{6,7}-a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{3,7}\,a_{5,6}+a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{4,4}\,a_{3,7}\,a_{5,6}$$
$$-a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{2,7}\,a_{5,6}+a_{3,1}\,a_{7,2}\,a_{6,3}\,a_{4,4}\,a_{2,7}\,a_{5,6}+a_{4,1}\,a_{3,2}\,a_{7,3}\,a_{6,4}\,a_{2,7}\,a_{5,6}+a_{6,1}\,a_{3,2}\,a_{7,3}\,a_{4,4}\,a_{2,7}\,a_{5,6}\,,$$
$$a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{5,6}\,a_{7,7}-a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{5,6}\,a_{6,7}-a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{1,7}\,a_{5,6}+a_{3,1}\,a_{7,2}\,a_{6,3}\,a_{4,4}\,a_{1,7}\,a_{5,6}$$
$$-a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{1,4}\,a_{5,6}\,a_{7,7}+a_{4,1}\,a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{5,6}\,a_{6,7}+a_{4,1}\,a_{3,2}\,a_{7,3}\,a_{6,4}\,a_{1,7}\,a_{5,6}-a_{4,1}\,a_{7,2}\,a_{6,3}\,a_{1,4}\,a_{3,7}\,a_{5,6}$$
$$+a_{6,1}\,a_{3,2}\,a_{7,3}\,a_{4,4}\,a_{1,7}\,a_{5,6}-a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{3,7}\,a_{5,6}\,,\,a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{5,6}\,a_{7,7}-a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{5,6}\,a_{6,7}$$
$$-a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{1,7}\,a_{5,6}+a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{4,4}\,a_{1,7}\,a_{5,6}+a_{4,1}\,a_{7,2}\,a_{6,3}\,a_{1,4}\,a_{2,7}\,a_{5,6}+a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{2,7}\,a_{5,6}\,,$$
$$a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{1,4}\,a_{5,6}\,a_{7,7}-a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{5,6}\,a_{6,7}-a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{6,4}\,a_{1,7}\,a_{5,6}+a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{1,4}\,a_{3,7}\,a_{5,6}$$
$$+a_{3,1}\,a_{7,2}\,a_{6,3}\,a_{1,4}\,a_{2,7}\,a_{5,6}+a_{6,1}\,a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{2,7}\,a_{5,6}\,,\,0\,,$$
$$-a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{4,4}\,a_{1,7}\,a_{5,6}+a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{3,7}\,a_{5,6}+a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{2,7}\,a_{5,6}-a_{4,1}\,a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{2,7}\,a_{5,6}\,,$$
$$a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{4,4}\,a_{1,7}\,a_{5,6}-a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{3,7}\,a_{5,6}-a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{2,7}\,a_{5,6}+a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{1,4}\,a_{2,7}\,a_{5,6}\,]$$
$$[-a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{2,7}\,a_{6,5}+a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{2,7}\,a_{5,5}-a_{3,1}\,a_{7,2}\,a_{6,3}\,a_{4,4}\,a_{2,7}\,a_{5,5}+a_{3,1}\,a_{7,2}\,a_{6,3}\,a_{5,4}\,a_{2,7}\,a_{4,5}$$
$$-a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{5,4}\,a_{2,7}\,a_{7,5}+a_{4,1}\,a_{3,2}\,a_{7,3}\,a_{5,4}\,a_{2,7}\,a_{6,5}-a_{4,1}\,a_{3,2}\,a_{7,3}\,a_{6,4}\,a_{2,7}\,a_{5,5}-a_{4,1}\,a_{7,2}\,a_{6,3}\,a_{5,4}\,a_{2,7}\,a_{3,5}$$
$$-a_{6,1}\,a_{3,2}\,a_{7,3}\,a_{4,4}\,a_{2,7}\,a_{5,5}+a_{6,1}\,a_{3,2}\,a_{7,3}\,a_{5,4}\,a_{2,7}\,a_{4,5}-a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{2,7}\,a_{3,5}+a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{5,4}\,a_{2,7}\,a_{7,5}$$
$$-a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{4,4}\,a_{5,5}\,a_{7,7}+a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{4,4}\,a_{5,7}\,a_{7,5}+a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{5,4}\,a_{4,5}\,a_{7,7}+a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{4,5}\,a_{5,7}\,a_{6,7}$$
$$-a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{4,4}\,a_{5,7}\,a_{6,5}-a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{5,4}\,a_{4,5}\,a_{6,7}+a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{6,4}\,a_{4,5}\,a_{5,7}-a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{5,4}\,a_{3,5}\,a_{7,7}$$
$$+a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{5,4}\,a_{3,7}\,a_{7,5}+a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{3,5}\,a_{6,7}-a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{3,7}\,a_{6,5}-a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{3,5}\,a_{5,7}$$
$$+a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{3,7}\,a_{5,5}+a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{4,4}\,a_{3,5}\,a_{5,7}-a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{4,4}\,a_{3,7}\,a_{5,5}+a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{5,4}\,a_{3,7}\,a_{4,5}\,,$$
$$-a_{4,1}\,a_{7,2}\,a_{6,3}\,a_{5,4}\,a_{1,7}\,a_{3,5}+a_{3,1}\,a_{7,2}\,a_{6,3}\,a_{5,4}\,a_{1,7}\,a_{4,5}+a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{1,4}\,a_{5,5}\,a_{7,7}-a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{1,4}\,a_{5,7}\,a_{7,5}$$
$$+a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{5,4}\,a_{1,5}\,a_{7,7}-a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{5,4}\,a_{1,7}\,a_{7,5}-a_{4,1}\,a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{5,5}\,a_{6,7}+a_{4,1}\,a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{5,7}\,a_{6,5}$$
$$-a_{4,1}\,a_{3,2}\,a_{7,3}\,a_{5,4}\,a_{1,5}\,a_{6,7}+a_{4,1}\,a_{3,2}\,a_{7,3}\,a_{5,4}\,a_{1,7}\,a_{6,5}+a_{4,1}\,a_{3,2}\,a_{7,3}\,a_{6,4}\,a_{1,5}\,a_{5,7}-a_{4,1}\,a_{3,2}\,a_{7,3}\,a_{6,4}\,a_{1,7}\,a_{5,5}$$
$$-a_{4,1}\,a_{7,2}\,a_{6,3}\,a_{1,4}\,a_{3,5}\,a_{5,7}+a_{4,1}\,a_{7,2}\,a_{6,3}\,a_{1,4}\,a_{3,7}\,a_{5,5}-a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{5,5}\,a_{7,7}+a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{5,7}\,a_{7,5}$$
$$-a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{5,4}\,a_{1,5}\,a_{7,7}+a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{5,4}\,a_{1,7}\,a_{7,5}+a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{5,5}\,a_{6,7}-a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{5,7}\,a_{6,5}$$
$$+a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{1,5}\,a_{6,7}-a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{1,7}\,a_{6,5}-a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{1,5}\,a_{5,7}+a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{1,7}\,a_{5,5}$$
$$+a_{3,1}\,a_{7,2}\,a_{6,3}\,a_{1,4}\,a_{4,5}\,a_{5,7}+a_{3,1}\,a_{7,2}\,a_{6,3}\,a_{4,4}\,a_{1,5}\,a_{5,7}-a_{3,1}\,a_{7,2}\,a_{6,3}\,a_{4,4}\,a_{1,7}\,a_{5,5}+a_{4,1}\,a_{7,2}\,a_{6,3}\,a_{5,4}\,a_{1,5}\,a_{3,7}$$
$$+a_{6,1}\,a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{4,5}\,a_{5,7}+a_{6,1}\,a_{3,2}\,a_{7,3}\,a_{4,4}\,a_{1,5}\,a_{5,7}-a_{6,1}\,a_{3,2}\,a_{7,3}\,a_{4,4}\,a_{1,7}\,a_{5,5}+a_{6,1}\,a_{3,2}\,a_{7,3}\,a_{5,4}\,a_{1,7}\,a_{4,5}$$
$$-a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{3,5}\,a_{5,7}+a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{3,7}\,a_{5,5}+a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{1,5}\,a_{3,7}-a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{1,7}\,a_{3,5}\,,$$
$$-a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{5,5}\,a_{7,7}+a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{5,7}\,a_{7,5}-a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{5,4}\,a_{1,5}\,a_{7,7}+a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{5,4}\,a_{1,7}\,a_{7,5}$$
$$+a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{5,5}\,a_{6,7}-a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{5,7}\,a_{6,5}+a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{1,5}\,a_{6,7}-a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{1,7}\,a_{6,5}$$
$$-a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{1,5}\,a_{5,7}+a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{1,7}\,a_{5,5}+a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{1,4}\,a_{4,5}\,a_{5,7}+a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{4,4}\,a_{1,5}\,a_{5,7}$$
$$-a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{4,4}\,a_{1,7}\,a_{5,5}+a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{5,4}\,a_{1,7}\,a_{4,5}-a_{4,1}\,a_{7,2}\,a_{6,3}\,a_{1,4}\,a_{2,7}\,a_{5,5}-a_{4,1}\,a_{7,2}\,a_{6,3}\,a_{5,4}\,a_{1,5}\,a_{2,7}$$
$$-a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{2,7}\,a_{5,5}-a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{1,5}\,a_{2,7}\,,\,-a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{1,4}\,a_{5,5}\,a_{7,7}+a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{1,4}\,a_{5,7}\,a_{7,5}$$
$$-a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{5,4}\,a_{1,5}\,a_{7,7}+a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{5,4}\,a_{1,7}\,a_{7,5}+a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{5,5}\,a_{6,7}-a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{5,7}\,a_{6,5}$$
$$+a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{5,4}\,a_{1,5}\,a_{6,7}-a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{5,4}\,a_{1,7}\,a_{6,5}-a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{6,4}\,a_{1,5}\,a_{5,7}+a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{6,4}\,a_{1,7}\,a_{5,5}$$
$$+a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{1,4}\,a_{3,5}\,a_{5,7}-a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{1,4}\,a_{3,7}\,a_{5,5}-a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{5,4}\,a_{1,5}\,a_{3,7}+a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{5,4}\,a_{1,7}\,a_{3,5}$$
$$-a_{3,1}\,a_{7,2}\,a_{6,3}\,a_{1,4}\,a_{2,7}\,a_{5,5}-a_{3,1}\,a_{7,2}\,a_{6,3}\,a_{5,4}\,a_{1,5}\,a_{2,7}-a_{6,1}\,a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{2,7}\,a_{5,5}-a_{6,1}\,a_{3,2}\,a_{7,3}\,a_{5,4}\,a_{1,5}\,a_{2,7}\,,$$
$$-a_{4,1}\,a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{2,7}\,a_{6,5}-a_{4,1}\,a_{3,2}\,a_{7,3}\,a_{6,4}\,a_{1,5}\,a_{2,7}+a_{4,1}\,a_{7,2}\,a_{6,3}\,a_{1,4}\,a_{2,7}\,a_{3,5}-a_{6,1}\,a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{2,7}\,a_{4,5}$$
$$-a_{6,1}\,a_{3,2}\,a_{7,3}\,a_{4,4}\,a_{1,5}\,a_{2,7}+a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{2,7}\,a_{3,5}+a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{1,4}\,a_{2,7}\,a_{7,5}-a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{1,4}\,a_{4,5}\,a_{7,7}$$
$$-a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{4,4}\,a_{1,5}\,a_{7,7}+a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{4,4}\,a_{1,7}\,a_{7,5}+a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{4,5}\,a_{6,7}+a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{4,4}\,a_{1,5}\,a_{6,7}$$
$$-a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{4,4}\,a_{1,7}\,a_{6,5}+a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{6,4}\,a_{1,7}\,a_{4,5}+a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{3,5}\,a_{7,7}-a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{3,7}\,a_{5,5}$$
$$-a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{3,5}\,a_{6,7}+a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{3,7}\,a_{6,5}+a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{1,5}\,a_{3,7}-a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{1,7}\,a_{3,5}$$

$$-a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{1,4}\,a_{3,7}\,a_{4,5}-a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{4,4}\,a_{1,5}\,a_{3,7}+a_{2,1}\,a_{7,2}\,a_{6,3}\,a_{4,4}\,a_{1,7}\,a_{3,5}-a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{2,7}\,a_{7,5}$$
$$+a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{2,7}\,a_{6,5}+a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{1,5}\,a_{2,7}-a_{3,1}\,a_{7,2}\,a_{6,3}\,a_{1,4}\,a_{2,7}\,a_{4,5}-a_{3,1}\,a_{7,2}\,a_{6,3}\,a_{4,4}\,a_{1,5}\,a_{2,7},$$
$$-a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{4,5}\,a_{5,7}-a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{4,4}\,a_{1,5}\,a_{5,7}+a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{4,4}\,a_{1,7}\,a_{5,5}-a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{5,4}\,a_{1,7}\,a_{4,5}$$
$$+a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{3,5}\,a_{5,7}-a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{3,7}\,a_{5,5}-a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{1,5}\,a_{3,7}+a_{2,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{1,7}\,a_{3,5}$$
$$-a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{2,7}\,a_{5,5}-a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{1,5}\,a_{2,7}+a_{4,1}\,a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{2,7}\,a_{5,5}+a_{4,1}\,a_{3,2}\,a_{7,3}\,a_{5,4}\,a_{1,5}\,a_{2,7},$$
$$a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{1,4}\,a_{4,5}\,a_{5,7}+a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{4,4}\,a_{1,5}\,a_{5,7}-a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{4,4}\,a_{1,7}\,a_{5,5}+a_{2,1}\,a_{3,2}\,a_{6,3}\,a_{5,4}\,a_{1,7}\,a_{4,5}$$
$$-a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{3,5}\,a_{5,7}+a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{3,7}\,a_{5,5}+a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{5,4}\,a_{1,5}\,a_{3,7}-a_{2,1}\,a_{4,2}\,a_{6,3}\,a_{5,4}\,a_{1,7}\,a_{3,5}$$
$$+a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{2,7}\,a_{5,5}+a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{5,4}\,a_{1,5}\,a_{2,7}-a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{1,4}\,a_{2,7}\,a_{5,5}-a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{5,4}\,a_{1,5}\,a_{2,7}]$$
$$[a_{2,1}\,(-a_{3,2}\,a_{6,3}\,a_{4,4}\,a_{5,6}\,a_{7,5}+a_{3,2}\,a_{7,3}\,a_{4,4}\,a_{5,6}\,a_{6,5}-a_{3,2}\,a_{7,3}\,a_{6,4}\,a_{4,5}\,a_{5,6}+a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{3,5}\,a_{5,6}-a_{7,2}\,a_{6,3}\,a_{4,4}\,a_{3,5}\,a_{5,6})$$
$$,-a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{5,6}\,a_{7,5}+a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{5,6}\,a_{6,5}+a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{1,5}\,a_{5,6}-a_{3,1}\,a_{7,2}\,a_{6,3}\,a_{1,4}\,a_{4,5}\,a_{5,6}$$
$$-a_{3,1}\,a_{7,2}\,a_{6,3}\,a_{4,4}\,a_{1,5}\,a_{5,6}+a_{4,1}\,a_{3,2}\,a_{6,3}\,a_{1,4}\,a_{5,6}\,a_{7,5}-a_{4,1}\,a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{5,6}\,a_{6,5}-a_{4,1}\,a_{3,2}\,a_{7,3}\,a_{6,4}\,a_{1,5}\,a_{5,6}$$
$$+a_{4,1}\,a_{7,2}\,a_{6,3}\,a_{1,4}\,a_{3,5}\,a_{5,6}-a_{6,1}\,a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{4,5}\,a_{5,6}-a_{6,1}\,a_{3,2}\,a_{7,3}\,a_{4,4}\,a_{1,5}\,a_{5,6}+a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{3,5}\,a_{5,6},$$
$$-a_{2,1}\,(a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{5,6}\,a_{7,5}-a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{5,6}\,a_{6,5}-a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{1,5}\,a_{5,6}+a_{7,2}\,a_{6,3}\,a_{1,4}\,a_{4,5}\,a_{5,6}+a_{7,2}\,a_{6,3}\,a_{4,4}\,a_{1,5}\,a_{5,6}),$$
$$a_{2,1}\,(-a_{3,2}\,a_{6,3}\,a_{1,4}\,a_{5,6}\,a_{7,5}+a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{5,6}\,a_{6,5}+a_{3,2}\,a_{7,3}\,a_{6,4}\,a_{1,5}\,a_{5,6}-a_{7,2}\,a_{6,3}\,a_{1,4}\,a_{3,5}\,a_{5,6}),0,$$
$$a_{2,1}\,(a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{4,5}\,a_{5,6}+a_{3,2}\,a_{7,3}\,a_{4,4}\,a_{1,5}\,a_{5,6}-a_{4,2}\,a_{7,3}\,a_{1,4}\,a_{3,5}\,a_{5,6}),$$
$$-a_{2,1}\,(a_{3,2}\,a_{6,3}\,a_{1,4}\,a_{4,5}\,a_{5,6}+a_{3,2}\,a_{6,3}\,a_{4,4}\,a_{1,5}\,a_{5,6}-a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{3,5}\,a_{5,6})]$$

"72"

$$A:=\begin{bmatrix}0&0&1&-1&0&0&0\\-1&-1&0&1&0&1&0\\0&-1&0&0&0&1&0\\0&0&0&0&-1&1&1\\0&-1&0&1&0&1&0\\0&0&0&0&1&0&0\\1&1&-1&0&0&-1&0\end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

poly_coef_F0_to_Fn = [-1, -1, 0, 2, 1, 0, 0, 0]

positive_feedback = [0, 0, 1, 5, 4, 6, 5, 4]

negative_feedback = [-1, -1, -1, -3, -3, -6, -5, -4]

absolute_feedback = [1, 1, 2, 8, 7, 12, 10, 8]

wFn = [-1., -1., 0., 0.25, 0.14, 0., 0., 0.]

"Criterion ii"

$wD_6 = 0.$

ratio_to_model_C = 0.

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix}0&0&0&0&0&0&0\\-1&-1&0&0&0&0&-1\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\-1&-1&0&0&0&0&-1\\1&1&0&0&0&0&1\end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 4 & 4 & 6 & 0 & 4 & 0 & 4 \\ 1 & 1 & 4 & 0 & 2 & 0 & 1 \\ 6 & 2 & 4 & 0 & 4 & 0 & 2 \\ 2 & 2 & 4 & 0 & 4 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 \\ 1 & 1 & 4 & 0 & 2 & 0 & 1 \\ 1 & 1 & 4 & 8 & 2 & 8 & 1 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 0. & 0. & 0. & 1. & 0. & 1. & 0. \\ 1. & 1. & 0. & 1. & 0. & 1. & 1. \\ 0. & 0. & 0. & 1. & 0. & 1. & 0. \\ 0. & 0. & 0. & 1. & 0. & 1. & 0. \\ 1. & 1. & 1. & 1. & 1. & 0. & 1. \\ 1. & 1. & 0. & 1. & 0. & 1. & 1. \\ 1. & 1. & 0. & 0. & 0. & 0. & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 2 & 2 & 4 & 0 & 4 & 0 & 2 \\ 3 & 5 & 6 & 0 & 4 & 0 & 3 \\ 1 & 1 & 4 & 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 \\ 1 & 1 & 4 & 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 3 & 6 & 0 & 4 & 0 & 3 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 0. & 0. & 0. & 1. & 0. & 1. & 0. \\ 0.33 & 0.20 & 0. & 1. & 0. & 1. & 0.33 \\ 1. & 1. & 0. & 1. & 0. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 0. & 1. \\ 1. & 1. & 0. & 1. & 0. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 0.20 & 0.33 & 0. & 1. & 0. & 1. & 0.33 \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 6 & 2 & 4 & 0 & 4 & 0 & 2 \\ 3 & 3 & 6 & 0 & 4 & 0 & 3 \\ 1 & 1 & 4 & 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 8 & 0 & 8 & 0 \\ 1 & 1 & 4 & 0 & 6 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 \\ 5 & 3 & 6 & 0 & 4 & 0 & 5 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 0. & 0. & 0. & 1. & 0. & 1. & 0. \\ 0.33 & 0.33 & 0. & 1. & 0. & 1. & 0.33 \\ 1. & 1. & 0. & 1. & 0. & 1. & 1. \\ 1. & 1. & 1. & 0. & 1. & 0. & 1. \\ 1. & 1. & 0. & 1. & 0. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 0. & 1. \\ 0.20 & 0.33 & 0. & 1. & 0. & 1. & 0.20 \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & 0 & a_{1,3} & -a_{1,4} & 0 & 0 & 0 \\ -a_{2,1} & -a_{2,2} & 0 & a_{2,4} & 0 & a_{2,6} & 0 \\ 0 & -a_{3,2} & 0 & 0 & 0 & a_{3,6} & 0 \\ 0 & 0 & 0 & 0 & -a_{4,5} & a_{4,6} & a_{4,7} \\ 0 & -a_{5,2} & 0 & a_{5,4} & 0 & a_{5,6} & 0 \\ 0 & 0 & 0 & 0 & a_{6,5} & 0 & 0 \\ a_{7,1} & a_{7,2} & -a_{7,3} & 0 & 0 & -a_{7,6} & 0 \end{bmatrix}$$

"system feedback"

F_7

$$a_{7,6}\,a_{2,1}\,a_{3,2}\,a_{1,3}\,a_{5,4}\,a_{4,7}\,a_{6,5} - a_{7,3}\,a_{2,1}\,a_{3,2}\,a_{1,4}\,a_{6,5}\,a_{4,7}\,a_{5,6} + a_{7,3}\,a_{2,1}\,a_{5,2}\,a_{1,4}\,a_{6,5}\,a_{3,6}\,a_{4,7}$$
$$- a_{2,1}\,a_{7,2}\,a_{1,3}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{4,7} + a_{2,2}\,a_{7,1}\,a_{1,3}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{4,7} + a_{7,1}\,a_{3,2}\,a_{1,3}\,a_{2,4}\,a_{6,5}\,a_{4,7}\,a_{5,6}$$
$$- a_{2,6}\,a_{7,1}\,a_{3,2}\,a_{1,3}\,a_{5,4}\,a_{4,7}\,a_{6,5} - a_{7,1}\,a_{5,2}\,a_{1,3}\,a_{2,4}\,a_{6,5}\,a_{3,6}\,a_{4,7}$$

F_6

$$-a_{2,2}\,a_{7,3}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{4,7} + a_{5,2}\,a_{7,3}\,a_{2,4}\,a_{6,5}\,a_{3,6}\,a_{4,7} + a_{3,2}\,a_{7,3}\,a_{5,4}\,a_{6,5}\,a_{2,6}\,a_{4,7} - a_{3,2}\,a_{7,3}\,a_{2,4}\,a_{6,5}\,a_{4,7}\,a_{5,6}$$
$$+ a_{7,1}\,a_{1,3}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{4,7} + a_{2,1}\,a_{5,2}\,a_{1,4}\,a_{6,5}\,a_{4,7}\,a_{7,6} - a_{2,1}\,a_{7,2}\,a_{1,4}\,a_{6,5}\,a_{4,7}\,a_{5,6} + a_{7,1}\,a_{2,2}\,a_{1,4}\,a_{6,5}\,a_{4,7}\,a_{5,6}$$
$$- a_{7,1}\,a_{5,2}\,a_{1,4}\,a_{6,5}\,a_{2,6}\,a_{4,7} - a_{2,1}\,a_{3,2}\,a_{1,3}\,a_{5,4}\,a_{4,6}\,a_{6,5}$$

F_5

$$-a_{7,1}\,a_{1,3}\,a_{3,2}\,a_{2,4}\,a_{4,7} + a_{7,2}\,a_{5,4}\,a_{6,5}\,a_{2,6}\,a_{4,7} + a_{7,3}\,a_{3,2}\,a_{2,1}\,a_{1,4}\,a_{4,7} - a_{7,3}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{4,7} + a_{7,6}\,a_{6,5}\,a_{5,2}\,a_{2,4}\,a_{4,7}$$
$$- a_{2,2}\,a_{5,4}\,a_{6,5}\,a_{4,7}\,a_{7,6} + a_{7,1}\,a_{1,4}\,a_{4,7}\,a_{5,6}\,a_{6,5} - a_{7,2}\,a_{2,4}\,a_{4,7}\,a_{5,6}\,a_{6,5} + a_{6,5}\,a_{5,2}\,a_{2,1}\,a_{1,3}\,a_{3,6} - a_{6,5}\,a_{5,2}\,a_{2,1}\,a_{1,4}\,a_{4,6}$$
$$- a_{5,6}\,a_{6,5}\,a_{3,2}\,a_{2,1}\,a_{1,3} + a_{5,4}\,a_{4,5}\,a_{3,2}\,a_{2,1}\,a_{1,3}$$

F_4

$$a_{7,2}\,a_{2,1}\,a_{1,4}\,a_{4,7} + a_{7,3}\,a_{3,2}\,a_{2,4}\,a_{4,7} - a_{5,4}\,a_{6,5}\,a_{4,7}\,a_{7,6} - a_{2,2}\,a_{7,1}\,a_{1,4}\,a_{4,7} - a_{6,5}\,a_{5,2}\,a_{2,4}\,a_{4,6} + a_{6,5}\,a_{2,2}\,a_{5,4}\,a_{4,6}$$
$$+ a_{5,2}\,a_{4,5}\,a_{2,1}\,a_{1,4}$$

F_3

$$-a_{7,1}\,a_{1,4}\,a_{4,7} + a_{7,2}\,a_{2,4}\,a_{4,7} - a_{6,5}\,a_{5,2}\,a_{2,6} + a_{5,4}\,a_{4,6}\,a_{6,5} + a_{5,6}\,a_{6,5}\,a_{2,2} + a_{5,2}\,a_{2,4}\,a_{4,5} - a_{5,4}\,a_{4,5}\,a_{2,2} + a_{3,2}\,a_{2,1}\,a_{1,3}$$

F_2

$$a_{5,6}\,a_{6,5} - a_{5,4}\,a_{4,5}$$

F_1

$$-a_{2,2}$$

F_0

-1

"adjoint (-A)"

$$\left[a_{2,2}\,a_{7,3}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{4,7} + a_{3,2}\,a_{7,3}\,a_{2,4}\,a_{6,5}\,a_{4,7}\,a_{5,6} - a_{3,2}\,a_{7,3}\,a_{5,4}\,a_{6,5}\,a_{2,6}\,a_{4,7} - a_{5,2}\,a_{7,3}\,a_{2,4}\,a_{6,5}\,a_{3,6}\,a_{4,7}\,,\right.$$

$$-a_{3,2}\,a_{1,3}\,a_{5,4}\,a_{6,5}\,a_{4,7}\,a_{7,6}+a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{6,5}\,a_{4,7}\,a_{5,6}-a_{5,2}\,a_{7,3}\,a_{1,4}\,a_{6,5}\,a_{3,6}\,a_{4,7}+a_{7,2}\,a_{1,3}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{4,7},$$

$$a_{2,2}\,a_{1,3}\,a_{5,4}\,a_{6,5}\,a_{4,7}\,a_{7,6}-a_{2,2}\,a_{7,3}\,a_{1,4}\,a_{6,5}\,a_{4,7}\,a_{5,6}-a_{5,2}\,a_{1,3}\,a_{2,4}\,a_{6,5}\,a_{4,7}\,a_{7,6}+a_{5,2}\,a_{7,3}\,a_{1,4}\,a_{6,5}\,a_{2,6}\,a_{4,7}$$
$$+a_{7,2}\,a_{1,3}\,a_{2,4}\,a_{6,5}\,a_{4,7}\,a_{5,6}-a_{7,2}\,a_{1,3}\,a_{5,4}\,a_{6,5}\,a_{2,6}\,a_{4,7},0,$$

$$a_{2,2}\,a_{7,3}\,a_{1,4}\,a_{6,5}\,a_{3,6}\,a_{4,7}+a_{3,2}\,a_{1,3}\,a_{2,4}\,a_{6,5}\,a_{4,7}\,a_{7,6}-a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{6,5}\,a_{2,6}\,a_{4,7}-a_{7,2}\,a_{1,3}\,a_{2,4}\,a_{6,5}\,a_{3,6}\,a_{4,7},0,$$

$$a_{2,2}\,a_{1,3}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{4,7}+a_{3,2}\,a_{1,3}\,a_{2,4}\,a_{6,5}\,a_{4,7}\,a_{5,6}-a_{3,2}\,a_{1,3}\,a_{5,4}\,a_{6,5}\,a_{2,6}\,a_{4,7}-a_{5,2}\,a_{1,3}\,a_{2,4}\,a_{6,5}\,a_{3,6}\,a_{4,7}]$$

$$[-a_{2,1}\,a_{7,3}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{4,7},-a_{7,1}\,a_{1,3}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{4,7},$$

$$-a_{2,1}\,a_{1,3}\,a_{5,4}\,a_{6,5}\,a_{4,7}\,a_{7,6}+a_{2,1}\,a_{7,3}\,a_{1,4}\,a_{6,5}\,a_{4,7}\,a_{5,6}-a_{7,1}\,a_{1,3}\,a_{2,4}\,a_{6,5}\,a_{4,7}\,a_{5,6}+a_{7,1}\,a_{1,3}\,a_{5,4}\,a_{6,5}\,a_{2,6}\,a_{4,7},0,$$

$$-a_{2,1}\,a_{7,3}\,a_{1,4}\,a_{6,5}\,a_{3,6}\,a_{4,7}+a_{7,1}\,a_{1,3}\,a_{2,4}\,a_{6,5}\,a_{3,6}\,a_{4,7},0,-a_{2,1}\,a_{1,3}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{4,7}]$$

$$[a_{2,1}\,a_{3,2}\,a_{5,4}\,a_{6,5}\,a_{4,7}\,a_{7,6}-a_{2,1}\,a_{7,2}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{4,7}+a_{7,1}\,a_{2,2}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{4,7}+a_{7,1}\,a_{3,2}\,a_{2,4}\,a_{6,5}\,a_{4,7}\,a_{5,6}$$
$$-a_{7,1}\,a_{3,2}\,a_{5,4}\,a_{6,5}\,a_{2,6}\,a_{4,7}-a_{7,1}\,a_{5,2}\,a_{2,4}\,a_{6,5}\,a_{3,6}\,a_{4,7},-a_{7,1}\,(-a_{3,2}\,a_{1,4}\,a_{6,5}\,a_{4,7}\,a_{5,6}+a_{5,2}\,a_{1,4}\,a_{6,5}\,a_{3,6}\,a_{4,7}),$$

$$-a_{2,1}\,a_{5,2}\,a_{1,4}\,a_{6,5}\,a_{4,7}\,a_{7,6}+a_{2,1}\,a_{7,2}\,a_{1,4}\,a_{6,5}\,a_{4,7}\,a_{5,6}-a_{7,1}\,a_{2,2}\,a_{1,4}\,a_{6,5}\,a_{4,7}\,a_{5,6}+a_{7,1}\,a_{5,2}\,a_{1,4}\,a_{6,5}\,a_{2,6}\,a_{4,7},0,$$

$$a_{2,1}\,a_{3,2}\,a_{1,4}\,a_{6,5}\,a_{4,7}\,a_{7,6}-a_{2,1}\,a_{7,2}\,a_{1,4}\,a_{6,5}\,a_{3,6}\,a_{4,7}+a_{7,1}\,a_{2,2}\,a_{1,4}\,a_{6,5}\,a_{3,6}\,a_{4,7}-a_{7,1}\,a_{3,2}\,a_{1,4}\,a_{6,5}\,a_{2,6}\,a_{4,7},0,$$

$$-a_{2,1}\,(-a_{3,2}\,a_{1,4}\,a_{6,5}\,a_{4,7}\,a_{5,6}+a_{5,2}\,a_{1,4}\,a_{6,5}\,a_{3,6}\,a_{4,7})]$$

$$[-a_{2,1}\,(-a_{3,2}\,a_{7,3}\,a_{6,5}\,a_{4,7}\,a_{5,6}+a_{5,2}\,a_{7,3}\,a_{6,5}\,a_{3,6}\,a_{4,7}),a_{7,1}\,(a_{3,2}\,a_{1,3}\,a_{6,5}\,a_{4,7}\,a_{5,6}-a_{5,2}\,a_{1,3}\,a_{6,5}\,a_{3,6}\,a_{4,7}),$$

$$-a_{2,1}\,a_{5,2}\,a_{1,3}\,a_{6,5}\,a_{4,7}\,a_{7,6}+a_{2,1}\,a_{7,2}\,a_{1,3}\,a_{6,5}\,a_{4,7}\,a_{5,6}-a_{7,1}\,a_{2,2}\,a_{1,3}\,a_{6,5}\,a_{4,7}\,a_{5,6}+a_{7,1}\,a_{5,2}\,a_{1,3}\,a_{6,5}\,a_{2,6}\,a_{4,7},0,$$

$$a_{2,1}\,a_{3,2}\,a_{1,3}\,a_{6,5}\,a_{4,7}\,a_{7,6}-a_{2,1}\,a_{7,2}\,a_{1,3}\,a_{6,5}\,a_{3,6}\,a_{4,7}+a_{7,1}\,a_{2,2}\,a_{1,3}\,a_{6,5}\,a_{3,6}\,a_{4,7}-a_{7,1}\,a_{3,2}\,a_{1,3}\,a_{6,5}\,a_{2,6}\,a_{4,7},0,$$

$$a_{2,1}\,(a_{3,2}\,a_{1,3}\,a_{6,5}\,a_{4,7}\,a_{5,6}-a_{5,2}\,a_{1,3}\,a_{6,5}\,a_{3,6}\,a_{4,7})]$$

$$[0,0,0,0,0,a_{2,1}\,a_{3,2}\,a_{1,3}\,a_{5,4}\,a_{4,7}\,a_{7,6}-a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{4,7}\,a_{5,6}+a_{2,1}\,a_{5,2}\,a_{7,3}\,a_{1,4}\,a_{3,6}\,a_{4,7}$$
$$-a_{2,1}\,a_{7,2}\,a_{1,3}\,a_{5,4}\,a_{3,6}\,a_{4,7}+a_{7,1}\,a_{2,2}\,a_{1,3}\,a_{5,4}\,a_{3,6}\,a_{4,7}+a_{7,1}\,a_{3,2}\,a_{1,3}\,a_{2,4}\,a_{4,7}\,a_{5,6}-a_{7,1}\,a_{3,2}\,a_{1,3}\,a_{5,4}\,a_{2,6}\,a_{4,7}$$
$$-a_{7,1}\,a_{5,2}\,a_{1,3}\,a_{2,4}\,a_{3,6}\,a_{4,7},0]$$

$$[-a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{5,4}\,a_{4,7}\,a_{6,5},-a_{7,1}\,a_{3,2}\,a_{1,3}\,a_{5,4}\,a_{4,7}\,a_{6,5},$$

$$a_{2,1}\,a_{5,2}\,a_{7,3}\,a_{1,4}\,a_{4,7}\,a_{6,5}-a_{2,1}\,a_{7,2}\,a_{1,3}\,a_{5,4}\,a_{4,7}\,a_{6,5}+a_{7,1}\,a_{2,2}\,a_{1,3}\,a_{5,4}\,a_{4,7}\,a_{6,5}-a_{7,1}\,a_{5,2}\,a_{1,3}\,a_{2,4}\,a_{4,7}\,a_{6,5},0,$$

$$-a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{4,7}\,a_{6,5}+a_{7,1}\,a_{3,2}\,a_{1,3}\,a_{2,4}\,a_{4,7}\,a_{6,5},0,-a_{2,1}\,a_{3,2}\,a_{1,3}\,a_{5,4}\,a_{4,7}\,a_{6,5}]$$

$$[a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{5,4}\,a_{4,6}\,a_{6,5},a_{7,1}\,a_{3,2}\,a_{1,3}\,a_{5,4}\,a_{4,6}\,a_{6,5},$$

$$-a_{2,1}\,a_{5,2}\,a_{7,3}\,a_{1,4}\,a_{4,6}\,a_{6,5}+a_{2,1}\,a_{7,2}\,a_{1,3}\,a_{5,4}\,a_{4,6}\,a_{6,5}-a_{7,1}\,a_{2,2}\,a_{1,3}\,a_{5,4}\,a_{4,6}\,a_{6,5}+a_{7,1}\,a_{5,2}\,a_{1,3}\,a_{2,4}\,a_{4,6}\,a_{6,5},$$

$$a_{2,1}\,a_{3,2}\,a_{1,3}\,a_{5,4}\,a_{6,5}\,a_{7,6}-a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{5,6}\,a_{6,5}+a_{2,1}\,a_{5,2}\,a_{7,3}\,a_{1,4}\,a_{3,6}\,a_{6,5}-a_{2,1}\,a_{7,2}\,a_{1,3}\,a_{5,4}\,a_{3,6}\,a_{6,5}$$
$$+a_{7,1}\,a_{2,2}\,a_{1,3}\,a_{5,4}\,a_{3,6}\,a_{6,5}+a_{7,1}\,a_{3,2}\,a_{1,3}\,a_{2,4}\,a_{5,6}\,a_{6,5}-a_{7,1}\,a_{3,2}\,a_{1,3}\,a_{5,4}\,a_{2,6}\,a_{6,5}-a_{7,1}\,a_{5,2}\,a_{1,3}\,a_{2,4}\,a_{3,6}\,a_{6,5},$$

$$a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{4,6}\,a_{6,5}-a_{7,1}\,a_{3,2}\,a_{1,3}\,a_{2,4}\,a_{4,6}\,a_{6,5},a_{2,1}\,a_{3,2}\,a_{1,3}\,a_{5,4}\,a_{4,5}\,a_{7,6}-a_{2,1}\,a_{3,2}\,a_{7,3}\,a_{1,4}\,a_{4,5}\,a_{5,6}$$
$$+a_{2,1}\,a_{5,2}\,a_{7,3}\,a_{1,4}\,a_{3,6}\,a_{4,5}-a_{2,1}\,a_{7,2}\,a_{1,3}\,a_{5,4}\,a_{3,6}\,a_{4,5}+a_{7,1}\,a_{2,2}\,a_{1,3}\,a_{5,4}\,a_{3,6}\,a_{4,5}+a_{7,1}\,a_{3,2}\,a_{1,3}\,a_{2,4}\,a_{4,5}\,a_{5,6}$$
$$-a_{7,1}\,a_{3,2}\,a_{1,3}\,a_{5,4}\,a_{2,6}\,a_{4,5}-a_{7,1}\,a_{5,2}\,a_{1,3}\,a_{2,4}\,a_{3,6}\,a_{4,5},a_{2,1}\,a_{3,2}\,a_{1,3}\,a_{5,4}\,a_{4,6}\,a_{6,5}]$$

"73"

$$A := \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & -1 & -1 \\ -1 & 0 & 0 & 1 & 0 & -1 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & -1 & 0 & -1 & -1 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

poly_coef_F0_to_Fn = [-1, -2, 2, 1, -2, -2, -5, 8]

positive_feedback = [0, 0, 3, 4, 4, 6, 6, 10]

negative_feedback = [-1, -2, -1, -3, -6, -8, -11, -2]

absolute_feedback = [1, 2, 4, 7, 10, 14, 17, 12]

wFn = [-1., -1., 0.50, 0.14, -0.20, -0.14, -0.29, 0.67]

"Criterion ii"

wD_6 = -0.0035

ratio_to_model_C = -190.

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} -4 & 0 & 0 & 2 & -1 & -1 & -1 \\ 4 & 0 & 0 & 2 & -1 & -1 & -1 \\ 0 & -8 & 0 & 4 & -2 & 6 & -2 \\ -4 & 0 & 0 & 2 & 3 & 3 & -5 \\ 4 & 0 & 8 & -2 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -4 & 4 & -4 \\ 0 & 0 & 0 & -4 & 2 & -6 & 2 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 6 & 0 & 0 & 2 & 1 & 3 & 1 \\ 6 & 0 & 0 & 2 & 1 & 3 & 1 \\ 6 & 12 & 0 & 4 & 2 & 6 & 2 \\ 8 & 0 & 0 & 4 & 7 & 5 & 5 \\ 6 & 0 & 12 & 2 & 1 & 3 & 1 \\ 6 & 0 & 0 & 4 & 4 & 4 & 4 \\ 6 & 0 & 0 & 4 & 2 & 6 & 2 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 0.67 & 1. & 1. & 1. & 1. & 0.33 & 1. \\ 0.67 & 1. & 1. & 1. & 1. & 0.33 & 1. \\ 0. & 0.67 & 1. & 1. & 1. & 1. & 1. \\ 0.50 & 1. & 1. & 0.50 & 0.43 & 0.60 & 1. \\ 0.67 & 1. & 0.67 & 1. & 1. & 0.33 & 1. \\ 0. & 1. & 1. & 0. & 1. & 1. & 1. \\ 0. & 1. & 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 4 & 0 & 0 & -2 & 1 & 1 & 1 \\ 0 & 8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 2 & 2 & 2 \\ 4 & 0 & 0 & -2 & 5 & -3 & 5 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 8 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 6 & 0 & 0 & 2 & 1 & 3 & 1 \\ 0 & 12 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 8 & 2 & 6 & 2 \\ 8 & 0 & 0 & 4 & 5 & 5 & 5 \\ 0 & 0 & 0 & 0 & 0 & 12 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 12 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 0.67 & 1. & 1. & 1. & 1. & 0.33 & 1. \\ 1. & 0.67 & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 0. & 1. & 1. & 0.50 & 1. & 0.33 & 1. \\ 0.50 & 1. & 1. & 0.50 & 1. & 0.60 & 1. \\ 1. & 1. & 1. & 1. & 1. & 0.67 & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 0.67 \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} -4 & 0 & 0 & -2 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4 & 2 & 2 & 2 \\ 4 & 0 & 0 & -2 & -3 & -3 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 6 & 0 & 0 & 2 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 12 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 4 & 2 & 6 & 2 \\ 8 & 0 & 0 & 4 & 7 & 5 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 0.67 & 1. & 1. & 1. & 1. & 0.33 & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 0.67 & 1. & 1. & 1. & 1. \\ 0. & 1. & 1. & 1. & 1. & 0.33 & 1. \\ 0.50 & 1. & 1. & 0.50 & 0.43 & 0.60 & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} -a_{1,1} & a_{1,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -a_{2,3} & 0 & 0 & 0 & -a_{2,7} \\ a_{3,1} & 0 & 0 & 0 & a_{3,5} & 0 & 0 \\ a_{4,1} & a_{4,2} & 0 & 0 & 0 & -a_{4,6} & -a_{4,7} \\ -a_{5,1} & 0 & 0 & a_{5,4} & 0 & -a_{5,6} & 0 \\ -a_{6,1} & -a_{6,2} & 0 & 0 & 0 & 0 & -a_{6,7} \\ 0 & -a_{7,2} & 0 & -a_{7,4} & 0 & -a_{7,6} & -a_{7,7} \end{bmatrix}$$

"system feedback"

F_7

$$a_{7,2}\,a_{1,1}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,6}\,a_{6,7} + a_{7,4}\,a_{1,1}\,a_{4,2}\,a_{2,3}\,a_{3,5}\,a_{5,6}\,a_{6,7} + a_{7,4}\,a_{1,1}\,a_{6,2}\,a_{2,3}\,a_{3,5}\,a_{4,7}\,a_{5,6}$$
$$+ a_{7,4}\,a_{4,1}\,a_{1,2}\,a_{2,3}\,a_{3,5}\,a_{5,6}\,a_{6,7} + a_{5,1}\,a_{1,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{4,6}\,a_{6,7} + a_{7,4}\,a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{3,5}\,a_{4,7}\,a_{5,6}$$
$$+ a_{7,6}\,a_{1,1}\,a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{6,7} + a_{7,6}\,a_{1,1}\,a_{6,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,7} + a_{7,6}\,a_{4,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{6,7}$$
$$+ a_{7,6}\,a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,7} - a_{7,7}\,a_{1,1}\,a_{6,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,6} - a_{7,7}\,a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,6}$$

F_6

$$a_{4,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{5,6}\,a_{6,7} - a_{6,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,6}\,a_{7,7} + a_{6,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,7}\,a_{7,6} + a_{6,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{4,7}\,a_{5,6}$$
$$+ a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,6}\,a_{6,7} + a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{6,7}\,a_{7,6} - a_{1,1}\,a_{6,2}\,a_{2,3}\,a_{3,5}\,a_{5,6}\,a_{7,7} + a_{1,1}\,a_{7,2}\,a_{2,3}\,a_{3,5}\,a_{5,6}\,a_{6,7}$$
$$- a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{3,5}\,a_{5,6}\,a_{7,7} - a_{5,1}\,a_{1,2}\,a_{2,3}\,a_{3,5}\,a_{6,7}\,a_{7,6} - a_{3,1}\,a_{1,2}\,a_{2,3}\,a_{7,4}\,a_{4,6}\,a_{6,7} - a_{5,1}\,a_{1,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{4,7}$$
$$- a_{1,1}\,a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{7,7} - a_{1,1}\,a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,7} - a_{4,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{7,7} - a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,6}$$
$$- a_{1,1}\,a_{6,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,6}$$

F_5

$$-a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,7} + a_{7,2}\,a_{2,3}\,a_{3,5}\,a_{5,6}\,a_{6,7} + a_{7,4}\,a_{4,6}\,a_{6,1}\,a_{1,2}\,a_{2,7} + a_{1,1}\,a_{7,4}\,a_{4,6}\,a_{6,2}\,a_{2,7} + a_{3,1}\,a_{1,2}\,a_{2,3}\,a_{7,4}\,a_{4,7}$$
$$+ a_{3,1}\,a_{1,2}\,a_{2,3}\,a_{6,7}\,a_{7,6} - a_{6,2}\,a_{2,3}\,a_{3,5}\,a_{5,6}\,a_{7,7} + a_{7,7}\,a_{5,1}\,a_{1,2}\,a_{2,3}\,a_{3,5} - a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{7,7} - a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{3,5}\,a_{5,6}$$
$$- a_{6,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,6} - a_{6,2}\,a_{2,3}\,a_{3,5}\,a_{5,6}\,a_{1,1} - a_{5,4}\,a_{4,1}\,a_{1,2}\,a_{2,3}\,a_{3,5} - a_{1,1}\,a_{5,4}\,a_{4,2}\,a_{2,3}\,a_{3,5}$$

F_4

$$a_{7,4}\,a_{4,1}\,a_{1,2}\,a_{2,7} + a_{7,4}\,a_{4,6}\,a_{6,2}\,a_{2,7} - a_{7,6}\,a_{6,1}\,a_{1,2}\,a_{2,7} + a_{1,1}\,a_{7,4}\,a_{4,2}\,a_{2,7} - a_{1,1}\,a_{7,4}\,a_{4,6}\,a_{6,7} - a_{1,1}\,a_{7,6}\,a_{6,2}\,a_{2,7}$$
$$- a_{7,7}\,a_{3,1}\,a_{1,2}\,a_{2,3} - a_{6,2}\,a_{2,3}\,a_{3,5}\,a_{5,6} + a_{5,1}\,a_{1,2}\,a_{2,3}\,a_{3,5} - a_{5,4}\,a_{4,2}\,a_{2,3}\,a_{3,5}$$

F_3

$$a_{7,4}\,a_{4,2}\,a_{2,7}-a_{7,4}\,a_{4,6}\,a_{6,7}-a_{7,6}\,a_{6,2}\,a_{2,7}+a_{1,1}\,a_{7,2}\,a_{2,7}+a_{1,1}\,a_{7,4}\,a_{4,7}+a_{1,1}\,a_{6,7}\,a_{7,6}-a_{3,1}\,a_{1,2}\,a_{2,3}$$

$$F_2$$

$$a_{7,2}\,a_{2,7}+a_{7,4}\,a_{4,7}+a_{6,7}\,a_{7,6}-a_{7,7}\,a_{1,1}$$

$$F_1$$

$$-a_{7,7}-a_{1,1}$$

$$F_0$$

-1

"adjoint (-A)"

$$\begin{bmatrix}
-a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{6,7}\,a_{7,6}-a_{4,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{5,6}\,a_{6,7}+a_{6,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,6}\,a_{7,7}-a_{6,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,7}\,a_{7,6} \\
-a_{6,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{4,7}\,a_{5,6}-a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,6}\,a_{6,7}\,,\,0\,,\,0\,,\,a_{1,2}\,a_{2,3}\,(a_{5,4}\,a_{3,5}\,a_{6,7}\,a_{7,6}+a_{7,4}\,a_{3,5}\,a_{5,6}\,a_{6,7})\,, \\
-a_{1,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{4,6}\,a_{6,7}\,,\,a_{1,2}\,a_{2,3}\,(a_{5,4}\,a_{3,5}\,a_{4,6}\,a_{7,7}-a_{5,4}\,a_{3,5}\,a_{4,7}\,a_{7,6}-a_{7,4}\,a_{3,5}\,a_{4,7}\,a_{5,6})\,,-a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,6}\,a_{6,7}\,] \\
[\,a_{4,1}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{6,7}\,a_{7,6}+a_{4,1}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{5,6}\,a_{6,7}+a_{5,1}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{4,6}\,a_{6,7}-a_{6,1}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,6}\,a_{7,7} \\
+a_{6,1}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,7}\,a_{7,6}+a_{6,1}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{4,7}\,a_{5,6}\,,\,0\,,\,0\,,\,a_{1,1}\,a_{2,3}\,(a_{5,4}\,a_{3,5}\,a_{6,7}\,a_{7,6}+a_{7,4}\,a_{3,5}\,a_{5,6}\,a_{6,7})\,, \\
-a_{1,1}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{4,6}\,a_{6,7}\,,\,a_{1,1}\,a_{2,3}\,(a_{5,4}\,a_{3,5}\,a_{4,6}\,a_{7,7}-a_{5,4}\,a_{3,5}\,a_{4,7}\,a_{7,6}-a_{7,4}\,a_{3,5}\,a_{4,7}\,a_{5,6})\,,-a_{1,1}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,6}\,a_{6,7}\,] \\
[\,a_{4,1}\,a_{6,2}\,a_{5,4}\,a_{3,5}\,a_{2,7}\,a_{7,6}+a_{4,1}\,a_{6,2}\,a_{7,4}\,a_{3,5}\,a_{2,7}\,a_{5,6}+a_{5,1}\,a_{6,2}\,a_{7,4}\,a_{3,5}\,a_{2,7}\,a_{4,6}-a_{6,1}\,a_{4,2}\,a_{5,4}\,a_{3,5}\,a_{2,7}\,a_{7,6} \\
-a_{6,1}\,a_{4,2}\,a_{7,4}\,a_{3,5}\,a_{2,7}\,a_{5,6}-a_{6,1}\,a_{7,2}\,a_{5,4}\,a_{3,5}\,a_{2,7}\,a_{4,6}\,,-a_{1,1}\,a_{4,2}\,a_{5,4}\,a_{3,5}\,a_{6,7}\,a_{7,6}-a_{1,1}\,a_{4,2}\,a_{7,4}\,a_{3,5}\,a_{5,6}\,a_{6,7} \\
+a_{1,1}\,a_{6,2}\,a_{5,4}\,a_{3,5}\,a_{4,6}\,a_{7,7}-a_{1,1}\,a_{6,2}\,a_{5,4}\,a_{3,5}\,a_{4,7}\,a_{7,6}-a_{1,1}\,a_{6,2}\,a_{7,4}\,a_{3,5}\,a_{4,7}\,a_{5,6}-a_{1,1}\,a_{7,2}\,a_{5,4}\,a_{3,5}\,a_{4,6}\,a_{6,7} \\
-a_{4,1}\,a_{1,2}\,a_{5,4}\,a_{3,5}\,a_{6,7}\,a_{7,6}-a_{4,1}\,a_{1,2}\,a_{7,4}\,a_{3,5}\,a_{5,6}\,a_{6,7}-a_{5,1}\,a_{1,2}\,a_{7,4}\,a_{3,5}\,a_{4,6}\,a_{6,7}+a_{6,1}\,a_{1,2}\,a_{5,4}\,a_{3,5}\,a_{4,6}\,a_{7,7} \\
-a_{6,1}\,a_{1,2}\,a_{5,4}\,a_{3,5}\,a_{4,7}\,a_{7,6}-a_{6,1}\,a_{1,2}\,a_{7,4}\,a_{3,5}\,a_{4,7}\,a_{5,6}\,,\,0\,, \\
a_{1,1}\,a_{6,2}\,a_{5,4}\,a_{3,5}\,a_{2,7}\,a_{7,6}+a_{1,1}\,a_{6,2}\,a_{7,4}\,a_{3,5}\,a_{2,7}\,a_{5,6}+a_{6,1}\,a_{1,2}\,a_{5,4}\,a_{3,5}\,a_{2,7}\,a_{7,6}+a_{6,1}\,a_{1,2}\,a_{7,4}\,a_{3,5}\,a_{2,7}\,a_{5,6}\,, \\
-a_{1,1}\,a_{6,2}\,a_{7,4}\,a_{3,5}\,a_{2,7}\,a_{4,6}-a_{6,1}\,a_{1,2}\,a_{7,4}\,a_{3,5}\,a_{2,7}\,a_{4,6}\,,\,a_{1,1}\,a_{4,2}\,a_{5,4}\,a_{3,5}\,a_{2,7}\,a_{7,6}+a_{1,1}\,a_{4,2}\,a_{7,4}\,a_{3,5}\,a_{2,7}\,a_{5,6} \\
+a_{1,1}\,a_{7,2}\,a_{5,4}\,a_{3,5}\,a_{2,7}\,a_{4,6}+a_{4,1}\,a_{1,2}\,a_{5,4}\,a_{3,5}\,a_{2,7}\,a_{7,6}+a_{4,1}\,a_{1,2}\,a_{7,4}\,a_{3,5}\,a_{2,7}\,a_{5,6}+a_{5,1}\,a_{1,2}\,a_{7,4}\,a_{3,5}\,a_{2,7}\,a_{4,6}\,, \\
-a_{1,1}\,a_{6,2}\,a_{5,4}\,a_{3,5}\,a_{2,7}\,a_{4,6}-a_{6,1}\,a_{1,2}\,a_{5,4}\,a_{3,5}\,a_{2,7}\,a_{4,6}\,] \\
[\,a_{4,1}\,a_{6,2}\,a_{2,3}\,a_{3,5}\,a_{5,6}\,a_{7,7}-a_{4,1}\,a_{7,2}\,a_{2,3}\,a_{3,5}\,a_{5,6}\,a_{6,7}-a_{5,1}\,a_{4,2}\,a_{2,3}\,a_{3,5}\,a_{6,7}\,a_{7,6}+a_{5,1}\,a_{6,2}\,a_{2,3}\,a_{3,5}\,a_{4,6}\,a_{7,7} \\
-a_{5,1}\,a_{6,2}\,a_{2,3}\,a_{3,5}\,a_{4,7}\,a_{7,6}-a_{5,1}\,a_{7,2}\,a_{2,3}\,a_{3,5}\,a_{4,6}\,a_{6,7}-a_{6,1}\,a_{4,2}\,a_{2,3}\,a_{3,5}\,a_{5,6}\,a_{7,7}-a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{3,5}\,a_{4,7}\,a_{5,6}\,,\,0\,,\,0\,, \\
a_{1,1}\,a_{6,2}\,a_{2,3}\,a_{3,5}\,a_{5,6}\,a_{7,7}-a_{1,1}\,a_{7,2}\,a_{2,3}\,a_{3,5}\,a_{5,6}\,a_{6,7}+a_{5,1}\,a_{1,2}\,a_{2,3}\,a_{3,5}\,a_{6,7}\,a_{7,6}+a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{3,5}\,a_{5,6}\,a_{7,7}\,, \\
a_{1,1}\,a_{4,2}\,a_{2,3}\,a_{3,5}\,a_{6,7}\,a_{7,6}-a_{1,1}\,a_{6,2}\,a_{2,3}\,a_{3,5}\,a_{4,6}\,a_{7,7}+a_{1,1}\,a_{6,2}\,a_{2,3}\,a_{3,5}\,a_{4,7}\,a_{7,6}+a_{1,1}\,a_{7,2}\,a_{2,3}\,a_{3,5}\,a_{4,6}\,a_{6,7} \\
+a_{4,1}\,a_{1,2}\,a_{2,3}\,a_{3,5}\,a_{6,7}\,a_{7,6}-a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{3,5}\,a_{4,6}\,a_{7,7}+a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{3,5}\,a_{4,7}\,a_{7,6}\,,\,a_{1,1}\,a_{4,2}\,a_{2,3}\,a_{3,5}\,a_{5,6}\,a_{7,7} \\
+a_{1,1}\,a_{7,2}\,a_{2,3}\,a_{3,5}\,a_{4,7}\,a_{5,6}+a_{4,1}\,a_{1,2}\,a_{2,3}\,a_{3,5}\,a_{5,6}\,a_{7,7}+a_{5,1}\,a_{1,2}\,a_{2,3}\,a_{3,5}\,a_{4,6}\,a_{7,7}-a_{5,1}\,a_{1,2}\,a_{2,3}\,a_{3,5}\,a_{4,7}\,a_{7,6}\,, \\
-a_{1,1}\,a_{4,2}\,a_{2,3}\,a_{3,5}\,a_{5,6}\,a_{6,7}-a_{1,1}\,a_{6,2}\,a_{2,3}\,a_{3,5}\,a_{4,7}\,a_{5,6}-a_{4,1}\,a_{1,2}\,a_{2,3}\,a_{3,5}\,a_{5,6}\,a_{6,7}-a_{5,1}\,a_{1,2}\,a_{2,3}\,a_{3,5}\,a_{4,6}\,a_{6,7} \\
-a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{3,5}\,a_{4,7}\,a_{5,6}\,] \\
[\,a_{3,1}\,(a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{6,7}\,a_{7,6}+a_{4,2}\,a_{2,3}\,a_{7,4}\,a_{5,6}\,a_{6,7}-a_{6,2}\,a_{2,3}\,a_{5,4}\,a_{4,6}\,a_{7,7}+a_{6,2}\,a_{2,3}\,a_{5,4}\,a_{4,7}\,a_{7,6}+a_{6,2}\,a_{2,3}\,a_{7,4}\,a_{4,7}\,a_{5,6} \\
+a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{4,6}\,a_{6,7})\,,\,0\,,\,a_{1,1}\,a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{6,7}\,a_{7,6}+a_{1,1}\,a_{4,2}\,a_{2,3}\,a_{7,4}\,a_{5,6}\,a_{6,7}-a_{1,1}\,a_{6,2}\,a_{2,3}\,a_{5,4}\,a_{4,6}\,a_{7,7} \\
+a_{1,1}\,a_{6,2}\,a_{2,3}\,a_{5,4}\,a_{4,7}\,a_{7,6}+a_{1,1}\,a_{6,2}\,a_{2,3}\,a_{7,4}\,a_{4,7}\,a_{5,6}+a_{1,1}\,a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{4,6}\,a_{6,7}+a_{4,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{6,7}\,a_{7,6} \\
+a_{4,1}\,a_{1,2}\,a_{2,3}\,a_{7,4}\,a_{5,6}\,a_{6,7}+a_{5,1}\,a_{1,2}\,a_{2,3}\,a_{7,4}\,a_{4,6}\,a_{6,7}-a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{4,6}\,a_{7,7}+a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{4,7}\,a_{7,6} \\
+a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{7,4}\,a_{4,7}\,a_{5,6}\,,-a_{3,1}\,a_{1,2}\,a_{2,3}\,(a_{5,4}\,a_{6,7}\,a_{7,6}+a_{7,4}\,a_{5,6}\,a_{6,7})\,,\,a_{3,1}\,a_{1,2}\,a_{2,3}\,a_{7,4}\,a_{4,6}\,a_{6,7}\,, \\
-a_{3,1}\,a_{1,2}\,a_{2,3}\,(a_{5,4}\,a_{4,6}\,a_{7,7}-a_{5,4}\,a_{4,7}\,a_{7,6}-a_{7,4}\,a_{4,7}\,a_{5,6})\,,\,a_{3,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{4,6}\,a_{6,7}\,] \\
[\,a_{4,1}\,a_{6,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{7,7}-a_{4,1}\,a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{6,7}+a_{5,1}\,a_{4,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{6,7}+a_{5,1}\,a_{6,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{4,7} \\
-a_{6,1}\,a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{7,7}-a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,7}\,,\,0\,,\,0\,, \\
a_{1,1}\,a_{6,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{7,7}-a_{1,1}\,a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{6,7}-a_{5,1}\,a_{1,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{6,7}+a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{7,7}\,, \\
-a_{1,1}\,a_{4,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{6,7}-a_{1,1}\,a_{6,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{4,7}-a_{4,1}\,a_{1,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{6,7}-a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{4,7}\,, \\
a_{1,1}\,a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{7,7}+a_{1,1}\,a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,7}+a_{4,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{7,7}+a_{5,1}\,a_{1,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{4,7}\,, \\
-a_{1,1}\,a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{6,7}-a_{1,1}\,a_{6,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,7}-a_{4,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{6,7}-a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,7}\,] \\
[\,-a_{4,1}\,a_{6,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{7,6}-a_{4,1}\,a_{6,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{5,6}-a_{5,1}\,a_{6,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{4,6}+a_{6,1}\,a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{7,6} \\
+a_{6,1}\,a_{4,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{5,6}+a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,6}\,,\,0\,,\,0\,, \\
-a_{1,1}\,a_{6,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{7,6}-a_{1,1}\,a_{6,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{5,6}-a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{7,6}-a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{5,6}\,,
\end{bmatrix}$$

$$a_{1,1}\,a_{6,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{4,6} + a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{4,6}\,,\; -a_{1,1}\,a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{7,6} - a_{1,1}\,a_{4,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{5,6} - a_{1,1}\,a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,6} - a_{4,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{7,6} - a_{4,1}\,a_{1,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{5,6} - a_{5,1}\,a_{1,2}\,a_{2,3}\,a_{7,4}\,a_{3,5}\,a_{4,6}\,,\; a_{1,1}\,a_{6,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,6} + a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{3,5}\,a_{4,6}\,]$$

"74"

$$A := \begin{bmatrix} 0 & 0 & -1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 \\ 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

poly_coef_F0_to_Fn = [-1, 0, 0, 0, -2, 2, -2, -2]

positive_feedback = [0, 1, 1, 3, 3, 6, 3, 0]

negative_feedback = [-1, -1, -1, -3, -5, -4, -5, -2]

absolute_feedback = [1, 2, 2, 6, 8, 10, 8, 2]

wFn = [-1., 0., 0., 0., -0.25, 0.20, -0.25, -1.]

"Criterion ii"

$wD_6 = 0.00018$

ratio_to_model_C = 9.8

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} -1 & 1 & -2 & 0 & -1 & -2 & 1 \\ 0 & 0 & 0 & 2 & 0 & -2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & -2 & 2 \\ 1 & -1 & 0 & 0 & 1 & 0 & 1 \\ 1 & -1 & 0 & 0 & -1 & -2 & 3 \\ 2 & 0 & 0 & 0 & 0 & -2 & 4 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 1 & 1 & 2 & 0 & 1 & 4 & 5 \\ 0 & 0 & 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 \\ 1 & 1 & 0 & 0 & 1 & 2 & 3 \\ 1 & 1 & 0 & 0 & 1 & 2 & 3 \\ 2 & 0 & 0 & 0 & 0 & 2 & 4 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. & 0.50 & 0.20 \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 0. & 0.33 \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} -2 & 0 & 0 & 0 & 0 & 2 & -2 \\ -2 & 0 & 0 & 0 & 0 & 2 & -4 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & -2 & 0 & 2 & -2 \\ -1 & 1 & 0 & 0 & -1 & 2 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 2 & 2 \\ 2 & 0 & 0 & 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 2 & 0 & 2 & 2 \\ 1 & 1 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 2 & -2 \\ -2 & 2 & 0 & 0 & 0 & 2 & -4 \\ 0 & 0 & 2 & 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 \\ -1 & 1 & 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 2 & 2 \\ 2 & 2 & 0 & 0 & 0 & 2 & 4 \\ 0 & 0 & 2 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 \\ 1 & 1 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & 0 & -a_{1,3} & a_{1,4} & 0 & 0 & -a_{1,7} \\ 0 & 0 & 0 & 0 & a_{2,5} & a_{2,6} & -a_{2,7} \\ a_{3,1} & 0 & 0 & -a_{3,4} & a_{3,5} & 0 & 0 \\ 0 & -a_{4,2} & 0 & a_{4,4} & 0 & 0 & 0 \\ 0 & 0 & 0 & -a_{5,4} & -a_{5,5} & a_{5,6} & 0 \\ 0 & 0 & a_{6,3} & a_{6,4} & 0 & 0 & 0 \\ 0 & 0 & a_{7,3} & 0 & 0 & 0 & 0 \end{bmatrix}$$

"system feedback"

$$F_7$$

$$-a_{5,5}\,a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{1,7}\,a_{2,6} - a_{2,5}\,a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{1,7}\,a_{5,6}$$

$$F_6$$

$$-a_{6,3}\,a_{3,1}\,a_{1,4}\,a_{4,2}\,a_{2,5}\,a_{5,6} - a_{6,4}\,a_{4,2}\,a_{2,6}\,a_{5,5}\,a_{3,1}\,a_{1,3} - a_{6,4}\,a_{4,2}\,a_{2,5}\,a_{5,6}\,a_{3,1}\,a_{1,3} - a_{6,3}\,a_{3,1}\,a_{1,4}\,a_{4,2}\,a_{2,6}\,a_{5,5}$$
$$+ a_{7,3}\,a_{3,1}\,a_{1,4}\,a_{4,2}\,a_{2,7}\,a_{5,5} + a_{7,3}\,a_{3,1}\,a_{1,7}\,a_{5,4}\,a_{4,2}\,a_{2,5} + a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{3,5}\,a_{2,7}\,a_{5,6} - a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{1,7}\,a_{2,6}$$

$$F_5$$

$$a_{7,3}\,a_{3,1}\,a_{1,4}\,a_{4,2}\,a_{2,7} - a_{7,3}\,a_{3,5}\,a_{5,4}\,a_{4,2}\,a_{2,7} - a_{7,3}\,a_{3,4}\,a_{4,2}\,a_{2,7}\,a_{5,5} + a_{7,3}\,a_{3,1}\,a_{1,7}\,a_{5,5}\,a_{4,4} - a_{6,3}\,a_{3,1}\,a_{1,4}\,a_{4,2}\,a_{2,6}$$
$$+ a_{6,3}\,a_{3,4}\,a_{4,2}\,a_{2,5}\,a_{5,6} + a_{6,3}\,a_{3,5}\,a_{5,4}\,a_{4,2}\,a_{2,6} + a_{6,3}\,a_{3,4}\,a_{4,2}\,a_{2,6}\,a_{5,5} - a_{6,4}\,a_{4,2}\,a_{2,6}\,a_{3,1}\,a_{1,3} + a_{5,4}\,a_{4,2}\,a_{2,5}\,a_{3,1}\,a_{1,3}$$

$$F_4$$

$$-a_{7,3}\,a_{3,4}\,a_{4,2}\,a_{2,7} - a_{7,3}\,a_{3,1}\,a_{1,7}\,a_{5,5} + a_{7,3}\,a_{3,1}\,a_{1,7}\,a_{4,4} + a_{6,3}\,a_{3,4}\,a_{4,2}\,a_{2,6} - a_{6,4}\,a_{4,2}\,a_{2,5}\,a_{5,6} - a_{6,3}\,a_{3,5}\,a_{5,6}\,a_{4,4}$$
$$- a_{6,4}\,a_{4,2}\,a_{2,6}\,a_{5,5} + a_{5,5}\,a_{4,4}\,a_{3,1}\,a_{1,3}$$

$$F_3$$

$$-a_{7,3}\,a_{3,1}\,a_{1,7} + a_{6,3}\,a_{3,5}\,a_{5,6} - a_{6,4}\,a_{4,2}\,a_{2,6} + a_{5,4}\,a_{4,2}\,a_{2,5} - a_{5,5}\,a_{3,1}\,a_{1,3} + a_{4,4}\,a_{3,1}\,a_{1,3}$$

$$F_2$$

$$a_{5,5}\,a_{4,4} - a_{3,1}\,a_{1,3}$$

$$F_1$$

$$-a_{5,5} + a_{4,4}$$

$$F_0$$

-1

"adjoint (-A)"

$$[-a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{3,5}\,a_{2,7}\,a_{5,6},\ a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{3,5}\,a_{1,7}\,a_{5,6},\ -a_{4,2}\,a_{7,3}\,a_{6,4}\,(a_{2,5}\,a_{1,7}\,a_{5,6} + a_{5,5}\,a_{1,7}\,a_{2,6}),\ 0,$$
$$-a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{3,5}\,a_{1,7}\,a_{2,6},\ -a_{4,2}\,a_{7,3}\,(-a_{1,4}\,a_{3,5}\,a_{2,7}\,a_{5,6} + a_{3,4}\,a_{2,5}\,a_{1,7}\,a_{5,6} + a_{3,4}\,a_{5,5}\,a_{1,7}\,a_{2,6} + a_{5,4}\,a_{3,5}\,a_{1,7}\,a_{2,6}),$$
$$-a_{4,2}\,(a_{1,3}\,a_{6,4}\,a_{3,5}\,a_{2,7}\,a_{5,6} + a_{6,3}\,a_{1,4}\,a_{3,5}\,a_{2,7}\,a_{5,6} - a_{6,3}\,a_{3,4}\,a_{2,5}\,a_{1,7}\,a_{5,6} - a_{6,3}\,a_{3,4}\,a_{5,5}\,a_{1,7}\,a_{2,6} - a_{6,3}\,a_{5,4}\,a_{3,5}\,a_{1,7}\,a_{2,6})]$$
$$[0,\ 0,\ 0,\ a_{3,1}\,a_{7,3}\,a_{6,4}\,(a_{2,5}\,a_{1,7}\,a_{5,6} + a_{5,5}\,a_{1,7}\,a_{2,6}),\ 0,\ -a_{3,1}\,a_{7,3}\,a_{4,4}\,(a_{2,5}\,a_{1,7}\,a_{5,6} + a_{5,5}\,a_{1,7}\,a_{2,6}),$$
$$a_{3,1}\,a_{6,3}\,a_{4,4}\,(a_{2,5}\,a_{1,7}\,a_{5,6} + a_{5,5}\,a_{1,7}\,a_{2,6})]$$
$$[0,\ 0,\ 0,\ 0,\ 0,\ 0,\ -a_{3,1}\,a_{4,2}\,a_{6,4}\,(a_{2,5}\,a_{1,7}\,a_{5,6} + a_{5,5}\,a_{1,7}\,a_{2,6})]$$
$$[0,\ 0,\ 0,\ 0,\ 0,\ -a_{3,1}\,a_{4,2}\,a_{7,3}\,(a_{2,5}\,a_{1,7}\,a_{5,6} + a_{5,5}\,a_{1,7}\,a_{2,6}),\ a_{3,1}\,a_{4,2}\,a_{6,3}\,(a_{2,5}\,a_{1,7}\,a_{5,6} + a_{5,5}\,a_{1,7}\,a_{2,6})]$$
$$[a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{2,7}\,a_{5,6},\ -a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{1,7}\,a_{5,6},\ 0,\ 0,\ a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{1,7}\,a_{2,6},$$
$$a_{3,1}\,a_{4,2}\,a_{7,3}\,(-a_{1,4}\,a_{2,7}\,a_{5,6} + a_{5,4}\,a_{1,7}\,a_{2,6}),\ -a_{3,1}\,a_{4,2}\,(-a_{1,3}\,a_{6,4}\,a_{2,7}\,a_{5,6} - a_{6,3}\,a_{1,4}\,a_{2,7}\,a_{5,6} + a_{6,3}\,a_{5,4}\,a_{1,7}\,a_{2,6})]$$
$$[a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{2,7}\,a_{5,5},\ -a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{1,7}\,a_{5,5},\ 0,\ 0,\ -a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{1,7}\,a_{2,5},$$
$$-a_{3,1}\,a_{4,2}\,a_{7,3}\,(a_{1,4}\,a_{2,7}\,a_{5,5} + a_{5,4}\,a_{1,7}\,a_{2,5}),\ a_{3,1}\,a_{4,2}\,(a_{1,3}\,a_{6,4}\,a_{2,7}\,a_{5,5} + a_{6,3}\,a_{1,4}\,a_{2,7}\,a_{5,5} + a_{6,3}\,a_{5,4}\,a_{1,7}\,a_{2,5})]$$
$$[a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,(a_{2,5}\,a_{5,6} + a_{2,6}\,a_{5,5}),\ 0,\ 0,\ 0,\ 0,\ -a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{1,4}\,(a_{2,5}\,a_{5,6} + a_{2,6}\,a_{5,5}),$$
$$-a_{3,1}\,a_{4,2}\,(-a_{1,3}\,a_{6,4}\,a_{2,5}\,a_{5,6} - a_{1,3}\,a_{6,4}\,a_{2,6}\,a_{5,5} - a_{6,3}\,a_{1,4}\,a_{2,5}\,a_{5,6} - a_{6,3}\,a_{1,4}\,a_{2,6}\,a_{5,5})]$$

"75"

$$A := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

$poly\_coef\_F0\_to\_Fn = [\,-1,\ -1,\ 0,\ 2,\ 1,\ -5,\ 1,\ 1\,]$

$positive\_feedback = [\,0,\ 0,\ 1,\ 4,\ 3,\ 0,\ 5,\ 2\,]$

$negative\_feedback = [\,-1,\ -1,\ -1,\ -2,\ -2,\ -5,\ -4,\ -1\,]$

$absolute\_feedback = [\,1,\ 1,\ 2,\ 6,\ 5,\ 5,\ 9,\ 3\,]$

$wFn = [\,-1.,\ -1.,\ 0.,\ 0.33,\ 0.20,\ -1.,\ 0.11,\ 0.33\,]$

"Criterion ii"

$wD_6 = 0.0025$

$ratio\_to\_model\_C = 130.$

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 1 & 0 & 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & -1 & -1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 0 & 0 & -1 & 1 \\ -1 & 1 & 0 & -1 & 1 & 1 & -1 \\ 1 & -1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 3 & 2 & 3 & 0 & 3 & 2 & 2 \\ 3 & 1 & 0 & 0 & 3 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 2 \\ 0 & 2 & 0 & 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 3 & 3 & 1 & 1 \\ 3 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 0.33 & 0. & 0.33 & 1. & 0.33 & 0. & 0. \\ 0.33 & 1. & 1. & 1. & 0.33 & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 0. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 0.33 & 1. & 1. & 0.33 & 0.33 & 1. & 1. \\ 0.33 & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 3 & 2 & 0 & 0 & 3 & 2 & 2 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 3 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 0.33 & 1. & 1. & 1. & 1. & 1. \\ 0.33 & 0. & 1. & 1. & 0.33 & 0. & 0. \\ 1. & 1. & 1. & 0.33 & 1. & 1. & 1. \\ 0.33 & 1. & 1. & 1. & 0.33 & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 2 & 3 & 0 & 3 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 0.33 & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 0.33 & 0. & 0.33 & 1. & 0.33 & 0. & 0. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 0.33 & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 0.33 & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 0. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & a_{1,6} & a_{1,7} \\ 0 & 0 & 0 & -a_{2,4} & 0 & 0 & -a_{2,7} \\ a_{3,1} & -a_{3,2} & 0 & 0 & 0 & 0 & -a_{3,7} \\ 0 & -a_{4,2} & 0 & 0 & -a_{4,5} & 0 & 0 \\ 0 & -a_{5,2} & 0 & 0 & 0 & a_{5,6} & 0 \\ 0 & 0 & a_{6,3} & a_{6,4} & 0 & 0 & a_{6,7} \\ 0 & 0 & a_{7,3} & 0 & 0 & 0 & -a_{7,7} \end{bmatrix}$$

"system feedback"

$$F_7$$

$$a_{7,7}\,a_{3,1}\,a_{5,2}\,a_{6,3}\,a_{2,4}\,a_{1,6}\,a_{4,5} + a_{2,4}\,a_{3,1}\,a_{5,2}\,a_{7,3}\,a_{4,5}\,a_{1,6}\,a_{6,7} - a_{2,7}\,a_{3,1}\,a_{5,2}\,a_{7,3}\,a_{6,4}\,a_{1,6}\,a_{4,5}$$

$$F_6$$

$$a_{7,3}\,a_{3,1}\,a_{1,6}\,a_{6,4}\,a_{4,2}\,a_{2,7} - a_{7,3}\,a_{3,2}\,a_{2,4}\,a_{4,5}\,a_{5,6}\,a_{6,7} - a_{7,3}\,a_{3,1}\,a_{1,6}\,a_{6,7}\,a_{4,2}\,a_{2,4} + a_{3,1}\,a_{7,3}\,a_{6,4}\,a_{4,5}\,a_{1,7}\,a_{5,6}$$
$$+ a_{3,1}\,a_{5,2}\,a_{7,3}\,a_{2,4}\,a_{1,7}\,a_{4,5} + a_{7,3}\,a_{3,2}\,a_{2,7}\,a_{6,4}\,a_{4,5}\,a_{5,6} - a_{7,7}\,a_{6,3}\,a_{3,2}\,a_{2,4}\,a_{4,5}\,a_{5,6} - a_{7,7}\,a_{4,2}\,a_{2,4}\,a_{6,3}\,a_{3,1}\,a_{1,6}$$
$$+ a_{3,1}\,a_{5,2}\,a_{6,3}\,a_{2,4}\,a_{1,6}\,a_{4,5}$$

$$F_5$$

$$-a_{7,3}\,a_{4,2}\,a_{2,4}\,a_{3,1}\,a_{1,7} - a_{7,3}\,a_{3,7}\,a_{6,4}\,a_{4,5}\,a_{5,6} - a_{7,3}\,a_{3,7}\,a_{5,2}\,a_{2,4}\,a_{4,5} - a_{6,3}\,a_{3,2}\,a_{2,4}\,a_{4,5}\,a_{5,6} - a_{4,2}\,a_{2,4}\,a_{6,3}\,a_{3,1}\,a_{1,6}$$

$$F_4$$

$$a_{7,3}\,a_{3,1}\,a_{1,6}\,a_{6,7} + a_{7,3}\,a_{3,7}\,a_{4,2}\,a_{2,4} + a_{7,7}\,a_{6,3}\,a_{3,1}\,a_{1,6} - a_{7,7}\,a_{6,4}\,a_{4,5}\,a_{5,6} - a_{7,7}\,a_{5,2}\,a_{2,4}\,a_{4,5}$$

$$F_3$$

$$a_{7,3}\,a_{3,1}\,a_{1,7} + a_{7,3}\,a_{3,2}\,a_{2,7} + a_{7,7}\,a_{4,2}\,a_{2,4} + a_{6,3}\,a_{3,1}\,a_{1,6} - a_{6,4}\,a_{4,5}\,a_{5,6} - a_{5,2}\,a_{2,4}\,a_{4,5}$$

$$F_2$$

$$-a_{7,3}\,a_{3,7} + a_{4,2}\,a_{2,4}$$

$$F_1$$

$$-a_{7,7}$$

$$F_0$$

$$-1$$

"adjoint (-A)"

$$[-a_{3,2}\,(-a_{6,3}\,a_{2,4}\,a_{4,5}\,a_{5,6}\,a_{7,7} - a_{7,3}\,a_{2,4}\,a_{4,5}\,a_{5,6}\,a_{6,7} + a_{7,3}\,a_{6,4}\,a_{4,5}\,a_{2,7}\,a_{5,6})\,,$$
$$-a_{3,2}\,a_{7,3}\,a_{6,4}\,a_{4,5}\,a_{1,7}\,a_{5,6} + a_{5,2}\,a_{7,3}\,a_{6,4}\,a_{4,5}\,a_{1,6}\,a_{3,7}\,,$$
$$-a_{5,2}\,(-a_{6,3}\,a_{2,4}\,a_{4,5}\,a_{1,6}\,a_{7,7} - a_{7,3}\,a_{2,4}\,a_{4,5}\,a_{1,6}\,a_{6,7} + a_{7,3}\,a_{6,4}\,a_{4,5}\,a_{1,6}\,a_{2,7})\,,\ 0\,,$$
$$a_{3,2}\,(-a_{6,3}\,a_{2,4}\,a_{4,5}\,a_{1,6}\,a_{7,7} - a_{7,3}\,a_{2,4}\,a_{4,5}\,a_{1,6}\,a_{6,7} + a_{7,3}\,a_{6,4}\,a_{4,5}\,a_{1,6}\,a_{2,7})\,,$$
$$-a_{3,2}\,a_{7,3}\,a_{2,4}\,a_{4,5}\,a_{1,7}\,a_{5,6} + a_{5,2}\,a_{7,3}\,a_{2,4}\,a_{4,5}\,a_{1,6}\,a_{3,7}\,,\ a_{3,2}\,a_{6,3}\,a_{2,4}\,a_{4,5}\,a_{1,7}\,a_{5,6} - a_{5,2}\,a_{6,3}\,a_{2,4}\,a_{4,5}\,a_{1,6}\,a_{3,7}]$$
$$[-a_{3,1}\,(-a_{6,3}\,a_{2,4}\,a_{4,5}\,a_{5,6}\,a_{7,7} - a_{7,3}\,a_{2,4}\,a_{4,5}\,a_{5,6}\,a_{6,7} + a_{7,3}\,a_{6,4}\,a_{4,5}\,a_{2,7}\,a_{5,6})\,,\ -a_{3,1}\,a_{7,3}\,a_{6,4}\,a_{4,5}\,a_{1,7}\,a_{5,6}\,,\ 0\,,\ 0\,,$$
$$a_{3,1}\,(-a_{6,3}\,a_{2,4}\,a_{4,5}\,a_{1,6}\,a_{7,7} - a_{7,3}\,a_{2,4}\,a_{4,5}\,a_{1,6}\,a_{6,7} + a_{7,3}\,a_{6,4}\,a_{4,5}\,a_{1,6}\,a_{2,7})\,,\ -a_{3,1}\,a_{7,3}\,a_{2,4}\,a_{4,5}\,a_{1,7}\,a_{5,6}\,,$$
$$a_{3,1}\,a_{6,3}\,a_{2,4}\,a_{4,5}\,a_{1,7}\,a_{5,6}]$$
$$[0\,,\ a_{3,1}\,a_{5,2}\,a_{6,4}\,a_{4,5}\,a_{1,6}\,a_{7,7}\,,\ 0\,,\ 0\,,\ 0\,,\ a_{3,1}\,a_{5,2}\,a_{2,4}\,a_{4,5}\,a_{1,6}\,a_{7,7}\,,\ a_{3,1}\,a_{5,2}\,(a_{2,4}\,a_{4,5}\,a_{1,6}\,a_{6,7} - a_{6,4}\,a_{4,5}\,a_{1,6}\,a_{2,7})]$$
$$[0\,,\ a_{3,1}\,a_{5,2}\,(-a_{6,3}\,a_{4,5}\,a_{1,6}\,a_{7,7} - a_{7,3}\,a_{4,5}\,a_{1,6}\,a_{6,7})\,,\ 0\,,\ 0\,,\ 0\,,\ -a_{3,1}\,a_{5,2}\,a_{7,3}\,a_{4,5}\,a_{1,6}\,a_{2,7}\,,\ a_{3,1}\,a_{5,2}\,a_{6,3}\,a_{4,5}\,a_{1,6}\,a_{2,7}]$$
$$[-a_{3,1}\,a_{4,2}\,(a_{6,3}\,a_{2,4}\,a_{5,6}\,a_{7,7} + a_{7,3}\,a_{2,4}\,a_{5,6}\,a_{6,7} - a_{7,3}\,a_{6,4}\,a_{2,7}\,a_{5,6})\,,\ a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{1,7}\,a_{5,6}\,,\ 0\,,$$
$$a_{3,1}\,a_{5,2}\,(-a_{6,3}\,a_{2,4}\,a_{1,6}\,a_{7,7} - a_{7,3}\,a_{2,4}\,a_{1,6}\,a_{6,7} + a_{7,3}\,a_{6,4}\,a_{1,6}\,a_{2,7})\,,$$
$$-a_{3,1}\,a_{4,2}\,(-a_{6,3}\,a_{2,4}\,a_{1,6}\,a_{7,7} - a_{7,3}\,a_{2,4}\,a_{1,6}\,a_{6,7} + a_{7,3}\,a_{6,4}\,a_{1,6}\,a_{2,7})\,,\ a_{3,1}\,a_{4,2}\,a_{7,3}\,a_{2,4}\,a_{1,7}\,a_{5,6}\,,\ -a_{3,1}\,a_{4,2}\,a_{6,3}\,a_{2,4}\,a_{1,7}\,a_{5,6}$$
$$]$$
$$[-a_{3,1}\,a_{5,2}\,(-a_{6,3}\,a_{2,4}\,a_{4,5}\,a_{7,7} - a_{7,3}\,a_{2,4}\,a_{4,5}\,a_{6,7} + a_{7,3}\,a_{6,4}\,a_{2,7}\,a_{4,5})\,,\ -a_{3,1}\,a_{5,2}\,a_{7,3}\,a_{6,4}\,a_{1,7}\,a_{4,5}\,,\ 0\,,\ 0\,,\ 0\,,$$
$$-a_{3,1}\,a_{5,2}\,a_{7,3}\,a_{2,4}\,a_{1,7}\,a_{4,5}\,,\ a_{3,1}\,a_{5,2}\,a_{6,3}\,a_{2,4}\,a_{1,7}\,a_{4,5}]$$
$$[0\,,\ a_{3,1}\,a_{5,2}\,a_{7,3}\,a_{6,4}\,a_{1,6}\,a_{4,5}\,,\ 0\,,\ 0\,,\ 0\,,\ a_{3,1}\,a_{5,2}\,a_{7,3}\,a_{2,4}\,a_{1,6}\,a_{4,5}\,,\ -a_{3,1}\,a_{5,2}\,a_{6,3}\,a_{2,4}\,a_{1,6}\,a_{4,5}]$$

"81"

$$A := \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & -1 & 0 & 0 & 1 & -1 & -1 \\ 0 & -1 & 0 & -1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & -1 & 0 & 0 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

$poly\_coef\_F0\_to\_Fn = [\,-1, -3, -3, -5, -9, 8, 20, 24, 25\,]$

$positive\_feedback = [\,0, 0, 3, 10, 24, 63, 94, 92, 50\,]$

$negative\_feedback = [\,-1, -3, -6, -15, -33, -55, -74, -68, -25\,]$

$absolute\_feedback = [\,1, 3, 9, 25, 57, 118, 168, 160, 75\,]$

$wFn = [\,-1., -1., -0.33, -0.20, -0.16, 0.068, 0.12, 0.15, 0.33\,]$

"Criterion ii"

$wD_7 = 0.30\ 10^{-6}$

$ratio\_to\_model\_C = 0.93$

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 3 & -9 & -9 & 15 & 2 & 13 & 12 & -10 \\ 19 & -7 & -7 & -5 & -4 & -1 & 1 & -5 \\ 12 & -11 & -11 & 10 & 8 & 2 & -2 & -15 \\ -2 & 6 & 6 & -10 & 7 & 8 & -8 & 15 \\ -1 & 3 & 3 & -5 & -9 & 4 & -4 & -5 \\ -11 & 8 & 8 & -5 & 1 & -6 & 6 & -5 \\ -6 & -7 & -7 & -5 & -4 & -1 & 1 & -5 \\ -17 & 26 & 1 & -10 & -3 & -7 & 7 & 15 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 37 & 21 & 21 & 27 & 30 & 43 & 32 & 40 \\ 45 & 13 & 13 & 19 & 14 & 11 & 11 & 19 \\ 18 & 13 & 13 & 12 & 14 & 10 & 10 & 25 \\ 28 & 18 & 18 & 24 & 31 & 20 & 20 & 31 \\ 17 & 11 & 11 & 11 & 15 & 12 & 12 & 25 \\ 17 & 12 & 12 & 13 & 15 & 10 & 10 & 25 \\ 30 & 13 & 13 & 19 & 14 & 11 & 11 & 19 \\ 33 & 38 & 37 & 22 & 23 & 17 & 17 & 37 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 0.081 & 0.43 & 0.43 & 0.56 & 0.067 & 0.30 & 0.38 & 0.25 \\ 0.42 & 0.54 & 0.54 & 0.26 & 0.29 & 0.091 & 0.091 & 0.26 \\ 0.67 & 0.85 & 0.85 & 0.83 & 0.57 & 0.20 & 0.20 & 0.60 \\ 0.071 & 0.33 & 0.33 & 0.42 & 0.23 & 0.40 & 0.40 & 0.48 \\ 0.059 & 0.27 & 0.27 & 0.45 & 0.60 & 0.33 & 0.33 & 0.20 \\ 0.65 & 0.67 & 0.67 & 0.38 & 0.067 & 0.60 & 0.60 & 0.20 \\ 0.20 & 0.54 & 0.54 & 0.26 & 0.29 & 0.091 & 0.091 & 0.26 \\ 0.52 & 0.68 & 0.027 & 0.45 & 0.13 & 0.41 & 0.41 & 0.41 \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 6 & 7 & 7 & 5 & 4 & 1 & -1 & 5 \\ 6 & 7 & 7 & 5 & 4 & 1 & -1 & 5 \\ 11 & -8 & 17 & 5 & -1 & 6 & -6 & 5 \\ 0 & 0 & 0 & 25 & 0 & 0 & 0 & 0 \\ 1 & -3 & -3 & 5 & 9 & -4 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -11 & 8 & 8 & -5 & 1 & -6 & 6 & 20 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 25 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 30 & 13 & 13 & 19 & 14 & 11 & 11 & 19 \\ 30 & 13 & 13 & 19 & 14 & 11 & 11 & 19 \\ 17 & 12 & 63 & 13 & 15 & 10 & 10 & 25 \\ 0 & 0 & 0 & 75 & 0 & 0 & 0 & 0 \\ 17 & 11 & 11 & 11 & 15 & 12 & 12 & 25 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 31 & 20 & 20 & 19 & 25 & 22 & 22 & 38 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 75 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 0.20 & 0.54 & 0.54 & 0.26 & 0.29 & 0.091 & 0.091 & 0.26 \\ 0.20 & 0.54 & 0.54 & 0.26 & 0.29 & 0.091 & 0.091 & 0.26 \\ 0.65 & 0.67 & 0.27 & 0.38 & 0.067 & 0.60 & 0.60 & 0.20 \\ 1. & 1. & 1. & 0.33 & 1. & 1. & 1. & 1. \\ 0.059 & 0.27 & 0.27 & 0.45 & 0.60 & 0.33 & 0.33 & 0.20 \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 0.35 & 0.40 & 0.40 & 0.26 & 0.040 & 0.27 & 0.27 & 0.53 \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 0.33 \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} -19 & 7 & 7 & 5 & 4 & 1 & -1 & 5 \\ 6 & -18 & 7 & 5 & 4 & 1 & -1 & 5 \\ 11 & -8 & -8 & 5 & -1 & 6 & -6 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -3 & -3 & 5 & -16 & -4 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 & -25 & 0 & 0 \\ -11 & 8 & 8 & -5 & 1 & -6 & -19 & 20 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 45 & 13 & 13 & 19 & 14 & 11 & 11 & 19 \\ 30 & 62 & 13 & 19 & 14 & 11 & 11 & 19 \\ 17 & 12 & 12 & 13 & 15 & 10 & 10 & 25 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 17 & 11 & 11 & 11 & 60 & 12 & 12 & 25 \\ 0 & 0 & 0 & 0 & 0 & 75 & 0 & 0 \\ 31 & 20 & 20 & 19 & 25 & 22 & 53 & 38 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 0.42 & 0.54 & 0.54 & 0.26 & 0.29 & 0.091 & 0.091 & 0.26 \\ 0.20 & 0.29 & 0.54 & 0.26 & 0.29 & 0.091 & 0.091 & 0.26 \\ 0.65 & 0.67 & 0.67 & 0.38 & 0.067 & 0.60 & 0.60 & 0.20 \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 0.059 & 0.27 & 0.27 & 0.45 & 0.27 & 0.33 & 0.33 & 0.20 \\ 1. & 1. & 1. & 1. & 1. & 0.33 & 1. & 1. \\ 0.35 & 0.40 & 0.40 & 0.26 & 0.040 & 0.27 & 0.36 & 0.53 \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & a_{1,2} & 0 & 0 & 0 & 0 & -a_{1,7} & 0 \\ 0 & 0 & a_{2,3} & 0 & a_{2,5} & 0 & -a_{2,7} & a_{2,8} \\ 0 & 0 & -a_{3,3} & 0 & 0 & a_{3,6} & -a_{3,7} & -a_{3,8} \\ 0 & -a_{4,2} & 0 & -a_{4,4} & 0 & -a_{4,6} & -a_{4,7} & 0 \\ 0 & 0 & a_{5,3} & a_{5,4} & -a_{5,5} & a_{5,6} & 0 & 0 \\ a_{6,1} & 0 & 0 & a_{6,4} & a_{6,5} & 0 & 0 & 0 \\ a_{7,1} & a_{7,2} & -a_{7,3} & 0 & -a_{7,5} & a_{7,6} & 0 & 0 \\ 0 & 0 & -a_{8,3} & 0 & -a_{8,5} & -a_{8,6} & 0 & 0 \end{bmatrix}$$

"system feedback"

$$F_8$$

$$\begin{aligned} & a_{7,5}\,a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{4,6}\,a_{2,7}\,a_{3,8} + a_{7,5}\,a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{4,6}\,a_{2,8}\,a_{3,7} - a_{7,5}\,a_{6,1}\,a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{8,6}\,a_{1,7}\,a_{3,8} \\ & + a_{7,5}\,a_{6,1}\,a_{4,2}\,a_{3,3}\,a_{5,4}\,a_{8,6}\,a_{1,7}\,a_{2,8} + a_{7,5}\,a_{6,1}\,a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{3,6}\,a_{1,7}\,a_{2,8} - a_{7,3}\,a_{6,1}\,a_{4,2}\,a_{5,4}\,a_{8,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} \\ & - a_{7,5}\,a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{8,6}\,a_{3,8}\,a_{4,7} + a_{7,5}\,a_{6,1}\,a_{1,2}\,a_{3,3}\,a_{5,4}\,a_{8,6}\,a_{2,8}\,a_{4,7} + a_{7,5}\,a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{8,6}\,a_{2,7}\,a_{3,8} \\ & + a_{7,5}\,a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{8,6}\,a_{2,8}\,a_{3,7} - a_{7,5}\,a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{5,6}\,a_{2,7}\,a_{3,8} - a_{7,5}\,a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{5,6}\,a_{2,8}\,a_{3,7} \\ & + a_{7,3}\,a_{6,1}\,a_{4,2}\,a_{5,4}\,a_{2,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} + a_{7,5}\,a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{3,6}\,a_{2,8}\,a_{4,7} + a_{7,3}\,a_{6,1}\,a_{1,2}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{2,7}\,a_{3,8} \\ & + a_{7,3}\,a_{6,1}\,a_{1,2}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{3,7} + a_{7,3}\,a_{6,1}\,a_{1,2}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{2,7}\,a_{3,8} + a_{7,3}\,a_{6,1}\,a_{1,2}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{3,7} \\ & + a_{7,3}\,a_{6,1}\,a_{1,2}\,a_{5,4}\,a_{2,5}\,a_{8,6}\,a_{3,8}\,a_{4,7} - a_{7,3}\,a_{6,1}\,a_{1,2}\,a_{5,4}\,a_{8,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} - a_{7,3}\,a_{6,1}\,a_{1,2}\,a_{5,4}\,a_{8,5}\,a_{4,6}\,a_{2,7}\,a_{3,8} \\ & - a_{7,3}\,a_{6,1}\,a_{1,2}\,a_{5,4}\,a_{8,5}\,a_{4,6}\,a_{2,8}\,a_{3,7} + a_{3,8}\,a_{6,1}\,a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{2,5}\,a_{1,7}\,a_{7,6} - a_{3,8}\,a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{4,4}\,a_{5,5}\,a_{1,7}\,a_{8,6} \\ & - a_{3,8}\,a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{4,4}\,a_{8,5}\,a_{1,7}\,a_{5,6} + a_{3,8}\,a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{8,5}\,a_{1,7}\,a_{4,6} - a_{3,8}\,a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{4,4}\,a_{2,5}\,a_{1,7}\,a_{8,6} \\ & + a_{3,8}\,a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{4,4}\,a_{2,5}\,a_{1,7}\,a_{5,6} - a_{3,8}\,a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{5,4}\,a_{2,5}\,a_{1,7}\,a_{4,6} - a_{3,8}\,a_{7,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{6,5}\,a_{4,7}\,a_{8,6} \\ & - a_{3,8}\,a_{7,1}\,a_{1,2}\,a_{2,3}\,a_{6,4}\,a_{4,5}\,a_{4,7}\,a_{8,6} - a_{3,8}\,a_{7,1}\,a_{1,2}\,a_{2,3}\,a_{6,4}\,a_{8,5}\,a_{4,7}\,a_{5,6} + a_{3,8}\,a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{6,5}\,a_{2,7}\,a_{8,6} \\ & - a_{3,8}\,a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{2,5}\,a_{4,7}\,a_{8,6} + a_{3,8}\,a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{2,7}\,a_{4,6} - a_{3,8}\,a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{6,5}\,a_{2,7}\,a_{5,6} \\ & + a_{3,8}\,a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{2,7}\,a_{4,6} + a_{3,8}\,a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{2,5}\,a_{4,7}\,a_{5,6} + a_{3,8}\,a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{2,7}\,a_{4,6} \\ & - a_{3,8}\,a_{7,1}\,a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{6,5}\,a_{1,7}\,a_{8,6} - a_{3,8}\,a_{7,1}\,a_{4,2}\,a_{2,3}\,a_{6,4}\,a_{5,5}\,a_{1,7}\,a_{8,6} - a_{3,8}\,a_{7,1}\,a_{4,2}\,a_{2,3}\,a_{6,4}\,a_{8,5}\,a_{1,7}\,a_{5,6} \\ & - a_{3,8}\,a_{7,1}\,a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{2,5}\,a_{1,7}\,a_{8,6} + a_{3,8}\,a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{2,5}\,a_{1,7}\,a_{5,6} - a_{3,8}\,a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{8,5}\,a_{4,7}\,a_{7,6} \end{aligned}$$

$$+ a_{3,8}\, a_{6,1}\, a_{1,2}\, a_{5,3}\, a_{4,4}\, a_{8,5}\, a_{2,7}\, a_{7,6} + a_{3,8}\, a_{6,1}\, a_{1,2}\, a_{8,3}\, a_{4,4}\, a_{5,5}\, a_{2,7}\, a_{7,6} + a_{3,8}\, a_{6,1}\, a_{1,2}\, a_{8,3}\, a_{5,4}\, a_{2,5}\, a_{4,7}\, a_{7,6}$$
$$- a_{3,8}\, a_{6,1}\, a_{4,2}\, a_{2,3}\, a_{5,4}\, a_{8,5}\, a_{1,7}\, a_{7,6} + a_{3,3}\, a_{6,1}\, a_{7,2}\, a_{4,4}\, a_{5,5}\, a_{8,6}\, a_{1,7}\, a_{2,8} + a_{3,3}\, a_{6,1}\, a_{7,2}\, a_{4,4}\, a_{8,5}\, a_{5,6}\, a_{1,7}\, a_{2,8}$$
$$- a_{3,3}\, a_{6,1}\, a_{7,2}\, a_{5,4}\, a_{8,5}\, a_{4,6}\, a_{1,7}\, a_{2,8} + a_{3,3}\, a_{7,1}\, a_{1,2}\, a_{5,4}\, a_{6,5}\, a_{8,6}\, a_{2,8}\, a_{4,7} + a_{3,3}\, a_{7,1}\, a_{1,2}\, a_{6,4}\, a_{5,5}\, a_{8,6}\, a_{2,8}\, a_{4,7}$$
$$+ a_{3,3}\, a_{7,1}\, a_{1,2}\, a_{6,4}\, a_{8,5}\, a_{5,6}\, a_{2,8}\, a_{4,7} + a_{3,3}\, a_{7,1}\, a_{4,2}\, a_{5,4}\, a_{6,5}\, a_{8,6}\, a_{1,7}\, a_{2,8} + a_{3,3}\, a_{7,1}\, a_{4,2}\, a_{6,4}\, a_{5,5}\, a_{8,6}\, a_{1,7}\, a_{2,8}$$
$$+ a_{3,3}\, a_{7,1}\, a_{4,2}\, a_{6,4}\, a_{8,5}\, a_{5,6}\, a_{1,7}\, a_{2,8} + a_{3,3}\, a_{6,1}\, a_{1,2}\, a_{5,4}\, a_{8,5}\, a_{7,6}\, a_{2,8}\, a_{4,7} + a_{3,3}\, a_{6,1}\, a_{4,2}\, a_{5,4}\, a_{8,5}\, a_{7,6}\, a_{1,7}\, a_{2,8}$$
$$+ a_{3,7}\, a_{6,1}\, a_{1,2}\, a_{5,3}\, a_{4,4}\, a_{8,5}\, a_{2,8}\, a_{7,6} + a_{3,7}\, a_{6,1}\, a_{1,2}\, a_{8,3}\, a_{4,4}\, a_{5,5}\, a_{2,8}\, a_{7,6} + a_{3,7}\, a_{7,1}\, a_{1,2}\, a_{5,3}\, a_{4,4}\, a_{6,5}\, a_{2,8}\, a_{8,6}$$
$$+ a_{3,7}\, a_{7,1}\, a_{1,2}\, a_{5,3}\, a_{6,4}\, a_{8,5}\, a_{2,8}\, a_{4,6} - a_{3,7}\, a_{7,1}\, a_{1,2}\, a_{8,3}\, a_{4,4}\, a_{6,5}\, a_{2,8}\, a_{5,6} + a_{3,7}\, a_{7,1}\, a_{1,2}\, a_{8,3}\, a_{5,4}\, a_{6,5}\, a_{2,8}\, a_{4,6}$$
$$+ a_{3,7}\, a_{7,1}\, a_{1,2}\, a_{8,3}\, a_{6,4}\, a_{5,5}\, a_{2,8}\, a_{4,6} + a_{8,5}\, a_{7,1}\, a_{1,2}\, a_{5,3}\, a_{6,4}\, a_{3,6}\, a_{2,8}\, a_{4,7} + a_{8,5}\, a_{6,1}\, a_{7,2}\, a_{5,3}\, a_{4,4}\, a_{3,6}\, a_{1,7}\, a_{2,8}$$
$$+ a_{8,5}\, a_{7,1}\, a_{4,2}\, a_{5,3}\, a_{6,4}\, a_{3,6}\, a_{1,7}\, a_{2,8} + a_{8,3}\, a_{7,1}\, a_{1,2}\, a_{6,4}\, a_{5,5}\, a_{3,6}\, a_{2,8}\, a_{4,7} + a_{8,3}\, a_{7,1}\, a_{4,2}\, a_{6,4}\, a_{5,5}\, a_{3,6}\, a_{1,7}\, a_{2,8}$$
$$+ a_{8,3}\, a_{7,1}\, a_{4,2}\, a_{5,4}\, a_{6,5}\, a_{3,6}\, a_{1,7}\, a_{2,8} + a_{8,3}\, a_{7,1}\, a_{1,2}\, a_{5,4}\, a_{6,5}\, a_{3,6}\, a_{2,8}\, a_{4,7} + a_{8,3}\, a_{6,1}\, a_{7,2}\, a_{4,4}\, a_{5,5}\, a_{3,6}\, a_{1,7}\, a_{2,8}$$

$$F_7$$

$$a_{6,1}\, a_{7,2}\, a_{3,3}\, a_{4,4}\, a_{8,6}\, a_{1,7}\, a_{2,8} - a_{6,1}\, a_{7,2}\, a_{2,3}\, a_{4,4}\, a_{8,6}\, a_{1,7}\, a_{3,8} + a_{6,1}\, a_{1,2}\, a_{8,3}\, a_{4,4}\, a_{7,6}\, a_{2,8}\, a_{3,7}$$
$$+ a_{6,1}\, a_{1,2}\, a_{8,3}\, a_{4,4}\, a_{7,6}\, a_{2,7}\, a_{3,8} + a_{6,1}\, a_{1,2}\, a_{7,3}\, a_{4,4}\, a_{8,6}\, a_{2,8}\, a_{3,7} + a_{6,1}\, a_{1,2}\, a_{7,3}\, a_{4,4}\, a_{8,6}\, a_{2,7}\, a_{3,8}$$
$$+ a_{7,1}\, a_{4,2}\, a_{8,3}\, a_{6,4}\, a_{3,6}\, a_{1,7}\, a_{2,8} + a_{7,1}\, a_{4,2}\, a_{3,3}\, a_{6,4}\, a_{8,6}\, a_{1,7}\, a_{2,8} - a_{7,1}\, a_{4,2}\, a_{2,3}\, a_{6,4}\, a_{8,6}\, a_{1,7}\, a_{3,8}$$
$$+ a_{7,1}\, a_{1,2}\, a_{8,3}\, a_{6,4}\, a_{4,6}\, a_{2,8}\, a_{3,7} + a_{7,1}\, a_{1,2}\, a_{8,3}\, a_{6,4}\, a_{4,6}\, a_{2,7}\, a_{3,8} + a_{7,1}\, a_{1,2}\, a_{8,3}\, a_{6,4}\, a_{3,6}\, a_{2,8}\, a_{4,7}$$
$$+ a_{7,1}\, a_{1,2}\, a_{3,3}\, a_{6,4}\, a_{8,6}\, a_{2,8}\, a_{4,7} - a_{7,1}\, a_{1,2}\, a_{2,3}\, a_{6,4}\, a_{8,6}\, a_{3,8}\, a_{4,7} + a_{6,1}\, a_{7,2}\, a_{8,3}\, a_{4,4}\, a_{3,6}\, a_{1,7}\, a_{2,8}$$
$$- a_{6,1}\, a_{1,2}\, a_{3,3}\, a_{5,4}\, a_{2,5}\, a_{4,7}\, a_{7,6} + a_{6,1}\, a_{1,2}\, a_{3,3}\, a_{4,4}\, a_{7,5}\, a_{2,7}\, a_{5,6} - a_{6,1}\, a_{1,2}\, a_{3,3}\, a_{4,4}\, a_{5,5}\, a_{2,7}\, a_{7,6}$$
$$- a_{6,1}\, a_{1,2}\, a_{2,3}\, a_{5,4}\, a_{7,5}\, a_{3,7}\, a_{4,6} + a_{6,1}\, a_{1,2}\, a_{2,3}\, a_{4,4}\, a_{7,5}\, a_{3,7}\, a_{5,6} - a_{6,1}\, a_{1,2}\, a_{2,3}\, a_{5,4}\, a_{7,5}\, a_{3,6}\, a_{4,7}$$
$$- a_{6,1}\, a_{1,2}\, a_{2,3}\, a_{4,4}\, a_{5,5}\, a_{3,7}\, a_{7,6} - a_{8,5}\, a_{5,4}\, a_{4,6}\, a_{6,1}\, a_{1,2}\, a_{2,3}\, a_{3,8} - a_{8,5}\, a_{5,4}\, a_{4,7}\, a_{7,1}\, a_{1,2}\, a_{2,3}\, a_{3,8}$$
$$- a_{7,1}\, a_{1,2}\, a_{3,3}\, a_{6,4}\, a_{2,5}\, a_{4,7}\, a_{5,6} - a_{7,1}\, a_{1,2}\, a_{3,3}\, a_{5,4}\, a_{6,5}\, a_{2,7}\, a_{4,6} + a_{7,1}\, a_{1,2}\, a_{3,3}\, a_{4,4}\, a_{6,5}\, a_{2,7}\, a_{5,6}$$
$$- a_{7,1}\, a_{1,2}\, a_{2,3}\, a_{6,4}\, a_{5,5}\, a_{3,7}\, a_{4,6} - a_{7,1}\, a_{1,2}\, a_{2,3}\, a_{6,4}\, a_{5,5}\, a_{3,6}\, a_{4,7} - a_{7,1}\, a_{1,2}\, a_{2,3}\, a_{5,4}\, a_{6,5}\, a_{3,7}\, a_{4,6}$$
$$- a_{7,1}\, a_{1,2}\, a_{2,3}\, a_{5,4}\, a_{6,5}\, a_{3,6}\, a_{4,7} + a_{7,1}\, a_{1,2}\, a_{2,3}\, a_{4,4}\, a_{6,5}\, a_{3,7}\, a_{5,6} - a_{6,1}\, a_{7,2}\, a_{5,3}\, a_{4,4}\, a_{2,5}\, a_{1,7}\, a_{3,6}$$
$$+ a_{6,1}\, a_{7,2}\, a_{3,3}\, a_{5,4}\, a_{2,5}\, a_{1,7}\, a_{4,6} - a_{6,1}\, a_{7,2}\, a_{3,3}\, a_{4,4}\, a_{2,5}\, a_{1,7}\, a_{5,6} - a_{6,1}\, a_{7,2}\, a_{2,3}\, a_{4,4}\, a_{5,5}\, a_{1,7}\, a_{3,6}$$
$$+ a_{6,1}\, a_{4,2}\, a_{7,3}\, a_{5,4}\, a_{2,5}\, a_{1,7}\, a_{3,6} - a_{6,1}\, a_{4,2}\, a_{3,3}\, a_{5,4}\, a_{2,5}\, a_{1,7}\, a_{7,6} - a_{6,1}\, a_{4,2}\, a_{2,3}\, a_{5,4}\, a_{7,5}\, a_{1,7}\, a_{3,6}$$
$$+ a_{6,1}\, a_{1,2}\, a_{7,3}\, a_{5,4}\, a_{2,5}\, a_{3,7}\, a_{4,6} + a_{6,1}\, a_{1,2}\, a_{7,3}\, a_{5,4}\, a_{2,5}\, a_{3,6}\, a_{4,7} - a_{6,1}\, a_{1,2}\, a_{7,3}\, a_{4,4}\, a_{2,5}\, a_{3,7}\, a_{5,6}$$
$$+ a_{6,1}\, a_{1,2}\, a_{7,3}\, a_{4,4}\, a_{5,5}\, a_{2,7}\, a_{3,6} + a_{6,1}\, a_{1,2}\, a_{5,3}\, a_{4,4}\, a_{7,5}\, a_{2,7}\, a_{3,6} - a_{6,1}\, a_{1,2}\, a_{3,3}\, a_{5,4}\, a_{7,5}\, a_{2,7}\, a_{4,6}$$
$$- a_{6,1}\, a_{1,2}\, a_{5,3}\, a_{4,4}\, a_{2,5}\, a_{3,7}\, a_{7,6} - a_{7,1}\, a_{4,2}\, a_{5,3}\, a_{6,4}\, a_{2,5}\, a_{1,7}\, a_{3,6} - a_{7,1}\, a_{4,2}\, a_{3,3}\, a_{6,4}\, a_{2,5}\, a_{1,7}\, a_{5,6}$$
$$- a_{7,1}\, a_{4,2}\, a_{2,3}\, a_{6,4}\, a_{5,5}\, a_{1,7}\, a_{3,6} - a_{7,1}\, a_{4,2}\, a_{2,3}\, a_{5,4}\, a_{6,5}\, a_{1,7}\, a_{3,6} - a_{7,1}\, a_{1,2}\, a_{5,3}\, a_{6,4}\, a_{2,5}\, a_{3,7}\, a_{4,6}$$
$$- a_{7,1}\, a_{1,2}\, a_{5,3}\, a_{6,4}\, a_{2,5}\, a_{3,6}\, a_{4,7} + a_{7,1}\, a_{1,2}\, a_{5,3}\, a_{4,4}\, a_{6,5}\, a_{2,7}\, a_{3,6} - a_{7,1}\, a_{1,2}\, a_{3,3}\, a_{6,4}\, a_{5,5}\, a_{2,7}\, a_{4,6}$$
$$+ a_{8,6}\, a_{6,1}\, a_{1,2}\, a_{2,3}\, a_{3,8}\, a_{5,5}\, a_{4,4} + a_{8,3}\, a_{3,7}\, a_{7,1}\, a_{1,2}\, a_{2,8}\, a_{5,5}\, a_{4,4} - a_{8,5}\, a_{5,4}\, a_{4,2}\, a_{2,3}\, a_{3,8}\, a_{7,1}\, a_{1,7}$$
$$- a_{8,5}\, a_{5,6}\, a_{6,1}\, a_{1,2}\, a_{2,8}\, a_{4,4}\, a_{3,3} - a_{8,3}\, a_{3,6}\, a_{6,1}\, a_{1,2}\, a_{2,8}\, a_{5,5}\, a_{4,4} + a_{8,3}\, a_{3,8}\, a_{7,1}\, a_{1,2}\, a_{2,5}\, a_{5,4}\, a_{4,7}$$
$$+ a_{8,5}\, a_{5,3}\, a_{3,8}\, a_{2,7}\, a_{7,1}\, a_{1,2}\, a_{4,4} + a_{8,3}\, a_{3,8}\, a_{6,1}\, a_{1,2}\, a_{2,5}\, a_{5,4}\, a_{4,6} - a_{8,3}\, a_{3,8}\, a_{6,1}\, a_{1,2}\, a_{2,5}\, a_{5,6}\, a_{4,4}$$
$$- a_{8,6}\, a_{6,1}\, a_{1,2}\, a_{2,8}\, a_{5,5}\, a_{4,4}\, a_{3,3} + a_{8,5}\, a_{5,4}\, a_{4,2}\, a_{2,8}\, a_{7,1}\, a_{1,7}\, a_{3,3} + a_{8,5}\, a_{5,4}\, a_{4,7}\, a_{7,1}\, a_{1,2}\, a_{2,8}\, a_{3,3}$$
$$+ a_{8,5}\, a_{5,4}\, a_{4,6}\, a_{6,1}\, a_{1,2}\, a_{2,8}\, a_{3,3} + a_{8,5}\, a_{5,6}\, a_{6,1}\, a_{1,2}\, a_{2,3}\, a_{3,8}\, a_{4,4} - a_{8,5}\, a_{5,3}\, a_{3,6}\, a_{6,1}\, a_{1,2}\, a_{2,8}\, a_{4,4}$$
$$+ a_{8,5}\, a_{5,3}\, a_{3,7}\, a_{7,1}\, a_{1,2}\, a_{2,8}\, a_{4,4} + a_{8,6}\, a_{6,1}\, a_{1,2}\, a_{2,5}\, a_{5,3}\, a_{3,8}\, a_{4,4} + a_{8,3}\, a_{3,8}\, a_{2,7}\, a_{7,1}\, a_{1,2}\, a_{5,5}\, a_{4,4}$$
$$+ a_{8,3}\, a_{3,8}\, a_{7,1}\, a_{1,7}\, a_{5,4}\, a_{4,2}\, a_{2,5} - a_{4,2}\, a_{5,3}\, a_{6,4}\, a_{7,5}\, a_{8,6}\, a_{2,8}\, a_{3,7} - a_{4,2}\, a_{5,3}\, a_{6,4}\, a_{8,5}\, a_{7,6}\, a_{2,7}\, a_{3,8}$$
$$- a_{4,2}\, a_{5,3}\, a_{6,4}\, a_{8,5}\, a_{7,6}\, a_{2,8}\, a_{3,7} - a_{4,2}\, a_{7,3}\, a_{5,4}\, a_{6,5}\, a_{8,6}\, a_{2,7}\, a_{3,8} - a_{4,2}\, a_{7,3}\, a_{5,4}\, a_{6,5}\, a_{8,6}\, a_{2,8}\, a_{3,7}$$
$$- a_{4,2}\, a_{7,3}\, a_{6,4}\, a_{5,5}\, a_{8,6}\, a_{2,7}\, a_{3,8} - a_{4,2}\, a_{7,3}\, a_{6,4}\, a_{5,5}\, a_{8,6}\, a_{2,8}\, a_{3,7} - a_{4,2}\, a_{7,3}\, a_{6,4}\, a_{8,5}\, a_{5,6}\, a_{2,7}\, a_{3,8}$$
$$- a_{4,2}\, a_{7,3}\, a_{6,4}\, a_{8,5}\, a_{5,6}\, a_{2,8}\, a_{3,7} - a_{4,2}\, a_{8,3}\, a_{5,4}\, a_{6,5}\, a_{7,6}\, a_{2,7}\, a_{3,8} - a_{4,2}\, a_{8,3}\, a_{5,4}\, a_{6,5}\, a_{7,6}\, a_{2,8}\, a_{3,7}$$
$$- a_{4,2}\, a_{8,3}\, a_{6,4}\, a_{5,5}\, a_{7,6}\, a_{2,7}\, a_{3,8} - a_{4,2}\, a_{8,3}\, a_{6,4}\, a_{5,5}\, a_{7,6}\, a_{2,8}\, a_{3,7} + a_{4,2}\, a_{8,3}\, a_{6,4}\, a_{7,5}\, a_{5,6}\, a_{2,7}\, a_{3,8}$$
$$+ a_{4,2}\, a_{8,3}\, a_{6,4}\, a_{7,5}\, a_{5,6}\, a_{2,8}\, a_{3,7} - a_{7,2}\, a_{2,3}\, a_{5,4}\, a_{6,5}\, a_{8,6}\, a_{3,8}\, a_{4,7} - a_{7,2}\, a_{2,3}\, a_{6,4}\, a_{5,5}\, a_{8,6}\, a_{3,8}\, a_{4,7}$$
$$- a_{4,2}\, a_{5,3}\, a_{6,4}\, a_{7,5}\, a_{8,6}\, a_{2,7}\, a_{3,8} - a_{7,2}\, a_{2,3}\, a_{6,4}\, a_{8,5}\, a_{5,6}\, a_{3,8}\, a_{4,7} + a_{7,2}\, a_{3,3}\, a_{5,4}\, a_{6,5}\, a_{8,6}\, a_{2,8}\, a_{4,7}$$
$$+ a_{7,2}\, a_{3,3}\, a_{6,4}\, a_{5,5}\, a_{8,6}\, a_{2,8}\, a_{4,7} + a_{7,2}\, a_{3,3}\, a_{6,4}\, a_{8,5}\, a_{5,6}\, a_{2,8}\, a_{4,7} + a_{7,2}\, a_{5,3}\, a_{4,4}\, a_{6,5}\, a_{8,6}\, a_{2,7}\, a_{3,8}$$
$$+ a_{7,2}\, a_{5,3}\, a_{4,4}\, a_{6,5}\, a_{8,6}\, a_{2,8}\, a_{3,7} - a_{7,2}\, a_{5,3}\, a_{6,4}\, a_{2,5}\, a_{8,6}\, a_{3,8}\, a_{4,7} + a_{7,2}\, a_{5,3}\, a_{6,4}\, a_{8,5}\, a_{3,6}\, a_{2,8}\, a_{4,7}$$
$$+ a_{7,2}\, a_{5,3}\, a_{6,4}\, a_{8,5}\, a_{4,6}\, a_{2,7}\, a_{3,8} + a_{7,2}\, a_{5,3}\, a_{6,4}\, a_{8,5}\, a_{4,6}\, a_{2,8}\, a_{3,7} - a_{7,2}\, a_{8,3}\, a_{4,4}\, a_{6,5}\, a_{5,6}\, a_{2,7}\, a_{3,8}$$

$$
\begin{aligned}
&- a_{7,2}\,a_{8,3}\,a_{4,4}\,a_{6,5}\,a_{5,6}\,a_{2,8}\,a_{3,7} + a_{7,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} + a_{7,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{4,6}\,a_{2,7}\,a_{3,8}\\
&+ a_{7,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{4,6}\,a_{2,8}\,a_{3,7} + a_{7,2}\,a_{8,3}\,a_{6,4}\,a_{2,5}\,a_{5,6}\,a_{3,8}\,a_{4,7} + a_{7,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{3,6}\,a_{2,8}\,a_{4,7}\\
&+ a_{7,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{4,6}\,a_{2,7}\,a_{3,8} + a_{7,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{4,6}\,a_{2,8}\,a_{3,7} + a_{6,1}\,a_{7,3}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{7,3}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{7,3}\,a_{5,4}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{8,3}\,a_{4,4}\,a_{5,5}\,a_{7,6}\,a_{1,7}\,a_{3,8}\\
&- a_{6,1}\,a_{8,3}\,a_{4,4}\,a_{7,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{4,6}\,a_{1,7}\,a_{3,8} + a_{7,1}\,a_{5,3}\,a_{4,4}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{7,1}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{3,8} - a_{7,1}\,a_{8,3}\,a_{4,4}\,a_{6,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} + a_{7,1}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{4,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{7,1}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{4,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{5,3}\,a_{4,4}\,a_{7,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{7,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{1,2}\,a_{5,4}\,a_{7,5}\,a_{8,6}\,a_{2,8}\,a_{4,7} + a_{6,1}\,a_{1,2}\,a_{5,4}\,a_{8,5}\,a_{7,6}\,a_{2,8}\,a_{4,7} + a_{6,1}\,a_{4,2}\,a_{5,4}\,a_{7,5}\,a_{8,6}\,a_{1,7}\,a_{2,8}\\
&+ a_{6,1}\,a_{4,2}\,a_{5,4}\,a_{8,5}\,a_{7,6}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{7,2}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{7,2}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{2,8}\\
&- a_{6,1}\,a_{7,2}\,a_{5,4}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{2,8} + a_{7,1}\,a_{1,2}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{2,8}\,a_{4,7} + a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{4,7}\\
&+ a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{4,7} + a_{7,1}\,a_{4,2}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{2,8} + a_{7,1}\,a_{4,2}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{2,8}\\
&+ a_{7,1}\,a_{4,2}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{7,5}\,a_{8,6}\,a_{2,7}\,a_{3,8} + a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{7,5}\,a_{8,6}\,a_{2,8}\,a_{3,7}\\
&+ a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{8,5}\,a_{7,6}\,a_{2,7}\,a_{3,8} + a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{8,5}\,a_{7,6}\,a_{2,8}\,a_{3,7} + a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,5}\,a_{8,6}\,a_{2,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{3,7} + a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{8,5}\,a_{5,6}\,a_{2,7}\,a_{3,8} + a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{3,7}\\
&+ a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{5,5}\,a_{7,6}\,a_{2,7}\,a_{3,8} + a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{5,5}\,a_{7,6}\,a_{2,8}\,a_{3,7} - a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{7,5}\,a_{5,6}\,a_{2,7}\,a_{3,8}\\
&- a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{7,5}\,a_{5,6}\,a_{2,8}\,a_{3,7} - a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{7,2}\,a_{3,3}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{7,2}\,a_{3,3}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{2,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{8,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{2,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{5,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}\\
&+ a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,5}\,a_{8,6}\,a_{2,7}\,a_{3,8} + a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,5}\,a_{8,6}\,a_{2,8}\,a_{3,7} - a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,5}\,a_{5,6}\,a_{2,7}\,a_{3,8}\\
&- a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,5}\,a_{5,6}\,a_{2,8}\,a_{3,7}
\end{aligned}
$$

$$F_6$$

$$
\begin{aligned}
&-a_{6,1}\,a_{1,2}\,a_{2,5}\,a_{5,4}\,a_{4,6}\,a_{3,3} + a_{6,1}\,a_{1,2}\,a_{2,5}\,a_{5,6}\,a_{4,4}\,a_{3,3} + a_{6,1}\,a_{1,2}\,a_{2,5}\,a_{5,3}\,a_{3,6}\,a_{4,4} - a_{7,5}\,a_{5,4}\,a_{4,6}\,a_{2,7}\,a_{6,1}\,a_{1,2}\\
&+ a_{7,5}\,a_{5,6}\,a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{3,7} - a_{7,5}\,a_{5,6}\,a_{6,4}\,a_{4,2}\,a_{2,3}\,a_{3,7} + a_{7,5}\,a_{5,3}\,a_{3,6}\,a_{2,7}\,a_{6,1}\,a_{1,2} - a_{7,5}\,a_{5,3}\,a_{3,6}\,a_{6,4}\,a_{4,2}\,a_{2,7}\\
&- a_{7,3}\,a_{3,6}\,a_{6,5}\,a_{5,4}\,a_{4,2}\,a_{2,7} - a_{7,2}\,a_{5,3}\,a_{6,4}\,a_{2,5}\,a_{3,6}\,a_{4,7} - a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{4,7} - a_{7,2}\,a_{2,5}\,a_{5,3}\,a_{3,6}\,a_{6,1}\,a_{1,7}\\
&+ a_{7,2}\,a_{2,5}\,a_{5,4}\,a_{4,6}\,a_{6,1}\,a_{1,7} - a_{7,1}\,a_{1,2}\,a_{2,3}\,a_{3,6}\,a_{6,4}\,a_{4,7} - a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{2,5}\,a_{4,7}\,a_{5,6} + a_{7,6}\,a_{6,5}\,a_{5,4}\,a_{4,2}\,a_{2,3}\,a_{3,7}\\
&+ a_{7,6}\,a_{6,4}\,a_{4,2}\,a_{2,5}\,a_{5,3}\,a_{3,7} - a_{7,6}\,a_{6,1}\,a_{1,2}\,a_{2,5}\,a_{5,3}\,a_{3,7} - a_{7,6}\,a_{6,1}\,a_{1,2}\,a_{2,5}\,a_{5,4}\,a_{4,7} - a_{7,6}\,a_{6,1}\,a_{1,7}\,a_{5,4}\,a_{4,2}\,a_{2,5}\\
&- a_{7,6}\,a_{6,1}\,a_{1,7}\,a_{5,5}\,a_{4,4}\,a_{3,3} - a_{7,1}\,a_{1,7}\,a_{6,4}\,a_{4,6}\,a_{5,5}\,a_{3,3} + a_{7,3}\,a_{3,7}\,a_{6,4}\,a_{4,2}\,a_{2,5}\,a_{5,6} - a_{7,3}\,a_{3,7}\,a_{6,1}\,a_{1,2}\,a_{2,5}\,a_{5,6}\\
&+ a_{7,1}\,a_{1,2}\,a_{2,3}\,a_{3,7}\,a_{6,5}\,a_{5,6} - a_{7,1}\,a_{1,2}\,a_{2,3}\,a_{3,7}\,a_{5,5}\,a_{4,4} - a_{7,2}\,a_{2,5}\,a_{5,6}\,a_{6,1}\,a_{1,7}\,a_{4,4} - a_{7,2}\,a_{2,5}\,a_{5,6}\,a_{6,1}\,a_{1,7}\,a_{3,3}\\
&- a_{7,2}\,a_{6,4}\,a_{2,5}\,a_{4,7}\,a_{5,6}\,a_{3,3} - a_{2,7}\,a_{7,1}\,a_{1,2}\,a_{6,5}\,a_{5,4}\,a_{4,6} - a_{2,7}\,a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{4,6}\,a_{5,5} - a_{2,7}\,a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{4,6}\,a_{3,3}\\
&+ a_{2,7}\,a_{7,1}\,a_{1,2}\,a_{6,5}\,a_{5,6}\,a_{4,4} + a_{2,7}\,a_{7,1}\,a_{1,2}\,a_{6,5}\,a_{5,6}\,a_{3,3} - a_{7,1}\,a_{1,2}\,a_{2,5}\,a_{5,4}\,a_{4,7}\,a_{3,3} - a_{7,2}\,a_{2,3}\,a_{3,6}\,a_{6,1}\,a_{1,7}\,a_{5,5}\\
&- a_{7,2}\,a_{2,3}\,a_{3,6}\,a_{6,1}\,a_{1,7}\,a_{4,4} - a_{7,2}\,a_{2,3}\,a_{3,6}\,a_{6,4}\,a_{4,7}\,a_{5,5} + a_{7,6}\,a_{6,5}\,a_{5,4}\,a_{4,2}\,a_{2,7}\,a_{3,3} - a_{7,1}\,a_{1,2}\,a_{2,5}\,a_{5,3}\,a_{3,7}\,a_{4,4}\\
&- a_{2,7}\,a_{7,1}\,a_{1,2}\,a_{5,5}\,a_{4,4}\,a_{3,3} - a_{7,2}\,a_{2,3}\,a_{3,7}\,a_{6,5}\,a_{5,4}\,a_{4,6} - a_{7,2}\,a_{2,3}\,a_{3,7}\,a_{6,4}\,a_{4,6}\,a_{5,5} + a_{7,2}\,a_{2,3}\,a_{3,7}\,a_{6,5}\,a_{5,6}\,a_{4,4}\\
&+ a_{7,3}\,a_{3,6}\,a_{2,7}\,a_{6,1}\,a_{1,2}\,a_{5,5} + a_{7,3}\,a_{3,6}\,a_{2,7}\,a_{6,1}\,a_{1,2}\,a_{4,4} - a_{7,3}\,a_{3,6}\,a_{6,4}\,a_{4,2}\,a_{2,7}\,a_{5,5} - a_{7,6}\,a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{3,7}\,a_{5,5}\\
&- a_{7,6}\,a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{3,7}\,a_{4,4} + a_{7,6}\,a_{6,4}\,a_{4,2}\,a_{2,3}\,a_{3,7}\,a_{5,5} + a_{7,5}\,a_{5,3}\,a_{3,6}\,a_{6,1}\,a_{1,7}\,a_{4,4} - a_{7,5}\,a_{5,4}\,a_{4,6}\,a_{6,1}\,a_{1,7}\,a_{3,3}\\
&+ a_{7,5}\,a_{5,6}\,a_{2,7}\,a_{6,1}\,a_{1,2}\,a_{4,4} + a_{7,5}\,a_{5,6}\,a_{2,7}\,a_{6,1}\,a_{1,2}\,a_{3,3} - a_{7,5}\,a_{5,6}\,a_{6,4}\,a_{4,2}\,a_{2,7}\,a_{3,3} + a_{2,7}\,a_{7,1}\,a_{1,2}\,a_{6,5}\,a_{5,3}\,a_{3,6}\\
&+ a_{7,6}\,a_{6,4}\,a_{4,2}\,a_{2,7}\,a_{5,5}\,a_{3,3} - a_{7,6}\,a_{2,7}\,a_{6,1}\,a_{1,2}\,a_{5,5}\,a_{4,4} - a_{7,6}\,a_{2,7}\,a_{6,1}\,a_{1,2}\,a_{5,5}\,a_{3,3} - a_{7,6}\,a_{2,7}\,a_{6,1}\,a_{1,2}\,a_{4,4}\,a_{3,3}\\
&+ a_{7,5}\,a_{5,6}\,a_{6,1}\,a_{1,7}\,a_{4,4}\,a_{3,3} - a_{7,2}\,a_{2,5}\,a_{5,3}\,a_{3,7}\,a_{6,4}\,a_{4,6} + a_{7,3}\,a_{3,6}\,a_{6,1}\,a_{1,7}\,a_{5,5}\,a_{4,4} - a_{7,1}\,a_{1,7}\,a_{6,5}\,a_{5,4}\,a_{4,6}\,a_{3,3}\\
&+ a_{7,2}\,a_{2,7}\,a_{6,5}\,a_{5,3}\,a_{3,6}\,a_{4,4} + a_{7,2}\,a_{2,7}\,a_{6,5}\,a_{5,6}\,a_{4,4}\,a_{3,3} - a_{7,2}\,a_{2,7}\,a_{6,4}\,a_{4,6}\,a_{5,5}\,a_{3,3} - a_{7,2}\,a_{2,7}\,a_{6,5}\,a_{5,4}\,a_{4,6}\,a_{3,3}\\
&- a_{7,1}\,a_{1,7}\,a_{6,4}\,a_{4,2}\,a_{2,5}\,a_{5,6} - a_{7,1}\,a_{1,7}\,a_{6,4}\,a_{4,2}\,a_{2,3}\,a_{3,6} + a_{7,1}\,a_{1,7}\,a_{6,5}\,a_{5,3}\,a_{3,6}\,a_{4,4} + a_{7,1}\,a_{1,7}\,a_{6,5}\,a_{5,6}\,a_{4,4}\,a_{3,3}\\
&- a_{8,6}\,a_{6,4}\,a_{4,2}\,a_{2,5}\,a_{5,3}\,a_{3,8} - a_{8,6}\,a_{6,4}\,a_{4,2}\,a_{2,7}\,a_{7,3}\,a_{3,8} + a_{8,6}\,a_{6,4}\,a_{4,7}\,a_{7,1}\,a_{1,2}\,a_{2,8} - a_{8,6}\,a_{6,4}\,a_{4,7}\,a_{7,2}\,a_{2,3}\,a_{3,8}\\
&+ a_{8,6}\,a_{6,1}\,a_{1,2}\,a_{2,5}\,a_{5,3}\,a_{3,8} + a_{8,6}\,a_{6,1}\,a_{1,2}\,a_{2,7}\,a_{7,3}\,a_{3,8} - a_{8,6}\,a_{6,1}\,a_{1,7}\,a_{7,2}\,a_{2,3}\,a_{3,8} + a_{8,6}\,a_{6,1}\,a_{1,7}\,a_{7,5}\,a_{5,3}\,a_{3,8}\\
&+ a_{8,6}\,a_{6,5}\,a_{5,3}\,a_{3,7}\,a_{7,2}\,a_{2,8} - a_{8,6}\,a_{6,5}\,a_{5,4}\,a_{4,2}\,a_{2,3}\,a_{3,8} + a_{8,6}\,a_{6,5}\,a_{5,4}\,a_{4,7}\,a_{7,2}\,a_{2,8} + a_{8,5}\,a_{5,6}\,a_{6,1}\,a_{1,7}\,a_{7,3}\,a_{3,8}\\
&- a_{8,5}\,a_{5,3}\,a_{3,6}\,a_{6,1}\,a_{1,2}\,a_{2,8} + a_{8,5}\,a_{5,3}\,a_{3,6}\,a_{6,4}\,a_{4,2}\,a_{2,8} + a_{8,5}\,a_{5,3}\,a_{3,7}\,a_{7,1}\,a_{1,2}\,a_{2,8} - a_{8,5}\,a_{5,6}\,a_{6,4}\,a_{4,2}\,a_{2,3}\,a_{3,8}\\
&+ a_{8,5}\,a_{5,6}\,a_{6,4}\,a_{4,7}\,a_{7,2}\,a_{2,8} + a_{8,5}\,a_{5,6}\,a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{3,8} + a_{8,5}\,a_{5,6}\,a_{6,1}\,a_{1,7}\,a_{7,2}\,a_{2,8} + a_{8,5}\,a_{5,4}\,a_{4,6}\,a_{6,1}\,a_{1,2}\,a_{2,8}\\
&+ a_{8,5}\,a_{5,4}\,a_{4,7}\,a_{7,1}\,a_{1,2}\,a_{2,8} + a_{8,3}\,a_{3,6}\,a_{6,4}\,a_{4,7}\,a_{7,2}\,a_{2,8} + a_{8,3}\,a_{3,6}\,a_{6,5}\,a_{5,4}\,a_{4,2}\,a_{2,8} + a_{8,3}\,a_{3,6}\,a_{6,1}\,a_{1,7}\,a_{7,2}\,a_{2,8}
\end{aligned}
$$

$$+a_{8,3}a_{3,7}a_{7,6}a_{6,1}a_{1,2}a_{2,8}-a_{8,3}a_{3,7}a_{7,6}a_{6,4}a_{4,2}a_{2,8}+a_{8,6}a_{6,1}a_{1,7}a_{7,2}a_{2,8}a_{4,4}+a_{8,6}a_{6,1}a_{1,7}a_{7,2}a_{2,8}a_{3,3}$$
$$+a_{8,6}a_{6,1}a_{1,7}a_{7,2}a_{2,8}a_{5,5}+a_{6,1}a_{1,2}a_{2,3}a_{3,6}a_{5,5}a_{4,4}-a_{7,1}a_{1,7}a_{5,4}a_{4,2}a_{2,5}a_{3,3}+a_{8,3}a_{3,7}a_{7,1}a_{1,2}a_{2,8}a_{4,4}$$
$$-a_{8,5}a_{5,6}a_{6,1}a_{1,2}a_{2,8}a_{4,4}-a_{8,5}a_{5,6}a_{6,1}a_{1,2}a_{2,8}a_{3,3}+a_{8,5}a_{5,6}a_{6,4}a_{4,2}a_{2,8}a_{3,3}-a_{8,3}a_{3,6}a_{6,1}a_{1,2}a_{2,8}a_{5,5}$$
$$-a_{8,3}a_{3,6}a_{6,1}a_{1,2}a_{2,8}a_{4,4}+a_{8,3}a_{3,6}a_{6,4}a_{4,2}a_{2,8}a_{5,5}+a_{8,3}a_{3,7}a_{7,1}a_{1,2}a_{2,8}a_{5,5}+a_{8,3}a_{3,8}a_{7,1}a_{1,7}a_{6,4}a_{4,6}$$
$$-a_{8,3}a_{3,8}a_{7,1}a_{1,7}a_{6,5}a_{5,6}+a_{8,3}a_{3,8}a_{7,1}a_{1,7}a_{5,5}a_{4,4}+a_{8,3}a_{3,8}a_{7,2}a_{2,7}a_{6,4}a_{4,6}-a_{8,3}a_{3,8}a_{7,2}a_{2,7}a_{6,5}a_{5,6}$$
$$-a_{8,3}a_{3,8}a_{7,6}a_{6,4}a_{4,2}a_{2,7}+a_{8,3}a_{3,8}a_{7,6}a_{2,7}a_{6,1}a_{1,2}-a_{8,3}a_{3,8}a_{7,5}a_{5,6}a_{6,1}a_{1,7}+a_{8,3}a_{3,8}a_{2,7}a_{7,1}a_{1,2}a_{5,5}$$
$$+a_{8,3}a_{3,8}a_{2,7}a_{7,1}a_{1,2}a_{4,4}-a_{8,3}a_{3,8}a_{6,1}a_{1,2}a_{2,5}a_{5,6}+a_{8,3}a_{3,8}a_{7,6}a_{6,1}a_{1,7}a_{5,5}+a_{8,3}a_{3,8}a_{7,6}a_{6,1}a_{1,7}a_{4,4}$$
$$+a_{8,3}a_{3,8}a_{6,4}a_{4,2}a_{2,5}a_{5,6}+a_{8,5}a_{5,3}a_{3,8}a_{2,7}a_{7,1}a_{1,2}+a_{8,5}a_{5,3}a_{3,8}a_{7,6}a_{6,1}a_{1,7}+a_{8,5}a_{5,3}a_{3,8}a_{7,1}a_{1,7}a_{4,4}$$
$$+a_{8,5}a_{5,4}a_{4,2}a_{2,8}a_{7,1}a_{1,7}+a_{8,3}a_{3,7}a_{7,2}a_{2,8}a_{6,4}a_{4,6}-a_{8,3}a_{3,7}a_{7,2}a_{2,8}a_{6,5}a_{5,6}+a_{8,6}a_{6,4}a_{4,7}a_{7,2}a_{2,8}a_{5,5}$$
$$+a_{8,6}a_{6,4}a_{4,7}a_{7,2}a_{2,8}a_{3,3}-a_{8,6}a_{6,4}a_{4,2}a_{2,3}a_{3,8}a_{5,5}+a_{8,6}a_{6,5}a_{5,4}a_{4,2}a_{2,8}a_{3,3}+a_{8,6}a_{6,1}a_{1,2}a_{2,3}a_{3,8}a_{4,4}$$
$$+a_{8,6}a_{6,1}a_{1,7}a_{7,3}a_{3,8}a_{5,5}+a_{8,6}a_{6,1}a_{1,7}a_{7,3}a_{3,8}a_{4,4}+a_{8,6}a_{6,1}a_{1,2}a_{2,3}a_{3,8}a_{5,5}+a_{8,6}a_{6,1}a_{1,2}a_{2,8}a_{7,3}a_{3,7}$$
$$-a_{8,6}a_{6,1}a_{1,2}a_{2,8}a_{5,5}a_{4,4}-a_{8,6}a_{6,1}a_{1,2}a_{2,8}a_{5,5}a_{3,3}-a_{8,6}a_{6,1}a_{1,2}a_{2,8}a_{4,4}a_{3,3}+a_{8,6}a_{6,4}a_{4,2}a_{2,8}a_{7,1}a_{1,7}$$
$$-a_{8,6}a_{6,4}a_{4,2}a_{2,8}a_{7,3}a_{3,7}+a_{8,6}a_{6,4}a_{4,2}a_{2,8}a_{5,5}a_{3,3}+a_{8,6}a_{6,5}a_{5,3}a_{3,8}a_{7,1}a_{1,7}+a_{8,6}a_{6,5}a_{5,3}a_{3,8}a_{7,2}a_{2,7}$$
$$-a_{7,1}a_{1,2}a_{2,3}a_{3,7}a_{6,4}a_{4,6}-a_{4,2}a_{7,3}a_{5,4}a_{8,5}a_{2,7}a_{3,8}-a_{4,2}a_{7,3}a_{5,4}a_{8,5}a_{2,8}a_{3,7}+a_{4,2}a_{8,3}a_{5,4}a_{7,5}a_{2,7}a_{3,8}$$
$$+a_{4,2}a_{8,3}a_{5,4}a_{7,5}a_{2,8}a_{3,7}-a_{7,2}a_{2,3}a_{5,4}a_{8,5}a_{3,8}a_{4,7}+a_{7,2}a_{3,3}a_{5,4}a_{8,5}a_{2,8}a_{4,7}+a_{7,2}a_{5,3}a_{4,4}a_{8,5}a_{2,7}a_{3,8}$$
$$+a_{7,2}a_{5,3}a_{4,4}a_{8,5}a_{2,8}a_{3,7}+a_{7,2}a_{8,3}a_{4,4}a_{5,5}a_{2,7}a_{3,8}+a_{7,2}a_{8,3}a_{4,4}a_{5,5}a_{2,8}a_{3,7}+a_{7,2}a_{8,3}a_{5,4}a_{2,5}a_{3,8}a_{4,7}$$
$$+a_{7,3}a_{5,4}a_{6,5}a_{8,6}a_{3,8}a_{4,7}+a_{7,3}a_{6,4}a_{5,5}a_{8,6}a_{3,8}a_{4,7}+a_{7,3}a_{6,4}a_{8,5}a_{5,6}a_{3,8}a_{4,7}+a_{8,3}a_{5,4}a_{6,5}a_{7,6}a_{3,8}a_{4,7}$$
$$+a_{8,3}a_{6,4}a_{5,5}a_{7,6}a_{3,8}a_{4,7}-a_{8,3}a_{6,4}a_{7,5}a_{5,6}a_{3,8}a_{4,7}+a_{5,3}a_{6,4}a_{7,5}a_{8,6}a_{3,8}a_{4,7}+a_{5,3}a_{6,4}a_{8,5}a_{7,6}a_{3,8}a_{4,7}$$

$$F_5$$

$$-a_{6,5}a_{5,4}a_{4,2}a_{2,3}a_{3,6}+a_{6,1}a_{1,2}a_{2,5}a_{5,3}a_{3,6}-a_{6,1}a_{1,2}a_{2,5}a_{5,4}a_{4,6}-a_{6,4}a_{4,2}a_{2,5}a_{5,3}a_{3,6}+a_{6,1}a_{1,2}a_{2,3}a_{3,6}a_{4,4}$$
$$+a_{6,1}a_{1,2}a_{2,5}a_{5,6}a_{4,4}+a_{6,1}a_{1,2}a_{2,5}a_{5,6}a_{3,3}-a_{6,4}a_{4,2}a_{2,3}a_{3,6}a_{5,5}-a_{6,4}a_{4,2}a_{2,5}a_{5,6}a_{3,3}+a_{6,1}a_{1,2}a_{2,3}a_{3,6}a_{5,5}$$
$$+a_{7,5}a_{5,3}a_{3,6}a_{6,1}a_{1,7}+a_{7,5}a_{5,3}a_{3,6}a_{6,4}a_{4,7}-a_{7,5}a_{5,4}a_{4,2}a_{2,3}a_{3,7}-a_{7,5}a_{5,4}a_{4,6}a_{6,1}a_{1,7}+a_{7,5}a_{5,6}a_{2,7}a_{6,1}a_{1,2}$$
$$-a_{7,5}a_{5,6}a_{6,4}a_{4,2}a_{2,7}+a_{7,3}a_{3,6}a_{2,7}a_{6,1}a_{1,2}-a_{7,3}a_{3,6}a_{6,4}a_{4,2}a_{2,7}+a_{7,3}a_{5,4}a_{6,5}a_{3,6}a_{4,7}-a_{7,6}a_{6,1}a_{1,2}a_{2,3}a_{3,7}$$
$$+a_{7,6}a_{6,4}a_{4,2}a_{2,3}a_{3,7}+a_{7,6}a_{6,5}a_{5,4}a_{4,2}a_{2,7}-a_{7,1}a_{1,2}a_{2,5}a_{5,3}a_{3,7}-a_{7,1}a_{1,2}a_{2,5}a_{5,4}a_{4,7}-a_{7,2}a_{2,3}a_{3,6}a_{6,1}a_{1,7}$$
$$-a_{7,2}a_{2,3}a_{3,6}a_{6,4}a_{4,7}-a_{7,2}a_{2,5}a_{5,6}a_{6,1}a_{1,7}-a_{7,2}a_{6,4}a_{2,5}a_{4,7}a_{5,6}-a_{7,3}a_{3,7}a_{6,5}a_{5,6}a_{4,4}+a_{7,3}a_{3,7}a_{5,4}a_{4,2}a_{2,5}$$
$$-a_{7,1}a_{1,7}a_{6,5}a_{5,4}a_{4,6}-a_{7,1}a_{1,7}a_{6,4}a_{4,6}a_{5,5}-a_{7,1}a_{1,7}a_{6,4}a_{4,6}a_{3,3}+a_{7,1}a_{1,7}a_{6,5}a_{5,6}a_{4,4}+a_{7,1}a_{1,7}a_{6,5}a_{5,6}a_{3,3}$$
$$-a_{7,1}a_{1,7}a_{5,4}a_{4,2}a_{2,5}-a_{7,1}a_{1,7}a_{5,5}a_{4,4}a_{3,3}+a_{7,2}a_{2,7}a_{6,5}a_{5,3}a_{3,6}-a_{7,2}a_{2,7}a_{6,5}a_{5,4}a_{4,6}-a_{7,2}a_{2,7}a_{6,4}a_{4,6}a_{5,5}$$
$$-a_{7,2}a_{2,7}a_{6,4}a_{4,6}a_{3,3}+a_{7,2}a_{2,7}a_{6,5}a_{5,6}a_{4,4}+a_{7,2}a_{2,7}a_{6,5}a_{5,6}a_{3,3}-a_{7,2}a_{2,7}a_{5,5}a_{4,4}a_{3,3}+a_{7,3}a_{3,7}a_{6,5}a_{5,4}a_{4,6}$$
$$+a_{7,1}a_{1,7}a_{6,5}a_{5,3}a_{3,6}+a_{7,3}a_{3,7}a_{6,4}a_{4,6}a_{5,5}-a_{7,1}a_{1,2}a_{2,3}a_{3,7}a_{5,5}-a_{7,1}a_{1,2}a_{2,3}a_{3,7}a_{4,4}-a_{7,2}a_{2,5}a_{5,3}a_{3,7}a_{4,4}$$
$$-a_{7,2}a_{2,5}a_{5,4}a_{4,7}a_{3,3}-a_{7,6}a_{6,4}a_{4,7}a_{5,5}a_{3,3}-a_{2,7}a_{7,1}a_{1,2}a_{4,4}a_{3,3}-a_{7,2}a_{2,3}a_{3,7}a_{6,4}a_{4,6}+a_{7,2}a_{2,3}a_{3,7}a_{6,5}a_{5,6}$$
$$-a_{7,2}a_{2,3}a_{3,7}a_{5,5}a_{4,4}+a_{7,5}a_{5,3}a_{3,7}a_{6,4}a_{4,6}-a_{7,6}a_{6,1}a_{1,7}a_{5,5}a_{4,4}-a_{7,6}a_{6,1}a_{1,7}a_{5,5}a_{3,3}-a_{7,6}a_{6,1}a_{1,7}a_{4,4}a_{3,3}$$
$$-a_{7,6}a_{2,7}a_{6,1}a_{1,2}a_{4,4}-a_{7,6}a_{2,7}a_{6,1}a_{1,2}a_{3,3}+a_{7,6}a_{6,4}a_{4,2}a_{2,7}a_{5,5}+a_{7,6}a_{6,4}a_{4,2}a_{2,7}a_{3,3}-a_{7,6}a_{6,5}a_{5,3}a_{3,7}a_{4,4}$$
$$-a_{7,6}a_{6,5}a_{5,4}a_{4,7}a_{3,3}-a_{2,7}a_{7,1}a_{1,2}a_{5,5}a_{4,4}-a_{2,7}a_{7,1}a_{1,2}a_{5,5}a_{3,3}+a_{7,3}a_{3,6}a_{6,1}a_{1,7}a_{4,4}+a_{7,3}a_{3,6}a_{6,4}a_{4,7}a_{5,5}$$
$$-a_{7,5}a_{5,4}a_{4,2}a_{2,7}a_{3,3}+a_{7,5}a_{5,6}a_{6,1}a_{1,7}a_{4,4}+a_{7,5}a_{5,6}a_{6,1}a_{1,7}a_{3,3}+a_{7,5}a_{5,6}a_{6,4}a_{4,7}a_{3,3}+a_{7,3}a_{3,6}a_{6,1}a_{1,7}a_{5,5}$$
$$-a_{7,6}a_{2,7}a_{6,1}a_{1,2}a_{5,5}-a_{8,5}a_{5,4}a_{4,2}a_{2,3}a_{3,8}+a_{8,5}a_{5,4}a_{4,7}a_{7,2}a_{2,8}-a_{8,5}a_{5,6}a_{6,1}a_{1,2}a_{2,8}+a_{8,5}a_{5,6}a_{6,4}a_{4,2}a_{2,8}$$
$$-a_{8,3}a_{3,6}a_{6,1}a_{1,2}a_{2,8}+a_{8,3}a_{3,7}a_{7,1}a_{1,2}a_{2,8}+a_{8,5}a_{5,3}a_{3,8}a_{7,1}a_{1,7}+a_{8,3}a_{3,8}a_{7,1}a_{1,7}a_{6,4}a_{4,6}-a_{8,3}a_{3,8}a_{7,2}a_{2,7}a_{6,5}a_{5,6}$$
$$-a_{8,3}a_{3,8}a_{7,6}a_{6,4}a_{4,2}a_{2,7}+a_{8,3}a_{3,8}a_{7,6}a_{2,7}a_{6,1}a_{1,2}-a_{8,3}a_{3,8}a_{7,5}a_{5,6}a_{6,1}a_{1,7}+a_{8,3}a_{3,8}a_{2,7}a_{7,1}a_{1,2}a_{5,5}$$
$$+a_{8,3}a_{3,8}a_{2,7}a_{7,1}a_{1,2}a_{4,4}-a_{8,3}a_{3,8}a_{6,1}a_{1,2}a_{2,5}a_{5,6}+a_{8,3}a_{3,8}a_{7,6}a_{6,1}a_{1,7}a_{5,5}+a_{8,3}a_{3,8}a_{7,6}a_{6,1}a_{1,7}a_{4,4}$$
$$+a_{8,3}a_{3,8}a_{6,4}a_{4,2}a_{2,5}a_{5,6}+a_{8,5}a_{5,3}a_{3,8}a_{2,7}a_{7,1}a_{1,2}+a_{8,5}a_{5,3}a_{3,8}a_{7,6}a_{6,1}a_{1,7}+a_{8,5}a_{5,3}a_{3,8}a_{7,1}a_{1,7}a_{4,4}$$
$$+a_{8,5}a_{5,4}a_{4,2}a_{2,8}a_{7,1}a_{1,7}+a_{8,3}a_{3,7}a_{7,2}a_{2,8}a_{6,4}a_{4,6}-a_{8,3}a_{3,7}a_{7,2}a_{2,8}a_{6,5}a_{5,6}+a_{8,6}a_{6,4}a_{4,7}a_{7,2}a_{2,8}a_{5,5}$$
$$+a_{8,6}a_{6,4}a_{4,7}a_{7,2}a_{2,8}a_{3,3}-a_{8,6}a_{6,4}a_{4,2}a_{2,3}a_{3,8}a_{5,5}+a_{8,6}a_{6,5}a_{5,4}a_{4,2}a_{2,8}a_{3,3}+a_{8,6}a_{6,1}a_{1,2}a_{2,3}a_{3,8}a_{4,4}$$
$$+a_{8,6}a_{6,1}a_{1,7}a_{7,3}a_{3,8}a_{5,5}+a_{8,6}a_{6,1}a_{1,7}a_{7,3}a_{3,8}a_{4,4}+a_{8,6}a_{6,1}a_{1,2}a_{2,3}a_{3,8}a_{5,5}+a_{8,6}a_{6,1}a_{1,2}a_{2,8}a_{7,3}a_{3,7}$$
$$-a_{8,6}a_{6,1}a_{1,2}a_{2,8}a_{5,5}a_{4,4}-a_{8,6}a_{6,1}a_{1,2}a_{2,8}a_{5,5}a_{3,3}-a_{8,6}a_{6,1}a_{1,2}a_{2,8}a_{4,4}a_{3,3}+a_{8,6}a_{6,4}a_{4,2}a_{2,8}a_{7,1}a_{1,7}$$
$$-a_{8,6}a_{6,4}a_{4,2}a_{2,8}a_{7,3}a_{3,7}+a_{8,6}a_{6,4}a_{4,2}a_{2,8}a_{5,5}a_{3,3}+a_{8,6}a_{6,5}a_{5,3}a_{3,8}a_{7,1}a_{1,7}+a_{8,6}a_{6,5}a_{5,3}a_{3,8}a_{7,2}a_{2,7}$$
$$-a_{7,1}a_{1,2}a_{2,3}a_{3,7}a_{6,4}a_{4,6}-a_{4,2}a_{7,3}a_{5,4}a_{8,5}a_{2,7}a_{3,8}-a_{4,2}a_{7,3}a_{5,4}a_{8,5}a_{2,8}a_{3,7}+a_{4,2}a_{8,3}a_{5,4}a_{7,5}a_{2,7}a_{3,8}$$
$$+a_{4,2}a_{8,3}a_{5,4}a_{7,5}a_{2,8}a_{3,7}-a_{7,2}a_{2,3}a_{5,4}a_{8,5}a_{3,8}a_{4,7}+a_{7,2}a_{3,3}a_{5,4}a_{8,5}a_{2,8}a_{4,7}+a_{7,2}a_{5,3}a_{4,4}a_{8,5}a_{2,7}a_{3,8}$$
$$+a_{7,2}a_{5,3}a_{4,4}a_{8,5}a_{2,8}a_{3,7}+a_{7,2}a_{8,3}a_{4,4}a_{5,5}a_{2,7}a_{3,8}+a_{7,2}a_{8,3}a_{4,4}a_{5,5}a_{2,8}a_{3,7}+a_{7,2}a_{8,3}a_{5,4}a_{2,5}a_{3,8}a_{4,7}$$
$$+a_{7,3}a_{5,4}a_{6,5}a_{8,6}a_{3,8}a_{4,7}+a_{7,3}a_{6,4}a_{5,5}a_{8,6}a_{3,8}a_{4,7}+a_{7,3}a_{6,4}a_{8,5}a_{5,6}a_{3,8}a_{4,7}+a_{8,3}a_{5,4}a_{6,5}a_{7,6}a_{3,8}a_{4,7}$$
$$+a_{8,3}a_{6,4}a_{5,5}a_{7,6}a_{3,8}a_{4,7}-a_{8,3}a_{6,4}a_{7,5}a_{5,6}a_{3,8}a_{4,7}+a_{5,3}a_{6,4}a_{7,5}a_{8,6}a_{3,8}a_{4,7}+a_{5,3}a_{6,4}a_{8,5}a_{7,6}a_{3,8}a_{4,7}$$

$$F_4$$

$$-a_{5,4}a_{4,2}a_{2,5}a_{3,3}+a_{6,1}a_{1,2}a_{2,3}a_{3,6}-a_{6,4}a_{4,2}a_{2,5}a_{5,6}-a_{6,4}a_{4,2}a_{2,3}a_{3,6}+a_{6,1}a_{1,2}a_{2,5}a_{5,6}+a_{6,5}a_{5,3}a_{3,6}a_{4,4}$$

$$+ a_{6,5}\,a_{5,6}\,a_{4,4}\,a_{3,3} - a_{6,4}\,a_{4,6}\,a_{5,5}\,a_{3,3} - a_{6,5}\,a_{5,4}\,a_{4,6}\,a_{3,3} - a_{7,2}\,a_{2,5}\,a_{5,3}\,a_{3,7} - a_{7,1}\,a_{1,2}\,a_{2,3}\,a_{3,7} - a_{7,6}\,a_{6,1}\,a_{1,7}\,a_{5,5}$$
$$- a_{7,6}\,a_{6,1}\,a_{1,7}\,a_{4,4} - a_{7,6}\,a_{6,1}\,a_{1,7}\,a_{3,3} - a_{7,6}\,a_{6,4}\,a_{4,7}\,a_{5,5} - a_{7,6}\,a_{6,4}\,a_{4,7}\,a_{3,3} - a_{7,1}\,a_{1,7}\,a_{6,4}\,a_{4,6} + a_{7,1}\,a_{1,7}\,a_{6,5}\,a_{5,6}$$
$$- a_{7,1}\,a_{1,7}\,a_{5,5}\,a_{4,4} - a_{7,1}\,a_{1,7}\,a_{5,5}\,a_{3,3} - a_{7,1}\,a_{1,7}\,a_{4,4}\,a_{3,3} - a_{7,2}\,a_{2,7}\,a_{6,4}\,a_{4,6} + a_{7,2}\,a_{2,7}\,a_{6,5}\,a_{5,6} - a_{7,2}\,a_{2,7}\,a_{5,5}\,a_{4,4}$$
$$- a_{7,2}\,a_{2,7}\,a_{5,5}\,a_{3,3} - a_{7,2}\,a_{2,7}\,a_{4,4}\,a_{3,3} + a_{7,3}\,a_{3,7}\,a_{6,4}\,a_{4,6} - a_{7,3}\,a_{3,7}\,a_{6,5}\,a_{5,6} + a_{7,3}\,a_{3,7}\,a_{5,5}\,a_{4,4} - a_{7,2}\,a_{2,3}\,a_{3,7}\,a_{5,5}$$
$$- a_{7,2}\,a_{2,3}\,a_{3,7}\,a_{4,4} + a_{7,5}\,a_{5,3}\,a_{3,7}\,a_{4,4} + a_{7,5}\,a_{5,4}\,a_{4,7}\,a_{3,3} - a_{7,5}\,a_{5,4}\,a_{4,2}\,a_{2,7} + a_{7,3}\,a_{3,6}\,a_{6,4}\,a_{4,7} + a_{7,3}\,a_{3,6}\,a_{6,1}\,a_{1,7}$$
$$- a_{7,2}\,a_{2,5}\,a_{5,4}\,a_{4,7} - a_{7,6}\,a_{6,5}\,a_{5,4}\,a_{4,7} - a_{7,6}\,a_{6,5}\,a_{5,3}\,a_{3,7} + a_{7,6}\,a_{6,4}\,a_{4,2}\,a_{2,7} - a_{7,6}\,a_{2,7}\,a_{6,1}\,a_{1,2} + a_{7,5}\,a_{5,6}\,a_{6,4}\,a_{4,7}$$
$$+ a_{7,5}\,a_{5,6}\,a_{6,1}\,a_{1,7} + a_{8,3}\,a_{3,7}\,a_{7,2}\,a_{2,8} + a_{8,5}\,a_{5,4}\,a_{4,2}\,a_{2,8} - a_{8,6}\,a_{6,1}\,a_{1,2}\,a_{2,8} + a_{8,6}\,a_{6,4}\,a_{4,2}\,a_{2,8} + a_{8,6}\,a_{6,5}\,a_{5,3}\,a_{3,8}$$
$$- a_{8,3}\,a_{3,8}\,a_{6,5}\,a_{5,6} + a_{8,3}\,a_{3,8}\,a_{6,4}\,a_{4,6} + a_{8,5}\,a_{5,3}\,a_{3,8}\,a_{4,4} + a_{8,3}\,a_{3,8}\,a_{7,1}\,a_{1,7} + a_{8,3}\,a_{3,8}\,a_{7,2}\,a_{2,7} + a_{8,3}\,a_{3,8}\,a_{5,5}\,a_{4,4}$$
$$- a_{2,7}\,a_{7,1}\,a_{1,2}\,a_{5,5} - a_{2,7}\,a_{7,1}\,a_{1,2}\,a_{3,3} - a_{2,7}\,a_{7,1}\,a_{1,2}\,a_{4,4}$$

F_3

$$-a_{5,4}\,a_{4,2}\,a_{2,5} - a_{5,5}\,a_{4,4}\,a_{3,3} + a_{6,5}\,a_{5,3}\,a_{3,6} - a_{6,5}\,a_{5,4}\,a_{4,6} - a_{6,4}\,a_{4,6}\,a_{5,5} - a_{6,4}\,a_{4,6}\,a_{3,3} + a_{6,5}\,a_{5,6}\,a_{4,4} + a_{6,5}\,a_{5,6}\,a_{3,3}$$
$$- a_{7,2}\,a_{2,3}\,a_{3,7} + a_{7,5}\,a_{5,3}\,a_{3,7} + a_{7,5}\,a_{5,4}\,a_{4,7} - a_{7,6}\,a_{6,1}\,a_{1,7} - a_{7,6}\,a_{6,4}\,a_{4,7} - a_{2,7}\,a_{7,1}\,a_{1,2} - a_{7,1}\,a_{1,7}\,a_{5,5} - a_{7,1}\,a_{1,7}\,a_{4,4}$$
$$- a_{7,1}\,a_{1,7}\,a_{3,3} - a_{7,2}\,a_{2,7}\,a_{5,5} - a_{7,2}\,a_{2,7}\,a_{4,4} - a_{7,2}\,a_{2,7}\,a_{3,3} + a_{7,3}\,a_{3,7}\,a_{5,5} + a_{7,3}\,a_{3,7}\,a_{4,4} + a_{8,5}\,a_{5,3}\,a_{3,8} + a_{8,3}\,a_{3,8}\,a_{5,5}$$
$$+ a_{8,3}\,a_{3,8}\,a_{4,4}$$

F_2

$$a_{8,3}\,a_{3,8} - a_{7,1}\,a_{1,7} - a_{7,2}\,a_{2,7} + a_{7,3}\,a_{3,7} - a_{6,4}\,a_{4,6} + a_{6,5}\,a_{5,6} - a_{5,5}\,a_{4,4} - a_{5,5}\,a_{3,3} - a_{4,4}\,a_{3,3}$$

F_1

$$-a_{5,5} - a_{4,4} - a_{3,3}$$

F_0

-1

"adjoint (-A)"

$$[\,a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{7,5}\,a_{8,6}\,a_{2,8}\,a_{3,7} + a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{7,6}\,a_{2,7}\,a_{3,8} + a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{7,6}\,a_{2,8}\,a_{3,7}$$
$$+ a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{2,7}\,a_{3,8} + a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{2,8}\,a_{3,7} + a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{2,7}\,a_{3,8}$$
$$+ a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{3,7} + a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{2,7}\,a_{3,8} + a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{3,7}$$
$$+ a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{7,6}\,a_{2,7}\,a_{3,8} + a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{7,6}\,a_{2,8}\,a_{3,7} + a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{7,6}\,a_{2,7}\,a_{3,8}$$
$$+ a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{7,6}\,a_{2,8}\,a_{3,7} - a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{7,5}\,a_{5,6}\,a_{2,7}\,a_{3,8} - a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{7,5}\,a_{5,6}\,a_{2,8}\,a_{3,7}$$
$$+ a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{3,8}\,a_{4,7} + a_{7,2}\,a_{2,3}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{3,8}\,a_{4,7} + a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{7,5}\,a_{8,6}\,a_{2,7}\,a_{3,8}$$
$$+ a_{7,2}\,a_{2,3}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{3,8}\,a_{4,7} - a_{7,2}\,a_{3,3}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{2,8}\,a_{4,7} - a_{7,2}\,a_{3,3}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{4,7}$$
$$- a_{7,2}\,a_{3,3}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{4,7} - a_{7,2}\,a_{5,3}\,a_{4,4}\,a_{6,5}\,a_{8,6}\,a_{2,7}\,a_{3,8} - a_{7,2}\,a_{5,3}\,a_{4,4}\,a_{6,5}\,a_{8,6}\,a_{2,8}\,a_{3,7}$$
$$+ a_{7,2}\,a_{5,3}\,a_{6,4}\,a_{2,5}\,a_{8,6}\,a_{3,8}\,a_{4,7} - a_{7,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} - a_{7,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{4,6}\,a_{2,7}\,a_{3,8}$$
$$- a_{7,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{4,6}\,a_{2,8}\,a_{3,7} + a_{7,2}\,a_{8,3}\,a_{4,4}\,a_{6,5}\,a_{5,6}\,a_{2,7}\,a_{3,8} + a_{7,2}\,a_{8,3}\,a_{4,4}\,a_{6,5}\,a_{5,6}\,a_{2,8}\,a_{3,7}$$
$$- a_{7,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} - a_{7,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{4,6}\,a_{2,7}\,a_{3,8} - a_{7,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{4,6}\,a_{2,8}\,a_{3,7}$$
$$- a_{7,2}\,a_{8,3}\,a_{6,4}\,a_{2,5}\,a_{5,6}\,a_{3,8}\,a_{4,7} - a_{7,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} - a_{7,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{4,6}\,a_{2,7}\,a_{3,8}$$
$$- a_{7,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{4,6}\,a_{2,8}\,a_{3,7}\,,\, -a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{7,5}\,a_{8,6}\,a_{3,8}\,a_{4,7} - a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{7,6}\,a_{3,8}\,a_{4,7}$$
$$- a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{3,8}\,a_{4,7} - a_{1,2}\,a_{7,3}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{3,8}\,a_{4,7} - a_{1,2}\,a_{7,3}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{3,8}\,a_{4,7}$$
$$- a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{7,6}\,a_{3,8}\,a_{4,7} - a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{7,6}\,a_{3,8}\,a_{4,7} + a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{7,5}\,a_{5,6}\,a_{3,8}\,a_{4,7}$$
$$- a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{7,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} - a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{7,6}\,a_{1,7}\,a_{3,8} - a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}$$
$$- a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} - a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} - a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{7,6}\,a_{1,7}\,a_{3,8}$$
$$- a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{7,6}\,a_{1,7}\,a_{3,8} + a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{7,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} + a_{7,2}\,a_{5,3}\,a_{4,4}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}$$
$$+ a_{7,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{3,8} - a_{7,2}\,a_{8,3}\,a_{4,4}\,a_{6,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} + a_{7,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{4,6}\,a_{1,7}\,a_{3,8}$$
$$+ a_{7,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{4,6}\,a_{1,7}\,a_{3,8}\,,\, -a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{7,5}\,a_{8,6}\,a_{2,8}\,a_{4,7} - a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{7,6}\,a_{2,8}\,a_{4,7}$$
$$- a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{2,8}\,a_{4,7} - a_{1,2}\,a_{7,3}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{4,7} - a_{1,2}\,a_{7,3}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{4,7}$$
$$- a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{7,6}\,a_{2,8}\,a_{4,7} - a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{7,6}\,a_{2,8}\,a_{4,7} + a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{7,5}\,a_{5,6}\,a_{2,8}\,a_{4,7}$$
$$- a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{7,5}\,a_{8,6}\,a_{1,7}\,a_{2,8} - a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{7,6}\,a_{1,7}\,a_{2,8} - a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{2,8}$$
$$- a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{2,8} - a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{2,8} - a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{7,6}\,a_{1,7}\,a_{2,8}$$
$$- a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{7,6}\,a_{1,7}\,a_{2,8} + a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{7,5}\,a_{5,6}\,a_{1,7}\,a_{2,8} + a_{7,2}\,a_{5,3}\,a_{4,4}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{2,8}$$

$$+a_{7,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{2,8}-a_{7,2}\,a_{8,3}\,a_{4,4}\,a_{6,5}\,a_{5,6}\,a_{1,7}\,a_{2,8}+a_{7,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{4,6}\,a_{1,7}\,a_{2,8}$$
$$+a_{7,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{4,6}\,a_{1,7}\,a_{2,8}\,,\,a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{7,5}\,a_{8,6}\,a_{2,7}\,a_{3,8}+a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{7,5}\,a_{8,6}\,a_{2,8}\,a_{3,7}$$
$$+a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{7,6}\,a_{2,7}\,a_{3,8}+a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{7,6}\,a_{2,8}\,a_{3,7}+a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{2,7}\,a_{3,8}$$
$$+a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{2,8}\,a_{3,7}+a_{1,2}\,a_{7,3}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{2,7}\,a_{3,8}+a_{1,2}\,a_{7,3}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{3,7}$$
$$+a_{1,2}\,a_{7,3}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{2,7}\,a_{3,8}+a_{1,2}\,a_{7,3}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{3,7}+a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{7,6}\,a_{2,7}\,a_{3,8}$$
$$+a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{7,6}\,a_{2,8}\,a_{3,7}+a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{7,6}\,a_{2,7}\,a_{3,8}+a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{7,6}\,a_{2,8}\,a_{3,7}$$
$$-a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{7,5}\,a_{5,6}\,a_{2,7}\,a_{3,8}-a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{7,5}\,a_{5,6}\,a_{2,8}\,a_{3,7}-a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}$$
$$-a_{7,2}\,a_{2,3}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}-a_{7,2}\,a_{2,3}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{3,8}+a_{7,2}\,a_{3,3}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{2,8}$$
$$+a_{7,2}\,a_{3,3}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{2,8}+a_{7,2}\,a_{3,3}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{2,8}-a_{7,2}\,a_{5,3}\,a_{6,4}\,a_{2,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}$$
$$+a_{7,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}+a_{7,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}+a_{7,2}\,a_{8,3}\,a_{6,4}\,a_{2,5}\,a_{5,6}\,a_{1,7}\,a_{3,8}$$
$$+a_{7,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}\,,\,a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{6,5}\,a_{7,6}\,a_{2,7}\,a_{3,8}+a_{1,2}\,a_{7,3}\,a_{6,4}\,a_{8,5}\,a_{4,6}\,a_{2,8}\,a_{3,7}$$
$$+a_{1,2}\,a_{7,3}\,a_{6,4}\,a_{8,5}\,a_{4,6}\,a_{2,7}\,a_{3,8}+a_{1,2}\,a_{7,3}\,a_{6,4}\,a_{8,5}\,a_{3,6}\,a_{2,8}\,a_{4,7}-a_{1,2}\,a_{7,3}\,a_{6,4}\,a_{2,5}\,a_{8,6}\,a_{3,8}\,a_{4,7}$$
$$+a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{6,5}\,a_{8,6}\,a_{2,8}\,a_{3,7}+a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{6,5}\,a_{8,6}\,a_{2,7}\,a_{3,8}-a_{1,2}\,a_{3,3}\,a_{6,4}\,a_{8,5}\,a_{7,6}\,a_{2,8}\,a_{4,7}$$
$$-a_{1,2}\,a_{3,3}\,a_{6,4}\,a_{7,5}\,a_{8,6}\,a_{2,8}\,a_{4,7}+a_{1,2}\,a_{2,3}\,a_{6,4}\,a_{8,5}\,a_{7,6}\,a_{3,8}\,a_{4,7}+a_{1,2}\,a_{2,3}\,a_{6,4}\,a_{7,5}\,a_{8,6}\,a_{3,8}\,a_{4,7}$$
$$+a_{7,2}\,a_{8,3}\,a_{6,4}\,a_{2,5}\,a_{4,6}\,a_{1,7}\,a_{3,8}+a_{7,2}\,a_{8,3}\,a_{4,4}\,a_{6,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}+a_{7,2}\,a_{3,3}\,a_{6,4}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{2,8}$$
$$+a_{7,2}\,a_{3,3}\,a_{4,4}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{2,8}-a_{7,2}\,a_{2,3}\,a_{6,4}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{3,8}-a_{7,2}\,a_{2,3}\,a_{4,4}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}$$
$$-a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{7,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}-a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{2,5}\,a_{7,6}\,a_{1,7}\,a_{3,8}+a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{8,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}$$
$$-a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{2,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}-a_{4,2}\,a_{3,3}\,a_{6,4}\,a_{8,5}\,a_{7,6}\,a_{1,7}\,a_{2,8}-a_{4,2}\,a_{3,3}\,a_{6,4}\,a_{7,5}\,a_{8,6}\,a_{1,7}\,a_{2,8}$$
$$+a_{4,2}\,a_{2,3}\,a_{6,4}\,a_{8,5}\,a_{7,6}\,a_{1,7}\,a_{3,8}+a_{4,2}\,a_{2,3}\,a_{6,4}\,a_{7,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}-a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{7,5}\,a_{4,6}\,a_{2,8}\,a_{3,7}$$
$$-a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{7,5}\,a_{4,6}\,a_{2,7}\,a_{3,8}-a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{7,5}\,a_{3,6}\,a_{2,8}\,a_{4,7}-a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{2,5}\,a_{7,6}\,a_{3,8}\,a_{4,7}$$
$$+a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{6,5}\,a_{7,6}\,a_{2,8}\,a_{3,7}\,,\,-a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{7,5}\,a_{8,6}\,a_{3,8}\,a_{4,7}+a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{7,5}\,a_{8,6}\,a_{2,7}\,a_{3,8}$$
$$+a_{1,2}\,a_{3,3}\,a_{5,4}\,a_{8,5}\,a_{7,6}\,a_{2,8}\,a_{4,7}+a_{1,2}\,a_{3,3}\,a_{5,4}\,a_{7,5}\,a_{8,6}\,a_{2,8}\,a_{4,7}-a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{8,5}\,a_{7,6}\,a_{3,8}\,a_{4,7}$$
$$+a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{5,5}\,a_{7,6}\,a_{2,7}\,a_{3,8}-a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{8,5}\,a_{4,6}\,a_{2,8}\,a_{3,7}-a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{8,5}\,a_{4,6}\,a_{2,7}\,a_{3,8}$$
$$-a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{8,5}\,a_{3,6}\,a_{2,8}\,a_{4,7}+a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{2,5}\,a_{8,6}\,a_{3,8}\,a_{4,7}+a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{3,7}$$
$$+a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{2,7}\,a_{3,8}+a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{3,7}+a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{2,7}\,a_{3,8}$$
$$+a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{7,6}\,a_{2,8}\,a_{3,7}+a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{7,6}\,a_{2,7}\,a_{3,8}+a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{7,5}\,a_{8,6}\,a_{2,8}\,a_{3,7}$$
$$+a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{3,6}\,a_{2,8}\,a_{4,7}+a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{2,5}\,a_{7,6}\,a_{3,8}\,a_{4,7}-a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{7,5}\,a_{5,6}\,a_{2,8}\,a_{3,7}$$
$$-a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{7,5}\,a_{5,6}\,a_{2,7}\,a_{3,8}+a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{5,5}\,a_{7,6}\,a_{2,8}\,a_{3,7}+a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{2,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}$$
$$+a_{4,2}\,a_{3,3}\,a_{5,4}\,a_{8,5}\,a_{7,6}\,a_{1,7}\,a_{2,8}+a_{4,2}\,a_{3,3}\,a_{5,4}\,a_{7,5}\,a_{8,6}\,a_{1,7}\,a_{2,8}-a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{8,5}\,a_{7,6}\,a_{1,7}\,a_{3,8}$$
$$-a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{7,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}+a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{4,6}\,a_{2,8}\,a_{3,7}+a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{4,6}\,a_{2,7}\,a_{3,8}$$
$$-a_{7,2}\,a_{3,3}\,a_{5,4}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{2,8}+a_{7,2}\,a_{3,3}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{2,8}+a_{7,2}\,a_{3,3}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{2,8}$$
$$+a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{3,8}-a_{7,2}\,a_{2,3}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{3,8}-a_{7,2}\,a_{2,3}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}$$
$$+a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}+a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{2,5}\,a_{7,6}\,a_{1,7}\,a_{3,8}-a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{8,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}$$
$$-a_{7,2}\,a_{8,3}\,a_{5,4}\,a_{2,5}\,a_{4,6}\,a_{1,7}\,a_{3,8}+a_{7,2}\,a_{8,3}\,a_{4,4}\,a_{5,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}+a_{7,2}\,a_{8,3}\,a_{4,4}\,a_{2,5}\,a_{5,6}\,a_{1,7}\,a_{3,8}$$
$$+a_{7,2}\,a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}-a_{7,2}\,a_{5,3}\,a_{4,4}\,a_{2,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}\,,\,-a_{1,2}\,a_{2,3}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{3,8}\,a_{4,7}$$
$$-a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{3,8}\,a_{4,7}-a_{1,2}\,a_{2,3}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{3,8}\,a_{4,7}-a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{2,5}\,a_{8,6}\,a_{3,8}\,a_{4,7}$$
$$+a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{6,5}\,a_{8,6}\,a_{2,8}\,a_{3,7}+a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{6,5}\,a_{8,6}\,a_{2,7}\,a_{3,8}+a_{1,2}\,a_{3,3}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{4,7}$$
$$+a_{1,2}\,a_{3,3}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{4,7}+a_{1,2}\,a_{3,3}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{2,8}\,a_{4,7}-a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{6,5}\,a_{5,6}\,a_{2,7}\,a_{3,8}$$
$$+a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{4,6}\,a_{2,8}\,a_{3,7}+a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{4,6}\,a_{2,7}\,a_{3,8}+a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{3,6}\,a_{2,8}\,a_{4,7}$$
$$+a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{4,6}\,a_{2,8}\,a_{3,7}+a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{4,6}\,a_{2,7}\,a_{3,8}+a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{4,6}\,a_{2,8}\,a_{3,7}$$
$$+a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{4,6}\,a_{2,7}\,a_{3,8}+a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{3,6}\,a_{2,8}\,a_{4,7}-a_{4,2}\,a_{2,3}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{3,8}$$
$$-a_{4,2}\,a_{2,3}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}-a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}+a_{4,2}\,a_{3,3}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{2,8}$$
$$+a_{4,2}\,a_{3,3}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{2,8}-a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{2,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}+a_{4,2}\,a_{3,3}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{2,8}$$
$$+a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}+a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{2,5}\,a_{5,6}\,a_{1,7}\,a_{3,8}+a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}$$

$$
\begin{aligned}
&+ a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}\,,\; a_{1,2}\,a_{2,3}\,a_{6,4}\,a_{7,5}\,a_{5,6}\,a_{3,8}\,a_{4,7} - a_{1,2}\,a_{2,3}\,a_{6,4}\,a_{5,5}\,a_{7,6}\,a_{3,8}\,a_{4,7}\\
&- a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{6,5}\,a_{7,6}\,a_{3,8}\,a_{4,7} + a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{6,5}\,a_{7,6}\,a_{2,8}\,a_{3,7} + a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{6,5}\,a_{7,6}\,a_{2,7}\,a_{3,8}\\
&- a_{1,2}\,a_{3,3}\,a_{6,4}\,a_{7,5}\,a_{5,6}\,a_{2,8}\,a_{4,7} + a_{1,2}\,a_{3,3}\,a_{6,4}\,a_{5,5}\,a_{7,6}\,a_{2,8}\,a_{4,7} + a_{1,2}\,a_{3,3}\,a_{5,4}\,a_{6,5}\,a_{7,6}\,a_{2,8}\,a_{4,7}\\
&- a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{6,5}\,a_{4,6}\,a_{2,7}\,a_{3,8} - a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} + a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{6,5}\,a_{5,6}\,a_{2,8}\,a_{3,7}\\
&+ a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{6,5}\,a_{5,6}\,a_{2,7}\,a_{3,8} - a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{7,5}\,a_{4,6}\,a_{2,8}\,a_{3,7} - a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{7,5}\,a_{4,6}\,a_{2,7}\,a_{3,8}\\
&- a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{7,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} - a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{2,5}\,a_{7,6}\,a_{3,8}\,a_{4,7} - a_{1,2}\,a_{7,3}\,a_{6,4}\,a_{5,5}\,a_{4,6}\,a_{2,8}\,a_{3,7}\\
&- a_{1,2}\,a_{7,3}\,a_{6,4}\,a_{5,5}\,a_{4,6}\,a_{2,7}\,a_{3,8} - a_{1,2}\,a_{7,3}\,a_{6,4}\,a_{5,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} - a_{1,2}\,a_{7,3}\,a_{6,4}\,a_{2,5}\,a_{5,6}\,a_{3,8}\,a_{4,7}\\
&- a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{6,5}\,a_{4,6}\,a_{2,8}\,a_{3,7} - a_{4,2}\,a_{2,3}\,a_{6,4}\,a_{5,5}\,a_{7,6}\,a_{1,7}\,a_{3,8} - a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{6,5}\,a_{7,6}\,a_{1,7}\,a_{3,8}\\
&- a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{2,5}\,a_{7,6}\,a_{1,7}\,a_{3,8} - a_{4,2}\,a_{3,3}\,a_{6,4}\,a_{7,5}\,a_{5,6}\,a_{1,7}\,a_{2,8} + a_{4,2}\,a_{3,3}\,a_{6,4}\,a_{5,5}\,a_{7,6}\,a_{1,7}\,a_{2,8}\\
&+ a_{4,2}\,a_{3,3}\,a_{5,4}\,a_{6,5}\,a_{7,6}\,a_{1,7}\,a_{2,8} + a_{4,2}\,a_{2,3}\,a_{6,4}\,a_{7,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} + a_{7,2}\,a_{3,3}\,a_{4,4}\,a_{6,5}\,a_{5,6}\,a_{1,7}\,a_{2,8}\\
&+ a_{7,2}\,a_{2,3}\,a_{6,4}\,a_{5,5}\,a_{4,6}\,a_{1,7}\,a_{3,8} + a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{6,5}\,a_{4,6}\,a_{1,7}\,a_{3,8} - a_{7,2}\,a_{2,3}\,a_{4,4}\,a_{6,5}\,a_{5,6}\,a_{1,7}\,a_{3,8}\\
&- a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{5,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} - a_{4,2}\,a_{7,3}\,a_{6,4}\,a_{2,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} - a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}\\
&- a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{7,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} + a_{7,2}\,a_{5,3}\,a_{4,4}\,a_{6,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} - a_{7,2}\,a_{3,3}\,a_{6,4}\,a_{5,5}\,a_{4,6}\,a_{1,7}\,a_{2,8}\\
&- a_{7,2}\,a_{3,3}\,a_{5,4}\,a_{6,5}\,a_{4,6}\,a_{1,7}\,a_{2,8} + a_{7,2}\,a_{5,3}\,a_{6,4}\,a_{2,5}\,a_{4,6}\,a_{1,7}\,a_{3,8}\,]\\
&[-a_{6,1}\,a_{2,3}\,a_{5,4}\,a_{7,5}\,a_{8,6}\,a_{3,8}\,a_{4,7} - a_{6,1}\,a_{2,3}\,a_{5,4}\,a_{8,5}\,a_{7,6}\,a_{3,8}\,a_{4,7} + a_{6,1}\,a_{3,3}\,a_{5,4}\,a_{7,5}\,a_{8,6}\,a_{2,8}\,a_{4,7}\\
&+ a_{6,1}\,a_{3,3}\,a_{5,4}\,a_{8,5}\,a_{7,6}\,a_{2,8}\,a_{4,7} + a_{6,1}\,a_{5,3}\,a_{4,4}\,a_{7,5}\,a_{8,6}\,a_{2,7}\,a_{3,8} + a_{6,1}\,a_{5,3}\,a_{4,4}\,a_{7,5}\,a_{8,6}\,a_{2,8}\,a_{3,7}\\
&+ a_{6,1}\,a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{7,6}\,a_{2,7}\,a_{3,8} + a_{6,1}\,a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{7,6}\,a_{2,8}\,a_{3,7} + a_{6,1}\,a_{7,3}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{2,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{7,3}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{3,7} + a_{6,1}\,a_{7,3}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{2,7}\,a_{3,8} + a_{6,1}\,a_{7,3}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{3,7}\\
&+ a_{6,1}\,a_{7,3}\,a_{5,4}\,a_{2,5}\,a_{8,6}\,a_{3,8}\,a_{4,7} - a_{6,1}\,a_{7,3}\,a_{5,4}\,a_{8,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} - a_{7,1}\,a_{2,3}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{3,8}\,a_{4,7}\\
&- a_{7,1}\,a_{2,3}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{3,8}\,a_{4,7} - a_{7,1}\,a_{2,3}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{3,8}\,a_{4,7} + a_{7,1}\,a_{3,3}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{2,8}\,a_{4,7}\\
&+ a_{7,1}\,a_{3,3}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{4,7} + a_{7,1}\,a_{3,3}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{4,7} + a_{7,1}\,a_{5,3}\,a_{4,4}\,a_{6,5}\,a_{8,6}\,a_{2,7}\,a_{3,8}\\
&+ a_{7,1}\,a_{5,3}\,a_{4,4}\,a_{6,5}\,a_{8,6}\,a_{2,8}\,a_{3,7} - a_{7,1}\,a_{5,3}\,a_{6,4}\,a_{2,5}\,a_{8,6}\,a_{3,8}\,a_{4,7} + a_{7,1}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{3,6}\,a_{2,8}\,a_{4,7}\\
&+ a_{7,1}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{4,6}\,a_{2,7}\,a_{3,8} + a_{7,1}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{4,6}\,a_{2,8}\,a_{3,7} - a_{7,1}\,a_{8,3}\,a_{4,4}\,a_{6,5}\,a_{5,6}\,a_{2,7}\,a_{3,8}\\
&- a_{7,1}\,a_{8,3}\,a_{4,4}\,a_{6,5}\,a_{5,6}\,a_{2,8}\,a_{3,7} + a_{7,1}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{4,6}\,a_{2,7}\,a_{3,8} + a_{7,1}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{4,6}\,a_{2,8}\,a_{3,7}\\
&+ a_{7,1}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{5,6}\,a_{2,7}\,a_{3,8} - a_{7,1}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} + a_{7,1}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{4,6}\,a_{2,8}\,a_{3,7}\,,\\
&-a_{6,1}\,a_{5,3}\,a_{4,4}\,a_{7,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{7,6}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{7,3}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}\\
&- a_{6,1}\,a_{7,3}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{7,3}\,a_{5,4}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{8,3}\,a_{4,4}\,a_{5,5}\,a_{7,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{8,3}\,a_{4,4}\,a_{7,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{4,6}\,a_{1,7}\,a_{3,8} - a_{7,1}\,a_{5,3}\,a_{4,4}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}\\
&- a_{7,1}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{3,8} + a_{7,1}\,a_{8,3}\,a_{4,4}\,a_{6,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} - a_{7,1}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{4,6}\,a_{1,7}\,a_{3,8}\\
&- a_{7,1}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{4,6}\,a_{1,7}\,a_{3,8}\,,\; -a_{6,1}\,a_{5,3}\,a_{4,4}\,a_{7,5}\,a_{8,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{7,6}\,a_{1,7}\,a_{2,8}\\
&- a_{6,1}\,a_{7,3}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{7,3}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{7,3}\,a_{5,4}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{2,8}\\
&- a_{6,1}\,a_{8,3}\,a_{4,4}\,a_{5,5}\,a_{7,6}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{8,3}\,a_{4,4}\,a_{7,5}\,a_{5,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{4,6}\,a_{1,7}\,a_{2,8}\\
&- a_{7,1}\,a_{5,3}\,a_{4,4}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{2,8} - a_{7,1}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{4,6}\,a_{1,7}\,a_{2,8}\,,\; a_{6,1}\,a_{2,3}\,a_{5,4}\,a_{7,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{2,3}\,a_{5,4}\,a_{8,5}\,a_{7,6}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{3,3}\,a_{5,4}\,a_{7,5}\,a_{8,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{3,3}\,a_{5,4}\,a_{8,5}\,a_{7,6}\,a_{1,7}\,a_{2,8}\\
&- a_{6,1}\,a_{7,3}\,a_{5,4}\,a_{2,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{7,3}\,a_{5,4}\,a_{8,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{8,3}\,a_{5,4}\,a_{2,5}\,a_{7,6}\,a_{1,7}\,a_{3,8}\\
&- a_{6,1}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} + a_{7,1}\,a_{2,3}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} + a_{7,1}\,a_{2,3}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{7,1}\,a_{2,3}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} - a_{7,1}\,a_{3,3}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{2,8} - a_{7,1}\,a_{3,3}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{2,8}\\
&- a_{7,1}\,a_{3,3}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{2,8} + a_{7,1}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} - a_{7,1}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}\\
&- a_{7,1}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} - a_{7,1}\,a_{8,3}\,a_{6,4}\,a_{2,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} - a_{7,1}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}\,,\\
&a_{6,1}\,a_{2,3}\,a_{4,4}\,a_{7,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{2,3}\,a_{4,4}\,a_{8,5}\,a_{7,6}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{3,3}\,a_{4,4}\,a_{7,5}\,a_{8,6}\,a_{1,7}\,a_{2,8}
\end{aligned}
$$

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\begin{aligned}
&- a_{6,1}\,a_{3,3}\,a_{4,4}\,a_{8,5}\,a_{7,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{7,3}\,a_{4,4}\,a_{2,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{7,3}\,a_{4,4}\,a_{8,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}\\
&- a_{6,1}\,a_{8,3}\,a_{4,4}\,a_{2,5}\,a_{7,6}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{8,3}\,a_{4,4}\,a_{7,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} + a_{7,1}\,a_{2,3}\,a_{4,4}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{7,1}\,a_{2,3}\,a_{6,4}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{3,8} - a_{7,1}\,a_{3,3}\,a_{4,4}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{2,8} - a_{7,1}\,a_{3,3}\,a_{6,4}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{2,8}\\
&- a_{7,1}\,a_{8,3}\,a_{4,4}\,a_{6,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} - a_{7,1}\,a_{8,3}\,a_{6,4}\,a_{2,5}\,a_{4,6}\,a_{1,7}\,a_{3,8}\Big],\ a_{7,1}\,\big(a_{2,3}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} + a_{2,3}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{3,8}\\
&- a_{2,3}\,a_{5,4}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{3,8} - a_{3,3}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{2,8} - a_{3,3}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{2,8} + a_{3,3}\,a_{5,4}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{2,8}\\
&+ a_{5,3}\,a_{4,4}\,a_{2,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} - a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} - a_{8,3}\,a_{4,4}\,a_{2,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} - a_{8,3}\,a_{4,4}\,a_{5,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}\\
&+ a_{8,3}\,a_{5,4}\,a_{2,5}\,a_{4,6}\,a_{1,7}\,a_{3,8}\big),\ -a_{6,1}\,\big(a_{2,3}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} + a_{2,3}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} - a_{2,3}\,a_{5,4}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{3,8}\\
&- a_{3,3}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{2,8} - a_{3,3}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{2,8} + a_{3,3}\,a_{5,4}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{2,8} + a_{5,3}\,a_{4,4}\,a_{2,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}\\
&- a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} - a_{8,3}\,a_{4,4}\,a_{2,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} - a_{8,3}\,a_{4,4}\,a_{5,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} + a_{8,3}\,a_{5,4}\,a_{2,5}\,a_{4,6}\,a_{1,7}\,a_{3,8}\big),\\
&-a_{6,1}\,a_{2,3}\,a_{4,4}\,a_{5,5}\,a_{7,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{2,3}\,a_{4,4}\,a_{7,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{2,3}\,a_{5,4}\,a_{7,5}\,a_{4,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{3,3}\,a_{4,4}\,a_{5,5}\,a_{7,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{3,3}\,a_{4,4}\,a_{7,5}\,a_{5,6}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{3,3}\,a_{5,4}\,a_{7,5}\,a_{4,6}\,a_{1,7}\,a_{2,8}\\
&- a_{6,1}\,a_{5,3}\,a_{4,4}\,a_{2,5}\,a_{7,6}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{5,3}\,a_{4,4}\,a_{7,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{7,3}\,a_{4,4}\,a_{2,5}\,a_{5,6}\,a_{1,7}\,a_{3,8}\\
&- a_{6,1}\,a_{7,3}\,a_{4,4}\,a_{5,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{7,3}\,a_{5,4}\,a_{2,5}\,a_{4,6}\,a_{1,7}\,a_{3,8} + a_{7,1}\,a_{2,3}\,a_{4,4}\,a_{6,5}\,a_{5,6}\,a_{1,7}\,a_{3,8}\\
&- a_{7,1}\,a_{2,3}\,a_{5,4}\,a_{6,5}\,a_{4,6}\,a_{1,7}\,a_{3,8} - a_{7,1}\,a_{2,3}\,a_{6,4}\,a_{5,5}\,a_{4,6}\,a_{1,7}\,a_{3,8} - a_{7,1}\,a_{3,3}\,a_{4,4}\,a_{6,5}\,a_{5,6}\,a_{1,7}\,a_{2,8}\\
&+ a_{7,1}\,a_{3,3}\,a_{5,4}\,a_{6,5}\,a_{4,6}\,a_{1,7}\,a_{2,8} + a_{7,1}\,a_{3,3}\,a_{6,4}\,a_{5,5}\,a_{4,6}\,a_{1,7}\,a_{2,8} - a_{7,1}\,a_{5,3}\,a_{4,4}\,a_{6,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}\\
&- a_{7,1}\,a_{5,3}\,a_{6,4}\,a_{2,5}\,a_{4,6}\,a_{1,7}\,a_{3,8}\big]\\
&\big[a_{6,1}\,a_{4,2}\,a_{5,4}\,a_{7,5}\,a_{8,6}\,a_{2,7}\,a_{3,8} + a_{6,1}\,a_{4,2}\,a_{5,4}\,a_{7,5}\,a_{8,6}\,a_{2,8}\,a_{3,7} + a_{6,1}\,a_{4,2}\,a_{5,4}\,a_{8,5}\,a_{7,6}\,a_{2,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{4,2}\,a_{5,4}\,a_{8,5}\,a_{7,6}\,a_{2,8}\,a_{3,7} + a_{6,1}\,a_{7,2}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{2,7}\,a_{3,8} + a_{6,1}\,a_{7,2}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{3,7}\\
&+ a_{6,1}\,a_{7,2}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{2,7}\,a_{3,8} + a_{6,1}\,a_{7,2}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{3,7} + a_{6,1}\,a_{7,2}\,a_{5,4}\,a_{2,5}\,a_{8,6}\,a_{3,8}\,a_{4,7}\\
&- a_{6,1}\,a_{7,2}\,a_{5,4}\,a_{8,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} - a_{6,1}\,a_{7,2}\,a_{5,4}\,a_{8,5}\,a_{4,6}\,a_{2,7}\,a_{3,8} - a_{6,1}\,a_{7,2}\,a_{5,4}\,a_{8,5}\,a_{4,6}\,a_{2,8}\,a_{3,7}\\
&+ a_{7,1}\,a_{4,2}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{2,7}\,a_{3,8} + a_{7,1}\,a_{4,2}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{2,8}\,a_{3,7} + a_{7,1}\,a_{4,2}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{2,7}\,a_{3,8}\\
&+ a_{7,1}\,a_{4,2}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{3,7} + a_{7,1}\,a_{4,2}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{2,7}\,a_{3,8} + a_{7,1}\,a_{4,2}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{3,7},\\
&-a_{6,1}\,a_{1,2}\,a_{5,4}\,a_{7,5}\,a_{8,6}\,a_{3,8}\,a_{4,7} - a_{6,1}\,a_{1,2}\,a_{5,4}\,a_{8,5}\,a_{7,6}\,a_{3,8}\,a_{4,7} - a_{6,1}\,a_{4,2}\,a_{5,4}\,a_{7,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}\\
&- a_{6,1}\,a_{4,2}\,a_{5,4}\,a_{8,5}\,a_{7,6}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{7,2}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{7,2}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{7,2}\,a_{5,4}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{3,8} - a_{7,1}\,a_{1,2}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{3,8}\,a_{4,7} - a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{3,8}\,a_{4,7}\\
&- a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{3,8}\,a_{4,7} - a_{7,1}\,a_{4,2}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} - a_{7,1}\,a_{4,2}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}\\
&- a_{7,1}\,a_{4,2}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{3,8},\ -a_{6,1}\,a_{1,2}\,a_{5,4}\,a_{7,5}\,a_{8,6}\,a_{2,8}\,a_{4,7} - a_{6,1}\,a_{1,2}\,a_{5,4}\,a_{8,5}\,a_{7,6}\,a_{2,8}\,a_{4,7}\\
&- a_{6,1}\,a_{4,2}\,a_{5,4}\,a_{7,5}\,a_{8,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{4,2}\,a_{5,4}\,a_{8,5}\,a_{7,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{7,2}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{2,8}\\
&- a_{6,1}\,a_{7,2}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{7,2}\,a_{5,4}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{2,8} - a_{7,1}\,a_{1,2}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{2,8}\,a_{4,7}\\
&- a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{4,7} - a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{4,7} - a_{7,1}\,a_{4,2}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{2,8}\\
&- a_{7,1}\,a_{4,2}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{2,8} - a_{7,1}\,a_{4,2}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{2,8},\ a_{6,1}\,a_{1,2}\,a_{5,4}\,a_{7,5}\,a_{8,6}\,a_{2,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{1,2}\,a_{5,4}\,a_{7,5}\,a_{8,6}\,a_{2,8}\,a_{3,7} + a_{6,1}\,a_{1,2}\,a_{5,4}\,a_{8,5}\,a_{7,6}\,a_{2,7}\,a_{3,8} + a_{6,1}\,a_{1,2}\,a_{5,4}\,a_{8,5}\,a_{7,6}\,a_{2,8}\,a_{3,7}\\
&- a_{6,1}\,a_{7,2}\,a_{5,4}\,a_{2,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{7,2}\,a_{5,4}\,a_{8,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} + a_{7,1}\,a_{1,2}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{2,7}\,a_{3,8}\\
&+ a_{7,1}\,a_{1,2}\,a_{5,4}\,a_{6,5}\,a_{8,6}\,a_{2,8}\,a_{3,7} + a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{2,7}\,a_{3,8} + a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{3,7}\\
&+ a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{2,7}\,a_{3,8} + a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{3,7},\ a_{6,1}\,a_{1,2}\,a_{4,4}\,a_{7,5}\,a_{8,6}\,a_{2,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{1,2}\,a_{4,4}\,a_{7,5}\,a_{8,6}\,a_{2,8}\,a_{3,7} + a_{6,1}\,a_{1,2}\,a_{4,4}\,a_{8,5}\,a_{7,6}\,a_{2,7}\,a_{3,8} + a_{6,1}\,a_{1,2}\,a_{4,4}\,a_{8,5}\,a_{7,6}\,a_{2,8}\,a_{3,7}\\
&- a_{6,1}\,a_{7,2}\,a_{4,4}\,a_{2,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{7,2}\,a_{4,4}\,a_{8,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} + a_{7,1}\,a_{1,2}\,a_{4,4}\,a_{6,5}\,a_{8,6}\,a_{2,7}\,a_{3,8}\\
&+ a_{7,1}\,a_{1,2}\,a_{4,4}\,a_{6,5}\,a_{8,6}\,a_{2,8}\,a_{3,7} - a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{2,5}\,a_{8,6}\,a_{3,8}\,a_{4,7} + a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{8,5}\,a_{3,6}\,a_{2,8}\,a_{4,7}\\
&+ a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{8,5}\,a_{4,6}\,a_{2,7}\,a_{3,8} + a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{8,5}\,a_{4,6}\,a_{2,8}\,a_{3,7} - a_{7,1}\,a_{4,2}\,a_{6,4}\,a_{2,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{7,1}\,a_{4,2}\,a_{6,4}\,a_{8,5}\,a_{3,6}\,a_{1,7}\,a_{2,8},\ -a_{7,1}\,\big(-a_{1,2}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{2,7}\,a_{3,8} - a_{1,2}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{3,7} - a_{1,2}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{2,7}\,a_{3,8}\\
&- a_{1,2}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{3,7} - a_{1,2}\,a_{5,4}\,a_{2,5}\,a_{8,6}\,a_{3,8}\,a_{4,7} + a_{1,2}\,a_{5,4}\,a_{8,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} + a_{1,2}\,a_{5,4}\,a_{8,5}\,a_{4,6}\,a_{2,7}\,a_{3,8}\\
&+ a_{1,2}\,a_{5,4}\,a_{8,5}\,a_{4,6}\,a_{2,8}\,a_{3,7} - a_{4,2}\,a_{5,4}\,a_{2,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} + a_{4,2}\,a_{5,4}\,a_{8,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}\big),\ a_{6,1}\,\big(-a_{1,2}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{2,7}\,a_{3,8}\\
&- a_{1,2}\,a_{4,4}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{3,7} - a_{1,2}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{2,7}\,a_{3,8} - a_{1,2}\,a_{4,4}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{3,7} - a_{1,2}\,a_{5,4}\,a_{2,5}\,a_{8,6}\,a_{3,8}\,a_{4,7}\\
&+ a_{1,2}\,a_{5,4}\,a_{8,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} + a_{1,2}\,a_{5,4}\,a_{8,5}\,a_{4,6}\,a_{2,7}\,a_{3,8} + a_{1,2}\,a_{5,4}\,a_{8,5}\,a_{4,6}\,a_{2,8}\,a_{3,7} - a_{4,2}\,a_{5,4}\,a_{2,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{4,2}\,a_{5,4}\,a_{8,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}\big),\ -a_{7,1}\,a_{1,2}\,a_{5,4}\,a_{6,5}\,a_{4,6}\,a_{2,8}\,a_{3,7} - a_{7,1}\,a_{1,2}\,a_{5,4}\,a_{6,5}\,a_{4,6}\,a_{2,7}\,a_{3,8}
\end{aligned}
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\begin{aligned}
&- a_{7,1}\,a_{1,2}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} + a_{7,1}\,a_{1,2}\,a_{4,4}\,a_{6,5}\,a_{5,6}\,a_{2,8}\,a_{3,7} + a_{7,1}\,a_{1,2}\,a_{4,4}\,a_{6,5}\,a_{5,6}\,a_{2,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{7,2}\,a_{5,4}\,a_{2,5}\,a_{4,6}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{7,2}\,a_{4,4}\,a_{5,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{7,2}\,a_{4,4}\,a_{2,5}\,a_{5,6}\,a_{1,7}\,a_{3,8}\\
&- a_{6,1}\,a_{4,2}\,a_{5,4}\,a_{7,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{4,2}\,a_{5,4}\,a_{2,5}\,a_{7,6}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{1,2}\,a_{5,4}\,a_{7,5}\,a_{4,6}\,a_{2,8}\,a_{3,7}\\
&- a_{6,1}\,a_{1,2}\,a_{5,4}\,a_{7,5}\,a_{4,6}\,a_{2,7}\,a_{3,8} - a_{6,1}\,a_{1,2}\,a_{5,4}\,a_{7,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} - a_{6,1}\,a_{1,2}\,a_{5,4}\,a_{2,5}\,a_{7,6}\,a_{3,8}\,a_{4,7}\\
&+ a_{6,1}\,a_{1,2}\,a_{4,4}\,a_{7,5}\,a_{5,6}\,a_{2,8}\,a_{3,7} + a_{6,1}\,a_{1,2}\,a_{4,4}\,a_{7,5}\,a_{5,6}\,a_{2,7}\,a_{3,8} - a_{6,1}\,a_{1,2}\,a_{4,4}\,a_{5,5}\,a_{7,6}\,a_{2,8}\,a_{3,7}\\
&- a_{6,1}\,a_{1,2}\,a_{4,4}\,a_{5,5}\,a_{7,6}\,a_{2,7}\,a_{3,8} - a_{7,1}\,a_{4,2}\,a_{6,4}\,a_{2,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} - a_{7,1}\,a_{4,2}\,a_{6,4}\,a_{5,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}\\
&- a_{7,1}\,a_{4,2}\,a_{5,4}\,a_{6,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} - a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{5,5}\,a_{4,6}\,a_{2,8}\,a_{3,7} - a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{5,5}\,a_{4,6}\,a_{2,7}\,a_{3,8}\\
&- a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{2,5}\,a_{5,6}\,a_{3,8}\,a_{4,7} - a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{5,5}\,a_{3,6}\,a_{2,8}\,a_{4,7}\Big]
\end{aligned}
$$

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\begin{aligned}
&\Big[a_{6,1}\,a_{4,2}\,a_{8,3}\,a_{7,5}\,a_{5,6}\,a_{2,7}\,a_{3,8} + a_{6,1}\,a_{4,2}\,a_{8,3}\,a_{7,5}\,a_{5,6}\,a_{2,8}\,a_{3,7} - a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{5,5}\,a_{8,6}\,a_{3,8}\,a_{4,7}\\
&- a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{8,5}\,a_{5,6}\,a_{3,8}\,a_{4,7} + a_{6,1}\,a_{7,2}\,a_{3,3}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{4,7} + a_{6,1}\,a_{7,2}\,a_{3,3}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{4,7}\\
&- a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{2,5}\,a_{8,6}\,a_{3,8}\,a_{4,7} + a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{8,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} + a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{8,5}\,a_{4,6}\,a_{2,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{8,5}\,a_{4,6}\,a_{2,8}\,a_{3,7} + a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{2,5}\,a_{5,6}\,a_{3,8}\,a_{4,7} + a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{5,5}\,a_{3,6}\,a_{2,8}\,a_{4,7}\\
&+ a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{5,5}\,a_{4,6}\,a_{2,7}\,a_{3,8} + a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{5,5}\,a_{4,6}\,a_{2,8}\,a_{3,7} - a_{7,1}\,a_{4,2}\,a_{5,3}\,a_{6,5}\,a_{8,6}\,a_{2,7}\,a_{3,8}\\
&- a_{7,1}\,a_{4,2}\,a_{5,3}\,a_{6,5}\,a_{8,6}\,a_{2,8}\,a_{3,7} + a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{6,5}\,a_{5,6}\,a_{2,7}\,a_{3,8} + a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{6,5}\,a_{5,6}\,a_{2,8}\,a_{3,7}\\
&- a_{6,1}\,a_{4,2}\,a_{5,3}\,a_{7,5}\,a_{8,6}\,a_{2,7}\,a_{3,8} - a_{6,1}\,a_{4,2}\,a_{5,3}\,a_{7,5}\,a_{8,6}\,a_{2,8}\,a_{3,7} - a_{6,1}\,a_{4,2}\,a_{5,3}\,a_{8,5}\,a_{7,6}\,a_{2,7}\,a_{3,8}\\
&- a_{6,1}\,a_{4,2}\,a_{5,3}\,a_{8,5}\,a_{7,6}\,a_{2,8}\,a_{3,7} - a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{5,5}\,a_{8,6}\,a_{2,7}\,a_{3,8} - a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{3,7}\\
&- a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{8,5}\,a_{5,6}\,a_{2,7}\,a_{3,8} - a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{3,7} - a_{6,1}\,a_{4,2}\,a_{8,3}\,a_{5,5}\,a_{7,6}\,a_{2,7}\,a_{3,8}\\
&- a_{6,1}\,a_{4,2}\,a_{8,3}\,a_{5,5}\,a_{7,6}\,a_{2,8}\,a_{3,7}\,,\, a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{7,5}\,a_{8,6}\,a_{3,8}\,a_{4,7} + a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{8,5}\,a_{7,6}\,a_{3,8}\,a_{4,7}\\
&+ a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,5}\,a_{8,6}\,a_{3,8}\,a_{4,7} + a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{8,5}\,a_{5,6}\,a_{3,8}\,a_{4,7} + a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{5,5}\,a_{7,6}\,a_{3,8}\,a_{4,7}\\
&- a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{7,5}\,a_{5,6}\,a_{3,8}\,a_{4,7} + a_{6,1}\,a_{4,2}\,a_{5,3}\,a_{7,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{4,2}\,a_{5,3}\,a_{8,5}\,a_{7,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{4,2}\,a_{8,3}\,a_{5,5}\,a_{7,6}\,a_{1,7}\,a_{3,8}\\
&- a_{6,1}\,a_{4,2}\,a_{8,3}\,a_{7,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{5,5}\,a_{4,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,5}\,a_{8,6}\,a_{3,8}\,a_{4,7} - a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,5}\,a_{5,6}\,a_{3,8}\,a_{4,7} + a_{7,1}\,a_{4,2}\,a_{5,3}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}\\
&- a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{6,5}\,a_{5,6}\,a_{1,7}\,a_{3,8}\,,\, a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{7,5}\,a_{8,6}\,a_{2,8}\,a_{4,7} + a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{8,5}\,a_{7,6}\,a_{2,8}\,a_{4,7}\\
&+ a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{4,7} + a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{4,7} + a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{5,5}\,a_{7,6}\,a_{2,8}\,a_{4,7}\\
&- a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{7,5}\,a_{5,6}\,a_{2,8}\,a_{4,7} + a_{6,1}\,a_{4,2}\,a_{5,3}\,a_{7,5}\,a_{8,6}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{4,2}\,a_{5,3}\,a_{8,5}\,a_{7,6}\,a_{1,7}\,a_{2,8}\\
&+ a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{4,2}\,a_{8,3}\,a_{5,5}\,a_{7,6}\,a_{1,7}\,a_{2,8}\\
&- a_{6,1}\,a_{4,2}\,a_{8,3}\,a_{7,5}\,a_{5,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{5,5}\,a_{4,6}\,a_{1,7}\,a_{2,8}\\
&+ a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,5}\,a_{8,6}\,a_{2,8}\,a_{4,7} - a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,5}\,a_{5,6}\,a_{2,8}\,a_{4,7} + a_{7,1}\,a_{4,2}\,a_{5,3}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{2,8}\\
&- a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{6,5}\,a_{5,6}\,a_{1,7}\,a_{2,8}\,,\, -a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{7,5}\,a_{8,6}\,a_{2,7}\,a_{3,8} - a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{7,5}\,a_{8,6}\,a_{2,8}\,a_{3,7}\\
&- a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{8,5}\,a_{7,6}\,a_{2,7}\,a_{3,8} - a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{8,5}\,a_{7,6}\,a_{2,8}\,a_{3,7} - a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,5}\,a_{8,6}\,a_{2,7}\,a_{3,8}\\
&- a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{3,7} - a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{8,5}\,a_{5,6}\,a_{2,7}\,a_{3,8} - a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{3,7}\\
&- a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{5,5}\,a_{7,6}\,a_{2,7}\,a_{3,8} - a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{5,5}\,a_{7,6}\,a_{2,8}\,a_{3,7} + a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{7,5}\,a_{5,6}\,a_{2,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{7,5}\,a_{5,6}\,a_{2,8}\,a_{3,7} + a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{3,8}\\
&- a_{6,1}\,a_{7,2}\,a_{3,3}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{7,2}\,a_{3,3}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{2,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}\\
&- a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{8,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{2,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{5,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}\\
&- a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,5}\,a_{8,6}\,a_{2,7}\,a_{3,8} - a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,5}\,a_{8,6}\,a_{2,8}\,a_{3,7} + a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,5}\,a_{5,6}\,a_{2,7}\,a_{3,8}\\
&+ a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,5}\,a_{5,6}\,a_{2,8}\,a_{3,7}\,,\, a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,5}\,a_{4,6}\,a_{2,7}\,a_{3,8} + a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,5}\,a_{3,6}\,a_{2,8}\,a_{4,7}\\
&+ a_{7,1}\,a_{1,2}\,a_{3,3}\,a_{6,5}\,a_{8,6}\,a_{2,8}\,a_{4,7} - a_{7,1}\,a_{1,2}\,a_{2,3}\,a_{6,5}\,a_{8,6}\,a_{3,8}\,a_{4,7} - a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{2,5}\,a_{4,6}\,a_{1,7}\,a_{3,8}\\
&- a_{6,1}\,a_{7,2}\,a_{3,3}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{8,5}\,a_{4,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{4,2}\,a_{8,3}\,a_{7,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}\\
&+ a_{6,1}\,a_{4,2}\,a_{8,3}\,a_{2,5}\,a_{7,6}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{8,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{2,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{4,2}\,a_{3,3}\,a_{8,5}\,a_{7,6}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{4,2}\,a_{3,3}\,a_{7,5}\,a_{8,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{4,2}\,a_{2,3}\,a_{8,5}\,a_{7,6}\,a_{1,7}\,a_{3,8}\\
&- a_{6,1}\,a_{4,2}\,a_{2,3}\,a_{7,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{7,5}\,a_{4,6}\,a_{2,8}\,a_{3,7} + a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{7,5}\,a_{4,6}\,a_{2,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{7,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} + a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{2,5}\,a_{7,6}\,a_{3,8}\,a_{4,7} - a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{8,5}\,a_{4,6}\,a_{2,8}\,a_{3,7}\\
&- a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{8,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} + a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{2,5}\,a_{8,6}\,a_{3,8}\,a_{4,7}
\end{aligned}
$$

$$
\begin{aligned}
&+ a_{6,1}\,a_{1,2}\,a_{3,3}\,a_{8,5}\,a_{7,6}\,a_{2,8}\,a_{4,7} + a_{6,1}\,a_{1,2}\,a_{3,3}\,a_{7,5}\,a_{8,6}\,a_{2,8}\,a_{4,7} - a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{8,5}\,a_{7,6}\,a_{3,8}\,a_{4,7}\\
&- a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{7,5}\,a_{8,6}\,a_{3,8}\,a_{4,7} + a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{6,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} + a_{7,1}\,a_{4,2}\,a_{3,3}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{2,8}\\
&- a_{7,1}\,a_{4,2}\,a_{2,3}\,a_{6,5}\,a_{8,6}\,a_{1,7}\,a_{3,8} + a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,5}\,a_{4,6}\,a_{2,8}\,a_{3,7}\,,\, a_{7,1}\,(-a_{1,2}\,a_{2,3}\,a_{5,5}\,a_{8,6}\,a_{3,8}\,a_{4,7}\\
&- a_{1,2}\,a_{2,3}\,a_{8,5}\,a_{5,6}\,a_{3,8}\,a_{4,7} + a_{1,2}\,a_{3,3}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{4,7} + a_{1,2}\,a_{3,3}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{4,7} - a_{1,2}\,a_{5,3}\,a_{2,5}\,a_{8,6}\,a_{3,8}\,a_{4,7}\\
&+ a_{1,2}\,a_{5,3}\,a_{8,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} + a_{1,2}\,a_{5,3}\,a_{8,5}\,a_{4,6}\,a_{2,7}\,a_{3,8} + a_{1,2}\,a_{5,3}\,a_{8,5}\,a_{4,6}\,a_{2,8}\,a_{3,7} + a_{1,2}\,a_{8,3}\,a_{2,5}\,a_{5,6}\,a_{3,8}\,a_{4,7}\\
&+ a_{1,2}\,a_{8,3}\,a_{5,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} + a_{1,2}\,a_{8,3}\,a_{5,5}\,a_{4,6}\,a_{2,7}\,a_{3,8} + a_{1,2}\,a_{8,3}\,a_{5,5}\,a_{4,6}\,a_{2,8}\,a_{3,7} - a_{4,2}\,a_{2,3}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}\\
&- a_{4,2}\,a_{2,3}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} + a_{4,2}\,a_{3,3}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{2,8} + a_{4,2}\,a_{3,3}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{2,8} - a_{4,2}\,a_{5,3}\,a_{2,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{4,2}\,a_{5,3}\,a_{8,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} + a_{4,2}\,a_{8,3}\,a_{2,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} + a_{4,2}\,a_{8,3}\,a_{5,5}\,a_{3,6}\,a_{1,7}\,a_{2,8})\,,\, -a_{6,1}\,(-a_{1,2}\,a_{2,3}\,a_{5,5}\,a_{8,6}\,a_{3,8}\,a_{4,7}\\
&- a_{1,2}\,a_{2,3}\,a_{8,5}\,a_{5,6}\,a_{3,8}\,a_{4,7} + a_{1,2}\,a_{3,3}\,a_{5,5}\,a_{8,6}\,a_{2,8}\,a_{4,7} + a_{1,2}\,a_{3,3}\,a_{8,5}\,a_{5,6}\,a_{2,8}\,a_{4,7} - a_{1,2}\,a_{5,3}\,a_{2,5}\,a_{8,6}\,a_{3,8}\,a_{4,7}\\
&+ a_{1,2}\,a_{5,3}\,a_{8,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} + a_{1,2}\,a_{5,3}\,a_{8,5}\,a_{4,6}\,a_{2,7}\,a_{3,8} + a_{1,2}\,a_{5,3}\,a_{8,5}\,a_{4,6}\,a_{2,8}\,a_{3,7} + a_{1,2}\,a_{8,3}\,a_{2,5}\,a_{5,6}\,a_{3,8}\,a_{4,7}\\
&+ a_{1,2}\,a_{8,3}\,a_{5,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} + a_{1,2}\,a_{8,3}\,a_{5,5}\,a_{4,6}\,a_{2,7}\,a_{3,8} + a_{1,2}\,a_{8,3}\,a_{5,5}\,a_{4,6}\,a_{2,8}\,a_{3,7} - a_{4,2}\,a_{2,3}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}\\
&- a_{4,2}\,a_{2,3}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} + a_{4,2}\,a_{3,3}\,a_{5,5}\,a_{8,6}\,a_{1,7}\,a_{2,8} + a_{4,2}\,a_{3,3}\,a_{8,5}\,a_{5,6}\,a_{1,7}\,a_{2,8} - a_{4,2}\,a_{5,3}\,a_{2,5}\,a_{8,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{4,2}\,a_{5,3}\,a_{8,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} + a_{4,2}\,a_{8,3}\,a_{2,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} + a_{4,2}\,a_{8,3}\,a_{5,5}\,a_{3,6}\,a_{1,7}\,a_{2,8})\,,\, a_{7,1}\,a_{4,2}\,a_{5,3}\,a_{6,5}\,a_{3,6}\,a_{1,7}\,a_{2,8}\\
&+ a_{7,1}\,a_{4,2}\,a_{3,3}\,a_{6,5}\,a_{5,6}\,a_{1,7}\,a_{2,8} + a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,5}\,a_{4,6}\,a_{2,8}\,a_{3,7} - a_{7,1}\,a_{4,2}\,a_{2,3}\,a_{6,5}\,a_{5,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} + a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,5}\,a_{4,6}\,a_{2,7}\,a_{3,8} + a_{7,1}\,a_{1,2}\,a_{3,3}\,a_{6,5}\,a_{5,6}\,a_{2,8}\,a_{4,7}\\
&- a_{7,1}\,a_{1,2}\,a_{2,3}\,a_{6,5}\,a_{5,6}\,a_{3,8}\,a_{4,7} - a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{2,5}\,a_{4,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{7,2}\,a_{3,3}\,a_{5,5}\,a_{4,6}\,a_{1,7}\,a_{2,8}\\
&- a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{5,5}\,a_{4,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{5,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{2,5}\,a_{5,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{4,2}\,a_{5,3}\,a_{7,5}\,a_{3,6}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{4,2}\,a_{5,3}\,a_{2,5}\,a_{7,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{4,2}\,a_{3,3}\,a_{7,5}\,a_{5,6}\,a_{1,7}\,a_{2,8}\\
&- a_{6,1}\,a_{4,2}\,a_{3,3}\,a_{5,5}\,a_{7,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{4,2}\,a_{2,3}\,a_{7,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{4,2}\,a_{2,3}\,a_{5,5}\,a_{7,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,5}\,a_{4,6}\,a_{2,8}\,a_{3,7} + a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,5}\,a_{4,6}\,a_{2,7}\,a_{3,8} + a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,5}\,a_{3,6}\,a_{2,8}\,a_{4,7}\\
&+ a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{2,5}\,a_{5,6}\,a_{3,8}\,a_{4,7} + a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{7,5}\,a_{4,6}\,a_{2,8}\,a_{3,7} + a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{7,5}\,a_{4,6}\,a_{2,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{7,5}\,a_{3,6}\,a_{2,8}\,a_{4,7} + a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{2,5}\,a_{7,6}\,a_{3,8}\,a_{4,7} + a_{6,1}\,a_{1,2}\,a_{3,3}\,a_{7,5}\,a_{5,6}\,a_{2,8}\,a_{4,7}\\
&- a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{7,5}\,a_{5,6}\,a_{3,8}\,a_{4,7} - a_{6,1}\,a_{1,2}\,a_{3,3}\,a_{5,5}\,a_{7,6}\,a_{2,8}\,a_{4,7} + a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{5,5}\,a_{7,6}\,a_{3,8}\,a_{4,7}]\\
&[-a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{8,6}\,a_{2,7}\,a_{3,8} - a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{8,6}\,a_{2,8}\,a_{3,7} - a_{6,1}\,a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{7,6}\,a_{2,7}\,a_{3,8}\\
&- a_{6,1}\,a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{7,6}\,a_{2,8}\,a_{3,7} - a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{8,6}\,a_{3,8}\,a_{4,7} + a_{6,1}\,a_{7,2}\,a_{3,3}\,a_{5,4}\,a_{8,6}\,a_{2,8}\,a_{4,7}\\
&+ a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{4,4}\,a_{8,6}\,a_{2,7}\,a_{3,8} + a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{4,4}\,a_{8,6}\,a_{2,8}\,a_{3,7} - a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{4,4}\,a_{5,6}\,a_{2,7}\,a_{3,8}\\
&- a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{4,4}\,a_{5,6}\,a_{2,8}\,a_{3,7} + a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{5,4}\,a_{3,6}\,a_{2,8}\,a_{4,7} + a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{5,4}\,a_{4,6}\,a_{2,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{5,4}\,a_{4,6}\,a_{2,8}\,a_{3,7} + a_{7,1}\,a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{8,6}\,a_{2,7}\,a_{3,8} + a_{7,1}\,a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{8,6}\,a_{2,8}\,a_{3,7}\\
&- a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{5,6}\,a_{2,7}\,a_{3,8} - a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{5,6}\,a_{2,8}\,a_{3,7}\,,\, a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{8,6}\,a_{3,8}\,a_{4,7}\\
&+ a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{7,6}\,a_{3,8}\,a_{4,7} + a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{8,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{7,6}\,a_{1,7}\,a_{3,8}\\
&- a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{4,4}\,a_{8,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{4,4}\,a_{5,6}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{5,4}\,a_{4,6}\,a_{1,7}\,a_{3,8}\\
&- a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{8,6}\,a_{3,8}\,a_{4,7} + a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{5,6}\,a_{3,8}\,a_{4,7} - a_{7,1}\,a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{8,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{5,6}\,a_{1,7}\,a_{3,8}\,,\, a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{8,6}\,a_{2,8}\,a_{4,7} + a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{7,6}\,a_{2,8}\,a_{4,7}\\
&+ a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{8,6}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{7,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{4,4}\,a_{8,6}\,a_{1,7}\,a_{2,8}\\
&+ a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{4,4}\,a_{5,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{5,4}\,a_{4,6}\,a_{1,7}\,a_{2,8} - a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{8,6}\,a_{2,8}\,a_{4,7}\\
&+ a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{5,6}\,a_{2,8}\,a_{4,7} - a_{7,1}\,a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{8,6}\,a_{1,7}\,a_{2,8} + a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{5,6}\,a_{1,7}\,a_{2,8}\,,\\
&-a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{8,6}\,a_{2,7}\,a_{3,8} - a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{8,6}\,a_{2,8}\,a_{3,7} - a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{7,6}\,a_{2,7}\,a_{3,8}\\
&- a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{7,6}\,a_{2,8}\,a_{3,7} + a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{8,6}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{7,2}\,a_{3,3}\,a_{5,4}\,a_{8,6}\,a_{1,7}\,a_{2,8}\\
&- a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{5,4}\,a_{3,6}\,a_{1,7}\,a_{2,8} + a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{8,6}\,a_{2,7}\,a_{3,8} + a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{8,6}\,a_{2,8}\,a_{3,7}\\
&- a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{5,6}\,a_{2,7}\,a_{3,8} - a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{5,6}\,a_{2,8}\,a_{3,7}\,,\, -a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{8,6}\,a_{2,7}\,a_{3,8}\\
&- a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{8,6}\,a_{2,8}\,a_{3,7} - a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{7,6}\,a_{2,7}\,a_{3,8} - a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{7,6}\,a_{2,8}\,a_{3,7}\\
&+ a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{4,4}\,a_{8,6}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{7,2}\,a_{3,3}\,a_{4,4}\,a_{8,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{4,4}\,a_{3,6}\,a_{1,7}\,a_{2,8}\\
&+ a_{7,1}\,a_{1,2}\,a_{2,3}\,a_{6,4}\,a_{8,6}\,a_{3,8}\,a_{4,7} - a_{7,1}\,a_{1,2}\,a_{3,3}\,a_{6,4}\,a_{8,6}\,a_{2,8}\,a_{4,7} - a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{4,6}\,a_{3,6}\,a_{2,8}\,a_{4,7}\\
&- a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{4,6}\,a_{2,7}\,a_{3,8} - a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{4,6}\,a_{2,8}\,a_{3,7} + a_{7,1}\,a_{4,2}\,a_{2,3}\,a_{6,4}\,a_{8,6}\,a_{1,7}\,a_{3,8}\\
&- a_{7,1}\,a_{4,2}\,a_{3,3}\,a_{6,4}\,a_{8,6}\,a_{1,7}\,a_{2,8} - a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{3,6}\,a_{1,7}\,a_{2,8}\,,\, -a_{7,1}\,(a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{8,6}\,a_{3,8}\,a_{4,7}
\end{aligned}
$$

$$
\begin{aligned}
&- a_{1,2}\,a_{3,3}\,a_{5,4}\,a_{8,6}\,a_{2,8}\,a_{4,7} - a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{8,6}\,a_{2,7}\,a_{3,8} - a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{8,6}\,a_{2,8}\,a_{3,7} + a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{5,6}\,a_{2,7}\,a_{3,8}\\
&+ a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{5,6}\,a_{2,8}\,a_{3,7} - a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{3,6}\,a_{2,8}\,a_{4,7} - a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{4,6}\,a_{2,7}\,a_{3,8} - a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{4,6}\,a_{2,8}\,a_{3,7}\\
&+ a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{8,6}\,a_{1,7}\,a_{3,8} - a_{4,2}\,a_{3,3}\,a_{5,4}\,a_{8,6}\,a_{1,7}\,a_{2,8} - a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{3,6}\,a_{1,7}\,a_{2,8}\big)\,,\,a_{6,1}\,\big(a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{8,6}\,a_{3,8}\,a_{4,7}\\
&- a_{1,2}\,a_{3,3}\,a_{5,4}\,a_{8,6}\,a_{2,8}\,a_{4,7} - a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{8,6}\,a_{2,7}\,a_{3,8} - a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{8,6}\,a_{2,8}\,a_{3,7} + a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{5,6}\,a_{2,7}\,a_{3,8}\\
&+ a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{5,6}\,a_{2,8}\,a_{3,7} - a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{3,6}\,a_{2,8}\,a_{4,7} - a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{4,6}\,a_{2,7}\,a_{3,8} - a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{4,6}\,a_{2,8}\,a_{3,7}\\
&+ a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{8,6}\,a_{1,7}\,a_{3,8} - a_{4,2}\,a_{3,3}\,a_{5,4}\,a_{8,6}\,a_{1,7}\,a_{2,8} - a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{3,6}\,a_{1,7}\,a_{2,8}\big)\,,\,a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{4,6}\,a_{2,8}\,a_{3,7}\\
&+ a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{4,6}\,a_{2,7}\,a_{3,8} + a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{3,6}\,a_{2,8}\,a_{4,7} - a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{5,6}\,a_{2,8}\,a_{3,7}\\
&- a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{5,6}\,a_{2,7}\,a_{3,8} - a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{7,6}\,a_{2,8}\,a_{3,7} - a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{7,6}\,a_{2,7}\,a_{3,8}\\
&- a_{6,1}\,a_{1,2}\,a_{3,3}\,a_{5,4}\,a_{7,6}\,a_{2,8}\,a_{4,7} + a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{7,6}\,a_{3,8}\,a_{4,7} + a_{6,1}\,a_{7,2}\,a_{3,3}\,a_{5,4}\,a_{4,6}\,a_{1,7}\,a_{2,8}\\
&- a_{6,1}\,a_{7,2}\,a_{3,3}\,a_{4,4}\,a_{5,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{4,6}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{4,4}\,a_{5,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{3,6}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{4,2}\,a_{3,3}\,a_{5,4}\,a_{7,6}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{7,6}\,a_{1,7}\,a_{3,8}\\
&+ a_{7,1}\,a_{4,2}\,a_{2,3}\,a_{6,4}\,a_{5,6}\,a_{1,7}\,a_{3,8} - a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{4,6}\,a_{2,8}\,a_{3,7} - a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{4,6}\,a_{2,7}\,a_{3,8}\\
&- a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{3,6}\,a_{2,8}\,a_{4,7} - a_{7,1}\,a_{1,2}\,a_{3,3}\,a_{6,4}\,a_{5,6}\,a_{2,8}\,a_{4,7} + a_{7,1}\,a_{1,2}\,a_{2,3}\,a_{6,4}\,a_{5,6}\,a_{3,8}\,a_{4,7}\\
&- a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{4,4}\,a_{3,6}\,a_{1,7}\,a_{2,8} - a_{7,1}\,a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{3,6}\,a_{1,7}\,a_{2,8} - a_{7,1}\,a_{4,2}\,a_{3,3}\,a_{6,4}\,a_{5,6}\,a_{1,7}\,a_{2,8}\big]\\
&\big[a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{8,5}\,a_{2,7}\,a_{3,8} + a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{8,5}\,a_{2,8}\,a_{3,7} - a_{6,1}\,a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{2,7}\,a_{3,8}\\
&- a_{6,1}\,a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{2,8}\,a_{3,7} + a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{8,5}\,a_{3,8}\,a_{4,7} - a_{6,1}\,a_{7,2}\,a_{3,3}\,a_{5,4}\,a_{8,5}\,a_{2,8}\,a_{4,7}\\
&- a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{2,7}\,a_{3,8} - a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{2,8}\,a_{3,7} - a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{4,4}\,a_{5,5}\,a_{2,7}\,a_{3,8}\\
&- a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{4,4}\,a_{5,5}\,a_{2,8}\,a_{3,7} - a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{5,4}\,a_{2,5}\,a_{3,8}\,a_{4,7} - a_{7,1}\,a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{2,7}\,a_{3,8}\\
&- a_{7,1}\,a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{2,8}\,a_{3,7} - a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{2,7}\,a_{3,8} - a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{2,8}\,a_{3,7}\\
&- a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{2,7}\,a_{3,8} - a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{2,8}\,a_{3,7}\,,\,-a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{8,5}\,a_{3,8}\,a_{4,7}\\
&+ a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{3,8}\,a_{4,7} - a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{8,5}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{1,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{4,4}\,a_{5,5}\,a_{1,7}\,a_{3,8} + a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{3,8}\,a_{4,7}\\
&+ a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{3,8}\,a_{4,7} + a_{7,1}\,a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{1,7}\,a_{3,8} + a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{1,7}\,a_{3,8}\\
&+ a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{1,7}\,a_{3,8}\,,\,-a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{8,5}\,a_{2,8}\,a_{4,7}\\
&+ a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{2,8}\,a_{4,7} - a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{8,5}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{1,7}\,a_{2,8}\\
&+ a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{4,4}\,a_{5,5}\,a_{1,7}\,a_{2,8} + a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{2,8}\,a_{4,7}\\
&+ a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{2,8}\,a_{4,7} + a_{7,1}\,a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{1,7}\,a_{2,8} + a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{1,7}\,a_{2,8}\\
&+ a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{1,7}\,a_{2,8}\,,\,a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{8,5}\,a_{2,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{8,5}\,a_{2,8}\,a_{3,7} - a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{2,7}\,a_{3,8} - a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{2,8}\,a_{3,7}\\
&- a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{8,5}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{7,2}\,a_{3,3}\,a_{5,4}\,a_{8,5}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{5,4}\,a_{2,5}\,a_{1,7}\,a_{3,8}\\
&- a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{2,7}\,a_{3,8} - a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{2,8}\,a_{3,7} - a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{2,7}\,a_{3,8}\\
&- a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{2,8}\,a_{3,7} - a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{2,7}\,a_{3,8} - a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{2,8}\,a_{3,7}\,,\\
&a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{8,5}\,a_{2,7}\,a_{3,8} + a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{8,5}\,a_{2,8}\,a_{3,7} - a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{7,5}\,a_{2,7}\,a_{3,8}\\
&- a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{7,5}\,a_{2,8}\,a_{3,7} - a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{4,4}\,a_{8,5}\,a_{1,7}\,a_{3,8} + a_{6,1}\,a_{7,2}\,a_{3,3}\,a_{4,4}\,a_{8,5}\,a_{1,7}\,a_{2,8}\\
&+ a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{4,4}\,a_{2,5}\,a_{1,7}\,a_{3,8} - a_{7,1}\,a_{1,2}\,a_{2,3}\,a_{6,4}\,a_{8,5}\,a_{3,8}\,a_{4,7} + a_{7,1}\,a_{1,2}\,a_{3,3}\,a_{6,4}\,a_{8,5}\,a_{2,8}\,a_{4,7}\\
&- a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{2,5}\,a_{3,8}\,a_{4,7} + a_{7,1}\,a_{4,2}\,a_{2,3}\,a_{6,4}\,a_{8,5}\,a_{1,7}\,a_{3,8} - a_{7,1}\,a_{4,2}\,a_{3,3}\,a_{6,4}\,a_{8,5}\,a_{2,8}\,a_{4,7}\\
&+ a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{2,5}\,a_{1,7}\,a_{3,8}\,,\,a_{7,1}\,\big(a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{8,5}\,a_{3,8}\,a_{4,7} - a_{1,2}\,a_{3,3}\,a_{5,4}\,a_{8,5}\,a_{2,8}\,a_{4,7} - a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{2,7}\,a_{3,8} - a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{2,8}\,a_{3,7}\\
&- a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{5,5}\,a_{2,7}\,a_{3,8} - a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{5,5}\,a_{2,8}\,a_{3,7} - a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{2,5}\,a_{3,8}\,a_{4,7} + a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{8,5}\,a_{1,7}\,a_{3,8}\\
&- a_{4,2}\,a_{3,3}\,a_{5,4}\,a_{8,5}\,a_{1,7}\,a_{2,8} - a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{2,5}\,a_{1,7}\,a_{3,8}\big)\,,\,-a_{6,1}\,\big(a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{8,5}\,a_{3,8}\,a_{4,7} - a_{1,2}\,a_{3,3}\,a_{5,4}\,a_{8,5}\,a_{2,8}\,a_{4,7}\\
&- a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{2,7}\,a_{3,8} - a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{2,8}\,a_{3,7} - a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{5,5}\,a_{2,7}\,a_{3,8} - a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{5,5}\,a_{2,8}\,a_{3,7}\\
&- a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{2,5}\,a_{3,8}\,a_{4,7} + a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{8,5}\,a_{1,7}\,a_{3,8} - a_{4,2}\,a_{3,3}\,a_{5,4}\,a_{8,5}\,a_{1,7}\,a_{2,8} - a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{2,5}\,a_{1,7}\,a_{3,8}\big)\,,\\
&-a_{6,1}\,a_{4,2}\,a_{3,3}\,a_{5,4}\,a_{7,5}\,a_{1,7}\,a_{2,8} + a_{6,1}\,a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{7,5}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{2,5}\,a_{3,8}\,a_{4,7}\\
&- a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{5,5}\,a_{2,8}\,a_{3,7} - a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{5,5}\,a_{2,7}\,a_{3,8} - a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{7,5}\,a_{2,8}\,a_{3,7}\\
&- a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{7,5}\,a_{2,7}\,a_{3,8} - a_{6,1}\,a_{1,2}\,a_{3,3}\,a_{5,4}\,a_{7,5}\,a_{2,8}\,a_{4,7} + a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{7,5}\,a_{3,8}\,a_{4,7}
\end{aligned}
$$

$$
\begin{aligned}
&+ a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{4,4}\,a_{2,5}\,a_{1,7}\,a_{3,8} - a_{6,1}\,a_{7,2}\,a_{3,3}\,a_{4,4}\,a_{5,5}\,a_{1,7}\,a_{2,8} - a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{2,5}\,a_{1,7}\,a_{3,8}\\
&+ a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{4,4}\,a_{5,5}\,a_{1,7}\,a_{3,8} - a_{7,1}\,a_{4,2}\,a_{3,3}\,a_{5,4}\,a_{6,5}\,a_{1,7}\,a_{2,8} + a_{7,1}\,a_{4,2}\,a_{2,3}\,a_{6,4}\,a_{5,5}\,a_{1,7}\,a_{3,8}\\
&+ a_{7,1}\,a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{6,5}\,a_{1,7}\,a_{3,8} + a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{2,5}\,a_{3,8}\,a_{4,7} - a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{6,5}\,a_{2,8}\,a_{3,7}\\
&- a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{6,5}\,a_{2,7}\,a_{3,8} - a_{7,1}\,a_{1,2}\,a_{3,3}\,a_{5,4}\,a_{6,5}\,a_{2,8}\,a_{4,7} - a_{7,1}\,a_{1,2}\,a_{3,3}\,a_{6,4}\,a_{5,5}\,a_{2,8}\,a_{4,7}\\
&+ a_{7,1}\,a_{1,2}\,a_{2,3}\,a_{6,4}\,a_{5,5}\,a_{3,8}\,a_{4,7} + a_{7,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{6,5}\,a_{3,8}\,a_{4,7} + a_{7,1}\,a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{2,5}\,a_{1,7}\,a_{3,8}\\
&- a_{7,1}\,a_{4,2}\,a_{3,3}\,a_{6,4}\,a_{5,5}\,a_{1,7}\,a_{2,8}]\\
&[a_{6,1}\,a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{7,5}\,a_{3,8}\,a_{8,6} + a_{6,1}\,a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{8,5}\,a_{3,8}\,a_{7,6} - a_{6,1}\,a_{4,2}\,a_{3,3}\,a_{5,4}\,a_{7,5}\,a_{2,8}\,a_{8,6}\\
&- a_{6,1}\,a_{4,2}\,a_{3,3}\,a_{5,4}\,a_{8,5}\,a_{2,8}\,a_{7,6} - a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{2,5}\,a_{3,8}\,a_{8,6} + a_{6,1}\,a_{4,2}\,a_{7,3}\,a_{5,4}\,a_{8,5}\,a_{2,8}\,a_{3,6}\\
&- a_{6,1}\,a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{2,5}\,a_{3,8}\,a_{7,6} - a_{6,1}\,a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{2,8}\,a_{3,6} + a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{4,4}\,a_{5,5}\,a_{3,8}\,a_{8,6}\\
&+ a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{4,4}\,a_{8,5}\,a_{3,8}\,a_{5,6} - a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{8,5}\,a_{3,8}\,a_{4,6} - a_{6,1}\,a_{7,2}\,a_{3,3}\,a_{4,4}\,a_{5,5}\,a_{2,8}\,a_{8,6}\\
&- a_{6,1}\,a_{7,2}\,a_{3,3}\,a_{4,4}\,a_{8,5}\,a_{2,8}\,a_{5,6} + a_{6,1}\,a_{7,2}\,a_{3,3}\,a_{5,4}\,a_{8,5}\,a_{2,8}\,a_{4,6} + a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{4,4}\,a_{2,5}\,a_{3,8}\,a_{8,6}\\
&- a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{2,8}\,a_{3,6} - a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{4,4}\,a_{2,5}\,a_{3,8}\,a_{5,6} - a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{4,4}\,a_{5,5}\,a_{2,8}\,a_{3,6}\\
&+ a_{6,1}\,a_{7,2}\,a_{8,3}\,a_{5,4}\,a_{2,5}\,a_{3,8}\,a_{4,6} + a_{7,1}\,a_{4,2}\,a_{2,3}\,a_{5,4}\,a_{6,5}\,a_{3,8}\,a_{8,6} + a_{7,1}\,a_{4,2}\,a_{2,3}\,a_{6,4}\,a_{5,5}\,a_{3,8}\,a_{8,6}\\
&+ a_{7,1}\,a_{4,2}\,a_{2,3}\,a_{6,4}\,a_{8,5}\,a_{3,8}\,a_{5,6} - a_{7,1}\,a_{4,2}\,a_{3,3}\,a_{5,4}\,a_{6,5}\,a_{2,8}\,a_{8,6} - a_{7,1}\,a_{4,2}\,a_{3,3}\,a_{6,4}\,a_{5,5}\,a_{2,8}\,a_{8,6}\\
&- a_{7,1}\,a_{4,2}\,a_{3,3}\,a_{6,4}\,a_{8,5}\,a_{2,8}\,a_{5,6} + a_{7,1}\,a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{2,5}\,a_{3,8}\,a_{8,6} - a_{7,1}\,a_{4,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{2,8}\,a_{3,6}\\
&- a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{2,8}\,a_{3,6} - a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{2,5}\,a_{3,8}\,a_{5,6} - a_{7,1}\,a_{4,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{2,8}\,a_{3,6},\\
&-a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{7,5}\,a_{3,8}\,a_{8,6} - a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{3,8}\,a_{7,6} - a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{5,5}\,a_{3,8}\,a_{8,6}\\
&- a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{8,5}\,a_{3,8}\,a_{5,6} + a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{8,5}\,a_{3,8}\,a_{4,6} - a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{5,5}\,a_{3,8}\,a_{7,6}\\
&+ a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{7,5}\,a_{3,8}\,a_{5,6} - a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{3,8}\,a_{4,6} - a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{6,5}\,a_{3,8}\,a_{8,6}\\
&- a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{3,8}\,a_{4,6} + a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{6,5}\,a_{3,8}\,a_{5,6} - a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{3,8}\,a_{4,6}\\
&- a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{3,8}\,a_{4,6}, -a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{7,5}\,a_{2,8}\,a_{8,6} - a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{2,8}\,a_{7,6}\\
&- a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{8,5}\,a_{2,8}\,a_{8,6} - a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{8,5}\,a_{2,8}\,a_{5,6} + a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{8,5}\,a_{2,8}\,a_{4,6}\\
&- a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{5,5}\,a_{2,8}\,a_{7,6} + a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{7,5}\,a_{2,8}\,a_{5,6} - a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{2,8}\,a_{4,6}\\
&- a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{6,5}\,a_{2,8}\,a_{8,6} - a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{2,8}\,a_{4,6} + a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{6,5}\,a_{2,8}\,a_{5,6}\\
&- a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{2,8}\,a_{4,6} - a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{2,8}\,a_{4,6}, a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{7,5}\,a_{3,8}\,a_{8,6}\\
&+ a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{8,5}\,a_{3,8}\,a_{7,6} - a_{6,1}\,a_{1,2}\,a_{3,3}\,a_{5,4}\,a_{7,5}\,a_{2,8}\,a_{8,6} - a_{6,1}\,a_{1,2}\,a_{3,3}\,a_{5,4}\,a_{8,5}\,a_{2,8}\,a_{7,6}\\
&- a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{2,5}\,a_{3,8}\,a_{8,6} + a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{8,5}\,a_{2,8}\,a_{3,6} - a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{2,5}\,a_{3,8}\,a_{7,6}\\
&- a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{7,5}\,a_{2,8}\,a_{3,6} + a_{7,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{6,5}\,a_{3,8}\,a_{8,6} + a_{7,1}\,a_{1,2}\,a_{2,3}\,a_{6,4}\,a_{5,5}\,a_{3,8}\,a_{8,6}\\
&+ a_{7,1}\,a_{1,2}\,a_{2,3}\,a_{6,4}\,a_{8,5}\,a_{3,8}\,a_{5,6} - a_{7,1}\,a_{1,2}\,a_{3,3}\,a_{5,4}\,a_{6,5}\,a_{2,8}\,a_{8,6} - a_{7,1}\,a_{1,2}\,a_{3,3}\,a_{6,4}\,a_{5,5}\,a_{2,8}\,a_{8,6}\\
&- a_{7,1}\,a_{1,2}\,a_{3,3}\,a_{6,4}\,a_{8,5}\,a_{2,8}\,a_{5,6} + a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{2,5}\,a_{3,8}\,a_{8,6} - a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{2,8}\,a_{3,6}\\
&- a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{2,8}\,a_{3,6} - a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{2,5}\,a_{3,8}\,a_{5,6} - a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{2,8}\,a_{3,6},\\
&a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{4,4}\,a_{7,5}\,a_{3,8}\,a_{8,6} + a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{4,4}\,a_{8,5}\,a_{3,8}\,a_{7,6} - a_{6,1}\,a_{1,2}\,a_{3,3}\,a_{4,4}\,a_{7,5}\,a_{2,8}\,a_{8,6}\\
&- a_{6,1}\,a_{1,2}\,a_{3,3}\,a_{4,4}\,a_{8,5}\,a_{2,8}\,a_{7,6} - a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{2,5}\,a_{3,8}\,a_{8,6} + a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{8,5}\,a_{2,8}\,a_{3,6}\\
&- a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{2,5}\,a_{3,8}\,a_{7,6} - a_{6,1}\,a_{1,2}\,a_{8,3}\,a_{4,4}\,a_{7,5}\,a_{2,8}\,a_{3,6} + a_{7,1}\,a_{1,2}\,a_{2,3}\,a_{4,4}\,a_{6,5}\,a_{3,8}\,a_{8,6}\\
&+ a_{7,1}\,a_{1,2}\,a_{2,3}\,a_{6,4}\,a_{5,5}\,a_{3,8}\,a_{8,6} + a_{7,1}\,a_{1,2}\,a_{2,3}\,a_{6,4}\,a_{8,5}\,a_{3,8}\,a_{5,6} - a_{7,1}\,a_{1,2}\,a_{3,3}\,a_{5,4}\,a_{6,5}\,a_{2,8}\,a_{8,6}\\
&- a_{7,1}\,a_{1,2}\,a_{3,3}\,a_{6,4}\,a_{5,5}\,a_{2,8}\,a_{8,6} - a_{7,1}\,a_{1,2}\,a_{3,3}\,a_{6,4}\,a_{8,5}\,a_{2,8}\,a_{5,6} + a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{2,5}\,a_{3,8}\,a_{8,6}\\
&- a_{7,1}\,a_{1,2}\,a_{5,3}\,a_{6,4}\,a_{8,5}\,a_{2,8}\,a_{3,6} - a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{5,4}\,a_{6,5}\,a_{2,8}\,a_{3,6} - a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{2,5}\,a_{3,8}\,a_{5,6}\\
&- a_{7,1}\,a_{1,2}\,a_{8,3}\,a_{6,4}\,a_{5,5}\,a_{2,8}\,a_{3,6}, a_{7,1}\,a_{1,2}\,(a_{2,3}\,a_{4,4}\,a_{5,5}\,a_{3,8}\,a_{8,6} + a_{2,3}\,a_{4,4}\,a_{8,5}\,a_{3,8}\,a_{5,6}\\
&- a_{2,3}\,a_{5,4}\,a_{8,5}\,a_{3,8}\,a_{4,6} - a_{3,3}\,a_{4,4}\,a_{5,5}\,a_{2,8}\,a_{8,6} - a_{3,3}\,a_{4,4}\,a_{8,5}\,a_{2,8}\,a_{5,6} + a_{3,3}\,a_{5,4}\,a_{8,5}\,a_{2,8}\,a_{4,6} + a_{5,3}\,a_{4,4}\,a_{2,5}\,a_{3,8}\,a_{8,6}\\
&- a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{2,8}\,a_{3,6} - a_{8,3}\,a_{4,4}\,a_{2,5}\,a_{3,8}\,a_{5,6} - a_{8,3}\,a_{4,4}\,a_{5,5}\,a_{2,8}\,a_{3,6} + a_{8,3}\,a_{5,4}\,a_{2,5}\,a_{3,8}\,a_{4,6}), -a_{6,1}\,a_{1,2}\,(\\
&a_{2,3}\,a_{4,4}\,a_{5,5}\,a_{3,8}\,a_{8,6} + a_{2,3}\,a_{4,4}\,a_{8,5}\,a_{3,8}\,a_{5,6} - a_{2,3}\,a_{5,4}\,a_{8,5}\,a_{3,8}\,a_{4,6} - a_{3,3}\,a_{4,4}\,a_{5,5}\,a_{2,8}\,a_{8,6} - a_{3,3}\,a_{4,4}\,a_{8,5}\,a_{2,8}\,a_{5,6}\\
&+ a_{3,3}\,a_{5,4}\,a_{8,5}\,a_{2,8}\,a_{4,6} + a_{5,3}\,a_{4,4}\,a_{2,5}\,a_{3,8}\,a_{8,6} - a_{5,3}\,a_{4,4}\,a_{8,5}\,a_{2,8}\,a_{3,6} - a_{8,3}\,a_{4,4}\,a_{2,5}\,a_{3,8}\,a_{5,6} - a_{8,3}\,a_{4,4}\,a_{5,5}\,a_{2,8}\,a_{3,6}\\
&+ a_{8,3}\,a_{5,4}\,a_{2,5}\,a_{3,8}\,a_{4,6}), -a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{4,4}\,a_{5,5}\,a_{3,8}\,a_{7,6} + a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{4,4}\,a_{7,5}\,a_{3,8}\,a_{5,6} - a_{6,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{7,5}\,a_{3,8}\,a_{4,6}\\
&+ a_{6,1}\,a_{1,2}\,a_{3,3}\,a_{4,4}\,a_{5,5}\,a_{2,8}\,a_{7,6} - a_{6,1}\,a_{1,2}\,a_{3,3}\,a_{4,4}\,a_{7,5}\,a_{2,8}\,a_{5,6} + a_{6,1}\,a_{1,2}\,a_{3,3}\,a_{5,4}\,a_{7,5}\,a_{2,8}\,a_{4,6}\\
&- a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{2,5}\,a_{3,8}\,a_{7,6} - a_{6,1}\,a_{1,2}\,a_{5,3}\,a_{4,4}\,a_{7,5}\,a_{3,8}\,a_{2,6} - a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{2,5}\,a_{3,8}\,a_{5,6}\\
&- a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{4,4}\,a_{5,5}\,a_{3,8}\,a_{2,6} + a_{6,1}\,a_{1,2}\,a_{7,3}\,a_{5,4}\,a_{2,5}\,a_{3,8}\,a_{4,6} + a_{7,1}\,a_{1,2}\,a_{2,3}\,a_{4,4}\,a_{6,5}\,a_{3,8}\,a_{5,6}\\
&- a_{7,1}\,a_{1,2}\,a_{2,3}\,a_{5,4}\,a_{6,5}\,a_{3,8}\,a_{4,6} - a_{7,1}\,a_{1,2}\,a_{3,3}\,a_{4,4}\,a_{6,5}\,a_{2,8}\,a_{5,6}
\end{aligned}
$$

$$+a_{7,1}a_{1,2}a_{3,3}a_{5,4}a_{6,5}a_{2,8}a_{4,6}+a_{7,1}a_{1,2}a_{3,3}a_{6,4}a_{5,5}a_{2,8}a_{4,6}-a_{7,1}a_{1,2}a_{5,3}a_{4,4}a_{6,5}a_{2,8}a_{3,6}$$
$$-a_{7,1}a_{1,2}a_{5,3}a_{6,4}a_{2,5}a_{3,8}a_{4,6}]$$
$$[a_{6,1}a_{4,2}a_{7,3}a_{5,4}a_{8,5}a_{2,7}a_{3,6}+a_{6,1}a_{4,2}a_{8,3}a_{5,4}a_{2,5}a_{3,7}a_{7,6}-a_{6,1}a_{4,2}a_{8,3}a_{5,4}a_{7,5}a_{2,7}a_{3,6}$$
$$-a_{6,1}a_{7,2}a_{2,3}a_{4,4}a_{5,5}a_{3,7}a_{8,6}-a_{6,1}a_{7,2}a_{2,3}a_{4,4}a_{8,5}a_{3,7}a_{5,6}+a_{6,1}a_{7,2}a_{2,3}a_{5,4}a_{8,5}a_{3,6}a_{4,7}$$
$$+a_{6,1}a_{7,2}a_{2,3}a_{5,4}a_{8,5}a_{3,7}a_{4,6}-a_{6,1}a_{7,2}a_{3,3}a_{4,4}a_{5,5}a_{2,7}a_{8,6}-a_{6,1}a_{7,2}a_{3,3}a_{4,4}a_{8,5}a_{2,7}a_{5,6}$$
$$-a_{6,1}a_{7,2}a_{3,3}a_{5,4}a_{2,5}a_{4,7}a_{8,6}+a_{6,1}a_{7,2}a_{3,3}a_{5,4}a_{8,5}a_{2,7}a_{4,6}-a_{6,1}a_{7,2}a_{5,3}a_{4,4}a_{2,5}a_{3,7}a_{8,6}$$
$$+a_{6,1}a_{4,2}a_{7,3}a_{5,4}a_{2,5}a_{3,7}a_{8,6}-a_{6,1}a_{7,2}a_{5,3}a_{4,4}a_{8,5}a_{2,7}a_{3,6}+a_{6,1}a_{7,2}a_{8,3}a_{4,4}a_{2,5}a_{3,7}a_{5,6}$$
$$-a_{6,1}a_{7,2}a_{8,3}a_{4,4}a_{5,5}a_{2,7}a_{3,6}-a_{6,1}a_{7,2}a_{8,3}a_{5,4}a_{2,5}a_{3,6}a_{4,7}-a_{6,1}a_{7,2}a_{8,3}a_{5,4}a_{2,5}a_{3,7}a_{4,6}$$
$$-a_{7,1}a_{4,2}a_{2,3}a_{5,4}a_{6,5}a_{3,7}a_{8,6}-a_{7,1}a_{4,2}a_{2,3}a_{6,4}a_{5,5}a_{3,7}a_{8,6}-a_{7,1}a_{4,2}a_{2,3}a_{6,4}a_{8,5}a_{3,7}a_{5,6}$$
$$-a_{7,1}a_{4,2}a_{3,3}a_{5,4}a_{6,5}a_{2,7}a_{8,6}-a_{7,1}a_{4,2}a_{3,3}a_{6,4}a_{5,5}a_{2,7}a_{8,6}-a_{7,1}a_{4,2}a_{3,3}a_{6,4}a_{8,5}a_{2,7}a_{5,6}$$
$$-a_{7,1}a_{4,2}a_{5,3}a_{6,4}a_{2,5}a_{3,7}a_{8,6}-a_{7,1}a_{4,2}a_{5,3}a_{6,4}a_{8,5}a_{2,7}a_{3,6}-a_{7,1}a_{4,2}a_{8,3}a_{5,4}a_{6,5}a_{2,7}a_{3,6}$$
$$+a_{7,1}a_{4,2}a_{8,3}a_{6,4}a_{2,5}a_{3,7}a_{5,6}-a_{7,1}a_{4,2}a_{8,3}a_{6,4}a_{5,5}a_{2,7}a_{3,6}-a_{6,1}a_{4,2}a_{2,3}a_{5,4}a_{7,5}a_{3,7}a_{8,6}$$
$$-a_{6,1}a_{4,2}a_{2,3}a_{5,4}a_{8,5}a_{3,7}a_{7,6}-a_{6,1}a_{4,2}a_{3,3}a_{5,4}a_{7,5}a_{2,7}a_{8,6}-a_{6,1}a_{4,2}a_{3,3}a_{5,4}a_{8,5}a_{2,7}a_{7,6},$$
$$a_{7,1}a_{4,2}a_{8,3}a_{5,4}a_{6,5}a_{1,7}a_{3,6}+a_{7,1}a_{4,2}a_{8,3}a_{6,4}a_{5,5}a_{1,7}a_{3,6}-a_{6,1}a_{4,2}a_{7,3}a_{5,4}a_{8,5}a_{1,7}a_{3,6}$$
$$+a_{6,1}a_{4,2}a_{8,3}a_{5,4}a_{7,5}a_{1,7}a_{3,6}+a_{6,1}a_{7,2}a_{3,3}a_{4,4}a_{5,5}a_{1,7}a_{8,6}+a_{6,1}a_{7,2}a_{3,3}a_{4,4}a_{8,5}a_{1,7}a_{5,6}$$
$$-a_{6,1}a_{7,2}a_{3,3}a_{5,4}a_{8,5}a_{1,7}a_{4,6}+a_{6,1}a_{7,2}a_{5,3}a_{4,4}a_{8,5}a_{1,7}a_{3,6}+a_{6,1}a_{7,2}a_{8,3}a_{4,4}a_{5,5}a_{1,7}a_{3,6}$$
$$+a_{7,1}a_{1,2}a_{3,3}a_{5,4}a_{6,5}a_{4,7}a_{8,6}+a_{7,1}a_{1,2}a_{3,3}a_{6,4}a_{5,5}a_{4,7}a_{8,6}+a_{7,1}a_{1,2}a_{3,3}a_{6,4}a_{8,5}a_{4,7}a_{5,6}$$
$$+a_{7,1}a_{1,2}a_{5,3}a_{4,4}a_{6,5}a_{3,7}a_{8,6}+a_{7,1}a_{1,2}a_{5,3}a_{6,4}a_{8,5}a_{3,6}a_{4,7}+a_{7,1}a_{1,2}a_{5,3}a_{6,4}a_{8,5}a_{3,7}a_{4,6}$$
$$-a_{7,1}a_{1,2}a_{8,3}a_{4,4}a_{6,5}a_{3,7}a_{5,6}+a_{7,1}a_{1,2}a_{8,3}a_{5,4}a_{6,5}a_{3,6}a_{4,7}+a_{7,1}a_{1,2}a_{8,3}a_{5,4}a_{6,5}a_{3,7}a_{4,6}$$
$$+a_{7,1}a_{1,2}a_{8,3}a_{6,4}a_{5,5}a_{3,6}a_{4,7}+a_{7,1}a_{1,2}a_{8,3}a_{6,4}a_{5,5}a_{3,7}a_{4,6}+a_{7,1}a_{4,2}a_{3,3}a_{5,4}a_{6,5}a_{1,7}a_{8,6}$$
$$+a_{7,1}a_{4,2}a_{3,3}a_{6,4}a_{5,5}a_{1,7}a_{8,6}+a_{7,1}a_{4,2}a_{3,3}a_{6,4}a_{8,5}a_{1,7}a_{5,6}+a_{6,1}a_{4,2}a_{3,3}a_{5,4}a_{8,5}a_{1,7}a_{7,6}$$
$$+a_{7,1}a_{4,2}a_{5,3}a_{6,4}a_{8,5}a_{1,7}a_{3,6}+a_{6,1}a_{1,2}a_{3,3}a_{5,4}a_{7,5}a_{4,7}a_{8,6}+a_{6,1}a_{1,2}a_{3,3}a_{5,4}a_{8,5}a_{4,7}a_{7,6}$$
$$+a_{6,1}a_{1,2}a_{5,3}a_{4,4}a_{7,5}a_{3,7}a_{8,6}+a_{6,1}a_{1,2}a_{5,3}a_{4,4}a_{8,5}a_{3,7}a_{7,6}+a_{6,1}a_{1,2}a_{7,3}a_{4,4}a_{5,5}a_{3,7}a_{8,6}$$
$$+a_{6,1}a_{1,2}a_{7,3}a_{4,4}a_{8,5}a_{3,7}a_{5,6}-a_{6,1}a_{1,2}a_{7,3}a_{5,4}a_{8,5}a_{3,6}a_{4,7}-a_{6,1}a_{1,2}a_{7,3}a_{5,4}a_{8,5}a_{3,7}a_{4,6}$$
$$+a_{6,1}a_{1,2}a_{8,3}a_{4,4}a_{5,5}a_{3,7}a_{7,6}-a_{6,1}a_{1,2}a_{8,3}a_{4,4}a_{7,5}a_{3,7}a_{5,6}+a_{6,1}a_{1,2}a_{8,3}a_{5,4}a_{7,5}a_{3,6}a_{4,7}$$
$$+a_{6,1}a_{1,2}a_{8,3}a_{5,4}a_{7,5}a_{3,7}a_{4,6}+a_{6,1}a_{4,2}a_{3,3}a_{5,4}a_{7,5}a_{1,7}a_{8,6},a_{7,1}a_{1,2}a_{2,3}a_{6,4}a_{5,5}a_{4,7}a_{8,6}$$
$$+a_{7,1}a_{1,2}a_{2,3}a_{6,4}a_{8,5}a_{4,7}a_{5,6}-a_{7,1}a_{1,2}a_{5,3}a_{4,4}a_{6,5}a_{2,7}a_{8,6}+a_{7,1}a_{1,2}a_{5,3}a_{6,4}a_{2,5}a_{4,7}a_{8,6}$$
$$-a_{7,1}a_{1,2}a_{5,3}a_{6,4}a_{8,5}a_{2,7}a_{4,6}+a_{7,1}a_{1,2}a_{8,3}a_{4,4}a_{6,5}a_{2,7}a_{5,6}-a_{7,1}a_{1,2}a_{8,3}a_{5,4}a_{6,5}a_{2,7}a_{4,6}$$
$$-a_{7,1}a_{1,2}a_{8,3}a_{6,4}a_{2,5}a_{4,7}a_{5,6}-a_{7,1}a_{1,2}a_{8,3}a_{6,4}a_{5,5}a_{2,7}a_{4,6}+a_{7,1}a_{4,2}a_{2,3}a_{5,4}a_{6,5}a_{1,7}a_{8,6}$$
$$+a_{7,1}a_{4,2}a_{2,3}a_{6,4}a_{5,5}a_{1,7}a_{8,6}+a_{7,1}a_{4,2}a_{2,3}a_{6,4}a_{8,5}a_{1,7}a_{5,6}+a_{7,1}a_{4,2}a_{5,3}a_{6,4}a_{2,5}a_{1,7}a_{8,6}$$
$$-a_{7,1}a_{4,2}a_{8,3}a_{6,4}a_{2,5}a_{1,7}a_{5,6}+a_{6,1}a_{1,2}a_{2,3}a_{5,4}a_{7,5}a_{4,7}a_{8,6}+a_{6,1}a_{1,2}a_{2,3}a_{5,4}a_{8,5}a_{4,7}a_{7,6}$$
$$-a_{6,1}a_{1,2}a_{5,3}a_{4,4}a_{7,5}a_{2,7}a_{8,6}-a_{6,1}a_{1,2}a_{5,3}a_{4,4}a_{8,5}a_{2,7}a_{7,6}-a_{6,1}a_{1,2}a_{7,3}a_{4,4}a_{5,5}a_{2,7}a_{8,6}$$
$$-a_{6,1}a_{1,2}a_{7,3}a_{4,4}a_{8,5}a_{2,7}a_{5,6}-a_{6,1}a_{1,2}a_{7,3}a_{5,4}a_{2,5}a_{4,7}a_{8,6}+a_{6,1}a_{1,2}a_{7,3}a_{5,4}a_{8,5}a_{2,7}a_{4,6}$$
$$-a_{6,1}a_{1,2}a_{8,3}a_{4,4}a_{5,5}a_{2,7}a_{7,6}+a_{6,1}a_{1,2}a_{8,3}a_{4,4}a_{7,5}a_{2,7}a_{5,6}-a_{6,1}a_{1,2}a_{8,3}a_{5,4}a_{2,5}a_{4,7}a_{7,6}$$
$$-a_{6,1}a_{1,2}a_{8,3}a_{5,4}a_{7,5}a_{2,7}a_{4,6}+a_{6,1}a_{4,2}a_{2,3}a_{5,4}a_{8,5}a_{1,7}a_{7,6}-a_{6,1}a_{4,2}a_{7,3}a_{5,4}a_{2,5}a_{1,7}a_{8,6}$$
$$+a_{6,1}a_{7,2}a_{2,3}a_{4,4}a_{5,5}a_{1,7}a_{8,6}+a_{6,1}a_{7,2}a_{2,3}a_{4,4}a_{8,5}a_{1,7}a_{5,6}-a_{6,1}a_{7,2}a_{2,3}a_{5,4}a_{8,5}a_{1,7}a_{4,6}$$
$$+a_{6,1}a_{7,2}a_{5,3}a_{4,4}a_{2,5}a_{1,7}a_{8,6}+a_{6,1}a_{7,2}a_{8,3}a_{4,4}a_{5,5}a_{1,7}a_{3,6}-a_{7,1}a_{1,2}a_{2,3}a_{5,4}a_{6,5}a_{3,7}a_{8,6}$$
$$-a_{7,1}a_{1,2}a_{2,3}a_{6,4}a_{5,5}a_{3,7}a_{8,6}-a_{7,1}a_{1,2}a_{2,3}a_{6,4}a_{8,5}a_{3,7}a_{5,6}-a_{7,1}a_{1,2}a_{3,3}a_{5,4}a_{6,5}a_{2,7}a_{8,6}$$
$$-a_{7,1}a_{1,2}a_{3,3}a_{6,4}a_{5,5}a_{2,7}a_{8,6}-a_{7,1}a_{1,2}a_{5,3}a_{6,4}a_{2,5}a_{3,7}a_{8,6}-a_{7,1}a_{1,2}a_{5,3}a_{6,4}a_{8,5}a_{2,7}a_{3,6}$$
$$-a_{7,1}a_{1,2}a_{8,3}a_{6,4}a_{2,5}a_{3,7}a_{5,6}+a_{7,1}a_{1,2}a_{8,3}a_{6,4}a_{5,5}a_{2,7}a_{3,6}+a_{6,1}a_{1,2}a_{2,3}a_{5,4}a_{7,5}a_{3,7}a_{8,6}$$
$$+a_{6,1}a_{1,2}a_{2,3}a_{5,4}a_{8,5}a_{3,7}a_{7,6}+a_{6,1}a_{1,2}a_{3,3}a_{5,4}a_{7,5}a_{2,7}a_{8,6}+a_{6,1}a_{1,2}a_{3,3}a_{5,4}a_{8,5}a_{2,7}a_{7,6}$$
$$+a_{6,1}a_{1,2}a_{5,3}a_{4,4}a_{7,5}a_{3,7}a_{8,6}+a_{6,1}a_{1,2}a_{5,3}a_{4,4}a_{8,5}a_{3,7}a_{7,6}+a_{6,1}a_{1,2}a_{7,3}a_{4,4}a_{5,5}a_{3,7}a_{8,6}$$
$$+a_{6,1}a_{1,2}a_{7,3}a_{4,4}a_{8,5}a_{3,7}a_{5,6}-a_{6,1}a_{1,2}a_{7,3}a_{5,4}a_{8,5}a_{3,6}a_{4,7}-a_{6,1}a_{1,2}a_{7,3}a_{5,4}a_{8,5}a_{3,7}a_{4,6}$$
$$+a_{6,1}a_{1,2}a_{8,3}a_{4,4}a_{5,5}a_{3,7}a_{7,6}-a_{6,1}a_{1,2}a_{8,3}a_{4,4}a_{7,5}a_{3,7}a_{5,6}+a_{6,1}a_{1,2}a_{8,3}a_{5,4}a_{7,5}a_{3,6}a_{4,7}$$
$$+a_{6,1}a_{1,2}a_{8,3}a_{5,4}a_{7,5}a_{3,7}a_{4,6}+a_{6,1}a_{4,2}a_{3,3}a_{5,4}a_{7,5}a_{1,7}a_{8,6},$$
$$-a_{6,1}a_{1,2}a_{2,3}a_{5,4}a_{7,5}a_{3,7}a_{8,6}-a_{6,1}a_{1,2}a_{2,3}a_{5,4}a_{8,5}a_{3,7}a_{7,6}-a_{6,1}a_{1,2}a_{3,3}a_{5,4}a_{7,5}a_{2,7}a_{8,6}$$
$$-a_{6,1}a_{1,2}a_{3,3}a_{5,4}a_{8,5}a_{2,7}a_{7,6}+a_{6,1}a_{1,2}a_{7,3}a_{5,4}a_{2,5}a_{3,7}a_{8,6}+a_{6,1}a_{1,2}a_{7,3}a_{5,4}a_{8,5}a_{2,7}a_{3,6}$$
$$+a_{6,1}a_{1,2}a_{8,3}a_{5,4}a_{2,5}a_{3,7}a_{7,6}-a_{6,1}a_{1,2}a_{8,3}a_{5,4}a_{7,5}a_{2,7}a_{3,6}-a_{6,1}a_{7,2}a_{2,3}a_{5,4}a_{8,5}a_{1,7}a_{3,6}$$
$$+a_{6,1}a_{7,2}a_{3,3}a_{5,4}a_{2,5}a_{1,7}a_{8,6}+a_{6,1}a_{7,2}a_{8,3}a_{5,4}a_{2,5}a_{1,7}a_{3,6}-a_{7,1}a_{1,2}a_{2,3}a_{5,4}a_{6,5}a_{3,7}a_{8,6}$$
$$-a_{7,1}a_{1,2}a_{2,3}a_{6,4}a_{5,5}a_{3,7}a_{8,6}-a_{7,1}a_{1,2}a_{2,3}a_{6,4}a_{8,5}a_{3,7}a_{5,6}-a_{7,1}a_{1,2}a_{3,3}a_{5,4}a_{6,5}a_{2,7}a_{8,6}$$
$$-a_{7,1}a_{1,2}a_{3,3}a_{6,4}a_{5,5}a_{2,7}a_{8,6}-a_{7,1}a_{1,2}a_{5,3}a_{6,4}a_{8,5}a_{2,7}a_{5,6}-a_{7,1}a_{1,2}a_{5,3}a_{6,4}a_{2,5}a_{3,7}a_{8,6}$$
$$-a_{7,1}a_{1,2}a_{5,3}a_{6,4}a_{8,5}a_{2,7}a_{3,6}+a_{7,1}a_{1,2}a_{8,3}a_{6,4}a_{2,5}a_{3,7}a_{5,6}$$

$$- a_{7,1}\, a_{1,2}\, a_{8,3}\, a_{6,4}\, a_{5,5}\, a_{2,7}\, a_{3,6}\,,\; -a_{6,1}\, a_{1,2}\, a_{2,3}\, a_{4,4}\, a_{7,5}\, a_{3,7}\, a_{8,6} - a_{6,1}\, a_{1,2}\, a_{2,3}\, a_{4,4}\, a_{8,5}\, a_{3,7}\, a_{7,6}$$
$$- a_{6,1}\, a_{1,2}\, a_{3,3}\, a_{4,4}\, a_{7,5}\, a_{2,7}\, a_{8,6} - a_{6,1}\, a_{1,2}\, a_{3,3}\, a_{4,4}\, a_{8,5}\, a_{2,7}\, a_{7,6} + a_{6,1}\, a_{1,2}\, a_{7,3}\, a_{4,4}\, a_{2,5}\, a_{3,7}\, a_{8,6}$$
$$+ a_{6,1}\, a_{1,2}\, a_{7,3}\, a_{4,4}\, a_{8,5}\, a_{2,7}\, a_{3,6} + a_{6,1}\, a_{1,2}\, a_{8,3}\, a_{4,4}\, a_{2,5}\, a_{3,7}\, a_{7,6} - a_{6,1}\, a_{1,2}\, a_{8,3}\, a_{4,4}\, a_{7,5}\, a_{2,7}\, a_{3,6}$$
$$- a_{6,1}\, a_{7,2}\, a_{2,3}\, a_{4,4}\, a_{8,5}\, a_{1,7}\, a_{3,6} + a_{6,1}\, a_{7,2}\, a_{3,3}\, a_{4,4}\, a_{2,5}\, a_{1,7}\, a_{8,6} + a_{6,1}\, a_{7,2}\, a_{8,3}\, a_{4,4}\, a_{2,5}\, a_{1,7}\, a_{3,6}$$
$$- a_{7,1}\, a_{1,2}\, a_{2,3}\, a_{4,4}\, a_{6,5}\, a_{3,7}\, a_{8,6} - a_{7,1}\, a_{1,2}\, a_{2,3}\, a_{6,4}\, a_{8,5}\, a_{3,6}\, a_{4,7} - a_{7,1}\, a_{1,2}\, a_{2,3}\, a_{6,4}\, a_{8,5}\, a_{3,7}\, a_{4,6}$$
$$- a_{7,1}\, a_{1,2}\, a_{3,3}\, a_{4,4}\, a_{6,5}\, a_{2,7}\, a_{8,6} + a_{7,1}\, a_{1,2}\, a_{3,3}\, a_{6,4}\, a_{2,5}\, a_{4,7}\, a_{8,6} - a_{7,1}\, a_{1,2}\, a_{3,3}\, a_{6,4}\, a_{8,5}\, a_{2,7}\, a_{4,6}$$
$$- a_{7,1}\, a_{1,2}\, a_{8,3}\, a_{4,4}\, a_{6,5}\, a_{2,7}\, a_{3,6} + a_{7,1}\, a_{1,2}\, a_{8,3}\, a_{6,4}\, a_{2,5}\, a_{3,6}\, a_{4,7} + a_{7,1}\, a_{1,2}\, a_{8,3}\, a_{6,4}\, a_{2,5}\, a_{3,7}\, a_{4,6}$$
$$- a_{7,1}\, a_{4,2}\, a_{2,3}\, a_{6,4}\, a_{8,5}\, a_{1,7}\, a_{3,6} + a_{7,1}\, a_{4,2}\, a_{3,3}\, a_{6,4}\, a_{2,5}\, a_{1,7}\, a_{8,6} + a_{7,1}\, a_{4,2}\, a_{8,3}\, a_{6,4}\, a_{2,5}\, a_{1,7}\, a_{3,6}\,,\; a_{7,1}\,($$
$$-a_{1,2}\, a_{2,3}\, a_{4,4}\, a_{5,5}\, a_{3,7}\, a_{8,6} - a_{1,2}\, a_{2,3}\, a_{4,4}\, a_{8,5}\, a_{3,7}\, a_{5,6} + a_{1,2}\, a_{2,3}\, a_{5,4}\, a_{8,5}\, a_{3,6}\, a_{4,7} + a_{1,2}\, a_{2,3}\, a_{5,4}\, a_{8,5}\, a_{3,7}\, a_{4,6}$$
$$- a_{1,2}\, a_{3,3}\, a_{4,4}\, a_{5,5}\, a_{2,7}\, a_{8,6} - a_{1,2}\, a_{3,3}\, a_{4,4}\, a_{8,5}\, a_{2,7}\, a_{5,6} - a_{1,2}\, a_{3,3}\, a_{5,4}\, a_{2,5}\, a_{4,7}\, a_{8,6} + a_{1,2}\, a_{3,3}\, a_{5,4}\, a_{8,5}\, a_{2,7}\, a_{4,6}$$
$$- a_{1,2}\, a_{5,3}\, a_{4,4}\, a_{2,5}\, a_{3,7}\, a_{8,6} - a_{1,2}\, a_{5,3}\, a_{4,4}\, a_{8,5}\, a_{2,7}\, a_{3,6} + a_{1,2}\, a_{8,3}\, a_{4,4}\, a_{2,5}\, a_{3,7}\, a_{5,6} - a_{1,2}\, a_{8,3}\, a_{4,4}\, a_{5,5}\, a_{2,7}\, a_{3,6}$$
$$- a_{1,2}\, a_{8,3}\, a_{5,4}\, a_{2,5}\, a_{3,6}\, a_{4,7} - a_{1,2}\, a_{8,3}\, a_{5,4}\, a_{2,5}\, a_{3,7}\, a_{4,6} + a_{4,2}\, a_{2,3}\, a_{5,4}\, a_{8,5}\, a_{1,7}\, a_{3,6} - a_{4,2}\, a_{3,3}\, a_{5,4}\, a_{2,5}\, a_{1,7}\, a_{8,6}$$
$$- a_{4,2}\, a_{8,3}\, a_{5,4}\, a_{2,5}\, a_{1,7}\, a_{3,6})\,,\; -a_{6,1}\,(-a_{1,2}\, a_{2,3}\, a_{4,4}\, a_{5,5}\, a_{3,7}\, a_{8,6} - a_{1,2}\, a_{2,3}\, a_{4,4}\, a_{8,5}\, a_{3,7}\, a_{5,6} + a_{1,2}\, a_{2,3}\, a_{5,4}\, a_{8,5}\, a_{3,6}\, a_{4,7}$$
$$+ a_{1,2}\, a_{2,3}\, a_{5,4}\, a_{8,5}\, a_{3,7}\, a_{4,6} - a_{1,2}\, a_{3,3}\, a_{4,4}\, a_{5,5}\, a_{2,7}\, a_{8,6} - a_{1,2}\, a_{3,3}\, a_{4,4}\, a_{8,5}\, a_{2,7}\, a_{5,6} - a_{1,2}\, a_{3,3}\, a_{5,4}\, a_{2,5}\, a_{4,7}\, a_{8,6}$$
$$+ a_{1,2}\, a_{3,3}\, a_{5,4}\, a_{8,5}\, a_{2,7}\, a_{4,6} - a_{1,2}\, a_{5,3}\, a_{4,4}\, a_{2,5}\, a_{3,7}\, a_{8,6} - a_{1,2}\, a_{5,3}\, a_{4,4}\, a_{8,5}\, a_{2,7}\, a_{3,6} + a_{1,2}\, a_{8,3}\, a_{4,4}\, a_{2,5}\, a_{3,7}\, a_{5,6}$$
$$- a_{1,2}\, a_{8,3}\, a_{4,4}\, a_{5,5}\, a_{2,7}\, a_{3,6} - a_{1,2}\, a_{8,3}\, a_{5,4}\, a_{2,5}\, a_{3,6}\, a_{4,7} - a_{1,2}\, a_{8,3}\, a_{5,4}\, a_{2,5}\, a_{3,7}\, a_{4,6} + a_{4,2}\, a_{2,3}\, a_{5,4}\, a_{8,5}\, a_{1,7}\, a_{3,6}$$
$$- a_{4,2}\, a_{3,3}\, a_{5,4}\, a_{2,5}\, a_{1,7}\, a_{8,6} - a_{4,2}\, a_{8,3}\, a_{5,4}\, a_{2,5}\, a_{1,7}\, a_{3,6})\,,\; a_{6,1}\, a_{1,2}\, a_{3,3}\, a_{5,4}\, a_{2,5}\, a_{4,7}\, a_{7,6} - a_{6,1}\, a_{1,2}\, a_{3,3}\, a_{4,4}\, a_{7,5}\, a_{2,7}\, a_{5,6}$$
$$+ a_{6,1}\, a_{1,2}\, a_{3,3}\, a_{4,4}\, a_{5,5}\, a_{2,7}\, a_{7,6} + a_{6,1}\, a_{1,2}\, a_{2,3}\, a_{5,4}\, a_{7,5}\, a_{3,7}\, a_{4,6} - a_{6,1}\, a_{1,2}\, a_{2,3}\, a_{4,4}\, a_{7,5}\, a_{3,7}\, a_{5,6}$$
$$+ a_{6,1}\, a_{1,2}\, a_{2,3}\, a_{5,4}\, a_{7,5}\, a_{3,6}\, a_{4,7} + a_{6,1}\, a_{1,2}\, a_{2,3}\, a_{4,4}\, a_{5,5}\, a_{3,7}\, a_{7,6} + a_{7,1}\, a_{1,2}\, a_{3,3}\, a_{6,4}\, a_{2,5}\, a_{4,7}\, a_{5,6}$$
$$+ a_{7,1}\, a_{1,2}\, a_{3,3}\, a_{5,4}\, a_{6,5}\, a_{2,7}\, a_{4,6} - a_{7,1}\, a_{1,2}\, a_{3,3}\, a_{4,4}\, a_{6,5}\, a_{2,7}\, a_{5,6} + a_{7,1}\, a_{1,2}\, a_{2,3}\, a_{6,4}\, a_{5,5}\, a_{3,7}\, a_{4,6}$$
$$+ a_{7,1}\, a_{1,2}\, a_{2,3}\, a_{6,4}\, a_{5,5}\, a_{3,6}\, a_{4,7} + a_{7,1}\, a_{1,2}\, a_{2,3}\, a_{5,4}\, a_{6,5}\, a_{3,6}\, a_{4,7}$$
$$- a_{7,1}\, a_{1,2}\, a_{2,3}\, a_{4,4}\, a_{6,5}\, a_{3,7}\, a_{5,6} + a_{6,1}\, a_{7,2}\, a_{5,3}\, a_{4,4}\, a_{2,5}\, a_{1,7}\, a_{3,6} - a_{6,1}\, a_{7,2}\, a_{3,3}\, a_{5,4}\, a_{2,5}\, a_{1,7}\, a_{4,6}$$
$$+ a_{6,1}\, a_{7,2}\, a_{3,3}\, a_{4,4}\, a_{2,5}\, a_{1,7}\, a_{5,6} + a_{6,1}\, a_{7,2}\, a_{2,3}\, a_{4,4}\, a_{5,5}\, a_{1,7}\, a_{3,6} - a_{6,1}\, a_{4,2}\, a_{7,3}\, a_{5,4}\, a_{2,5}\, a_{1,7}\, a_{3,6}$$
$$+ a_{6,1}\, a_{4,2}\, a_{3,3}\, a_{5,4}\, a_{2,5}\, a_{1,7}\, a_{7,6} + a_{6,1}\, a_{4,2}\, a_{2,3}\, a_{5,4}\, a_{7,5}\, a_{1,7}\, a_{3,6} - a_{6,1}\, a_{1,2}\, a_{7,3}\, a_{5,4}\, a_{2,5}\, a_{3,7}\, a_{4,6}$$
$$- a_{6,1}\, a_{1,2}\, a_{7,3}\, a_{5,4}\, a_{2,5}\, a_{3,6}\, a_{4,7} + a_{6,1}\, a_{1,2}\, a_{7,3}\, a_{4,4}\, a_{2,5}\, a_{3,7}\, a_{5,6} - a_{6,1}\, a_{1,2}\, a_{7,3}\, a_{4,4}\, a_{5,5}\, a_{2,7}\, a_{3,6}$$
$$- a_{6,1}\, a_{1,2}\, a_{5,3}\, a_{4,4}\, a_{7,5}\, a_{2,7}\, a_{3,6} + a_{6,1}\, a_{1,2}\, a_{3,3}\, a_{5,4}\, a_{7,5}\, a_{2,7}\, a_{4,6} + a_{6,1}\, a_{1,2}\, a_{5,3}\, a_{4,4}\, a_{2,5}\, a_{3,7}\, a_{7,6}$$
$$+ a_{7,1}\, a_{4,2}\, a_{5,3}\, a_{6,4}\, a_{2,5}\, a_{1,7}\, a_{3,6} + a_{7,1}\, a_{4,2}\, a_{3,3}\, a_{6,4}\, a_{2,5}\, a_{1,7}\, a_{5,6} + a_{7,1}\, a_{4,2}\, a_{2,3}\, a_{6,4}\, a_{5,5}\, a_{1,7}\, a_{3,6}$$
$$+ a_{7,1}\, a_{4,2}\, a_{2,3}\, a_{5,4}\, a_{6,5}\, a_{1,7}\, a_{3,6} + a_{7,1}\, a_{1,2}\, a_{5,3}\, a_{6,4}\, a_{2,5}\, a_{3,7}\, a_{4,6} + a_{7,1}\, a_{1,2}\, a_{5,3}\, a_{6,4}\, a_{2,5}\, a_{3,6}\, a_{4,7}$$
$$- a_{7,1}\, a_{1,2}\, a_{5,3}\, a_{4,4}\, a_{6,5}\, a_{2,7}\, a_{3,6} + a_{7,1}\, a_{1,2}\, a_{3,3}\, a_{6,4}\, a_{5,5}\, a_{2,7}\, a_{4,6}\,]$$

"82"

$$A := \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 & 0 & -1 & -1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

$poly\_coef\_F0\_to\_Fn = [\,-1,\ 1,\ 3,\ -3,\ -4,\ 7,\ -5,\ 14,\ -6\,]$

$positive\_feedback = [\,0,\ 2,\ 5,\ 6,\ 12,\ 26,\ 23,\ 30,\ 5\,]$

$negative\_feedback = [\,-1,\ -1,\ -2,\ -9,\ -16,\ -19,\ -28,\ -16,\ -11\,]$

$absolute\_feedback = [\,1,\ 3,\ 7,\ 15,\ 28,\ 45,\ 51,\ 46,\ 16\,]$

$wFn = [\,-1.,\ 0.33,\ 0.43,\ -0.20,\ -0.14,\ 0.16,\ -0.098,\ 0.30,\ -0.38\,]$

"Criterion ii"

$wD_7 = 0.25\ 10^{-5}$

$ratio\_to\_model\_C = 7.7$

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 0 & 2 & -2 & -2 & 0 & 0 & -2 & 0 \\ 6 & -6 & 0 & 3 & -3 & 0 & -3 & 3 \\ 0 & -6 & 0 & 0 & 0 & 6 & 6 & 6 \\ 0 & 8 & -2 & -5 & 3 & 0 & 1 & -9 \\ 0 & -2 & 2 & -1 & -3 & 0 & -1 & 3 \\ 0 & 0 & 0 & 3 & -3 & 0 & -3 & 3 \\ 0 & -6 & 0 & 3 & -3 & 0 & -3 & 3 \\ 0 & -4 & -2 & 1 & -3 & 0 & 1 & 3 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 0 & 4 & 4 & 4 & 2 & 0 & 4 & 2 \\ 16 & 8 & 4 & 5 & 3 & 0 & 5 & 3 \\ 0 & 32 & 20 & 22 & 14 & 16 & 24 & 20 \\ 0 & 10 & 6 & 7 & 5 & 0 & 7 & 11 \\ 0 & 6 & 6 & 7 & 3 & 0 & 5 & 3 \\ 0 & 8 & 4 & 5 & 3 & 0 & 5 & 3 \\ 0 & 8 & 4 & 5 & 3 & 0 & 5 & 3 \\ 0 & 6 & 6 & 5 & 3 & 0 & 7 & 3 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 1. & 0.50 & 0.50 & 0.50 & 0. & 1. & 0.50 & 0. \\ 0.38 & 0.75 & 0. & 0.60 & 1. & 1. & 0.60 & 1. \\ 1. & 0.19 & 0. & 0. & 0. & 0.38 & 0.25 & 0.30 \\ 1. & 0.80 & 0.33 & 0.71 & 0.60 & 1. & 0.14 & 0.82 \\ 1. & 0.33 & 0.33 & 0.14 & 1. & 1. & 0.20 & 1. \\ 1. & 0. & 0. & 0.60 & 1. & 1. & 0.60 & 1. \\ 1. & 0.75 & 0. & 0.60 & 1. & 1. & 0.60 & 1. \\ 1. & 0.67 & 0.33 & 0.20 & 1. & 1. & 0.14 & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} -6 & 6 & 0 & -3 & 3 & 0 & 3 & -3 \\ 0 & 0 & 0 & -3 & 3 & 0 & 3 & -3 \\ 0 & 2 & -2 & 1 & 3 & 0 & 1 & -3 \\ 0 & 0 & 0 & -3 & 3 & 0 & 3 & -3 \\ 0 & -2 & 2 & 2 & 0 & 0 & 2 & 0 \\ 0 & 4 & 2 & -1 & 3 & -6 & -1 & -3 \\ 0 & 4 & 2 & -1 & 3 & 0 & -1 & -3 \\ 0 & -2 & 2 & 2 & 0 & 0 & 2 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 16 & 8 & 4 & 5 & 3 & 0 & 5 & 3 \\ 0 & 8 & 4 & 5 & 3 & 0 & 5 & 3 \\ 0 & 6 & 6 & 7 & 3 & 0 & 5 & 3 \\ 0 & 8 & 4 & 5 & 3 & 0 & 5 & 3 \\ 0 & 4 & 4 & 4 & 2 & 0 & 4 & 2 \\ 0 & 6 & 6 & 5 & 3 & 16 & 7 & 3 \\ 0 & 6 & 6 & 5 & 3 & 0 & 7 & 3 \\ 0 & 4 & 4 & 4 & 2 & 0 & 4 & 2 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 0.38 & 0.75 & 0. & 0.60 & 1. & 1. & 0.60 & 1. \\ 1. & 0. & 0. & 0.60 & 1. & 1. & 0.60 & 1. \\ 1. & 0.33 & 0.33 & 0.14 & 1. & 1. & 0.20 & 1. \\ 1. & 0. & 0. & 0.60 & 1. & 1. & 0.60 & 1. \\ 1. & 0.50 & 0.50 & 0.50 & 0. & 1. & 0.50 & 0. \\ 1. & 0.67 & 0.33 & 0.20 & 1. & 0.38 & 0.14 & 1. \\ 1. & 0.67 & 0.33 & 0.20 & 1. & 1. & 0.14 & 1. \\ 1. & 0.50 & 0.50 & 0.50 & 0. & 1. & 0.50 & 0. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} 0 & 6 & 0 & -3 & 3 & 0 & 3 & -3 \\ 0 & 6 & 0 & -3 & 3 & 0 & 3 & -3 \\ 0 & 2 & 4 & 1 & 3 & 0 & 1 & -3 \\ 0 & 0 & 0 & 3 & 3 & 0 & 3 & -3 \\ 0 & -2 & 2 & 2 & 6 & 0 & 2 & 0 \\ 0 & 4 & 2 & -1 & 3 & 0 & -1 & -3 \\ 0 & 4 & 2 & -1 & 3 & 0 & 5 & -3 \\ 0 & -2 & 2 & 2 & 0 & 0 & 2 & 6 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 8 & 4 & 5 & 3 & 0 & 5 & 3 \\ 0 & 8 & 4 & 5 & 3 & 0 & 5 & 3 \\ 0 & 6 & 10 & 7 & 3 & 0 & 5 & 3 \\ 0 & 8 & 4 & 11 & 3 & 0 & 5 & 3 \\ 0 & 4 & 4 & 4 & 14 & 0 & 4 & 2 \\ 0 & 6 & 6 & 5 & 3 & 0 & 7 & 3 \\ 0 & 6 & 6 & 5 & 3 & 0 & 9 & 3 \\ 0 & 4 & 4 & 4 & 2 & 0 & 4 & 14 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 0.75 & 0. & 0.60 & 1. & 1. & 0.60 & 1. \\ 1. & 0.75 & 0. & 0.60 & 1. & 1. & 0.60 & 1. \\ 1. & 0.33 & 0.40 & 0.14 & 1. & 1. & 0.20 & 1. \\ 1. & 0. & 0. & 0.27 & 1. & 1. & 0.60 & 1. \\ 1. & 0.50 & 0.50 & 0.50 & 0.43 & 1. & 0.50 & 0. \\ 1. & 0.67 & 0.33 & 0.20 & 1. & 1. & 0.14 & 1. \\ 1. & 0.67 & 0.33 & 0.20 & 1. & 1. & 0.56 & 1. \\ 1. & 0.50 & 0.50 & 0.50 & 0. & 1. & 0.50 & 0.43 \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & -a_{1,2} & 0 & 0 & 0 & 0 & a_{1,7} & 0 \\ 0 & 0 & 0 & 0 & 0 & -a_{2,6} & a_{2,7} & 0 \\ a_{3,1} & 0 & 0 & 0 & -a_{3,5} & 0 & 0 & a_{3,8} \\ a_{4,1} & 0 & 0 & 0 & a_{4,5} & -a_{4,6} & 0 & 0 \\ -a_{5,1} & 0 & 0 & a_{5,4} & a_{5,5} & a_{5,6} & 0 & a_{5,8} \\ 0 & 0 & -a_{6,3} & -a_{6,4} & 0 & -a_{6,6} & -a_{6,7} & a_{6,8} \\ a_{7,1} & 0 & 0 & 0 & 0 & 0 & a_{7,7} & -a_{7,8} \\ -a_{8,1} & 0 & 0 & a_{8,4} & 0 & 0 & a_{8,7} & 0 \end{bmatrix}$$

"system feedback"

$$F_8$$

$$\begin{aligned} &-a_{5,6}\,a_{4,1}\,a_{1,2}\,a_{6,3}\,a_{8,4}\,a_{3,5}\,a_{2,7}\,a_{7,8} + a_{5,4}\,a_{4,1}\,a_{1,2}\,a_{6,3}\,a_{3,5}\,a_{2,6}\,a_{7,8}\,a_{8,7} - a_{5,4}\,a_{8,1}\,a_{1,2}\,a_{6,3}\,a_{3,5}\,a_{4,6}\,a_{2,7}\,a_{7,8} \\ &\quad + a_{8,4}\,a_{5,1}\,a_{1,2}\,a_{6,3}\,a_{3,5}\,a_{4,6}\,a_{2,7}\,a_{7,8} - a_{7,7}\,a_{3,1}\,a_{1,2}\,a_{6,3}\,a_{8,4}\,a_{4,5}\,a_{2,6}\,a_{5,8} - a_{7,7}\,a_{4,1}\,a_{1,2}\,a_{6,3}\,a_{8,4}\,a_{3,5}\,a_{2,6}\,a_{5,8} \\ &\quad - a_{7,7}\,a_{4,1}\,a_{1,2}\,a_{6,3}\,a_{8,4}\,a_{5,5}\,a_{2,6}\,a_{3,8} - a_{7,7}\,a_{5,1}\,a_{1,2}\,a_{6,3}\,a_{8,4}\,a_{4,5}\,a_{2,6}\,a_{3,8} - a_{7,1}\,a_{1,2}\,a_{6,3}\,a_{8,4}\,a_{5,5}\,a_{4,6}\,a_{2,7}\,a_{3,8} \\ &\quad + a_{7,7}\,a_{8,1}\,a_{1,2}\,a_{6,3}\,a_{5,4}\,a_{4,5}\,a_{2,6}\,a_{3,8} + a_{7,1}\,a_{1,2}\,a_{6,3}\,a_{5,4}\,a_{4,5}\,a_{2,6}\,a_{3,8}\,a_{8,7} - a_{7,1}\,a_{1,2}\,a_{6,3}\,a_{8,4}\,a_{3,5}\,a_{4,6}\,a_{2,7}\,a_{5,8} \\ &\quad - a_{7,1}\,a_{1,2}\,a_{6,3}\,a_{8,4}\,a_{4,5}\,a_{5,6}\,a_{2,7}\,a_{3,8} - a_{3,1}\,a_{1,2}\,a_{6,3}\,a_{8,4}\,a_{4,5}\,a_{5,6}\,a_{2,7}\,a_{7,8} + a_{3,1}\,a_{1,2}\,a_{6,3}\,a_{5,4}\,a_{4,5}\,a_{2,6}\,a_{7,8}\,a_{8,7} \\ &\quad - a_{3,1}\,a_{1,2}\,a_{6,3}\,a_{8,4}\,a_{5,5}\,a_{4,6}\,a_{2,7}\,a_{7,8} \end{aligned}$$

$$F_7$$

$$
\begin{aligned}
&a_{8,1}\,a_{6,3}\,a_{5,4}\,a_{3,5}\,a_{4,6}\,a_{1,7}\,a_{7,8} + a_{3,1}\,a_{6,3}\,a_{8,4}\,a_{4,5}\,a_{5,6}\,a_{1,7}\,a_{7,8} + a_{3,1}\,a_{6,3}\,a_{8,4}\,a_{5,5}\,a_{4,6}\,a_{1,7}\,a_{7,8}\\
&\quad + a_{7,1}\,a_{6,3}\,a_{8,4}\,a_{3,5}\,a_{4,6}\,a_{1,7}\,a_{5,8} + a_{7,1}\,a_{6,3}\,a_{8,4}\,a_{4,5}\,a_{5,6}\,a_{1,7}\,a_{3,8} + a_{7,1}\,a_{6,3}\,a_{8,4}\,a_{5,5}\,a_{4,6}\,a_{1,7}\,a_{3,8}\\
&\quad - a_{5,1}\,a_{6,3}\,a_{8,4}\,a_{3,5}\,a_{4,6}\,a_{1,7}\,a_{7,8} + a_{4,1}\,a_{6,3}\,a_{8,4}\,a_{3,5}\,a_{5,6}\,a_{1,7}\,a_{7,8} - a_{5,1}\,a_{1,2}\,a_{8,4}\,a_{4,5}\,a_{2,6}\,a_{6,7}\,a_{7,8}\\
&\quad + a_{5,1}\,a_{1,2}\,a_{8,4}\,a_{4,5}\,a_{2,6}\,a_{6,8}\,a_{7,7} - a_{5,1}\,a_{1,2}\,a_{8,4}\,a_{4,5}\,a_{6,6}\,a_{2,7}\,a_{7,8} - a_{7,1}\,a_{1,2}\,a_{5,4}\,a_{4,5}\,a_{2,6}\,a_{6,8}\,a_{8,7}\\
&\quad - a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{4,5}\,a_{2,6}\,a_{5,8}\,a_{8,7} + a_{7,1}\,a_{1,2}\,a_{8,4}\,a_{4,5}\,a_{2,6}\,a_{5,8}\,a_{6,7} + a_{7,1}\,a_{1,2}\,a_{8,4}\,a_{4,5}\,a_{5,6}\,a_{2,7}\,a_{6,8}\\
&\quad + a_{7,1}\,a_{1,2}\,a_{8,4}\,a_{4,5}\,a_{6,6}\,a_{2,7}\,a_{5,8} + a_{7,1}\,a_{1,2}\,a_{8,4}\,a_{5,5}\,a_{4,6}\,a_{2,7}\,a_{6,8} + a_{8,1}\,a_{1,2}\,a_{5,4}\,a_{4,5}\,a_{2,6}\,a_{6,7}\,a_{7,8}\\
&\quad - a_{8,1}\,a_{1,2}\,a_{5,4}\,a_{4,5}\,a_{2,6}\,a_{6,8}\,a_{7,7} + a_{8,1}\,a_{1,2}\,a_{5,4}\,a_{4,5}\,a_{6,6}\,a_{2,7}\,a_{7,8} - a_{8,1}\,a_{1,2}\,a_{6,4}\,a_{4,5}\,a_{2,6}\,a_{5,8}\,a_{7,7}\\
&\quad - a_{8,1}\,a_{1,2}\,a_{6,4}\,a_{4,5}\,a_{5,6}\,a_{2,7}\,a_{7,8} - a_{8,1}\,a_{1,2}\,a_{6,4}\,a_{5,5}\,a_{4,6}\,a_{2,7}\,a_{7,8} + a_{4,1}\,a_{1,2}\,a_{6,4}\,a_{5,5}\,a_{2,6}\,a_{7,8}\,a_{8,7}\\
&\quad - a_{4,1}\,a_{1,2}\,a_{8,4}\,a_{5,5}\,a_{2,6}\,a_{6,7}\,a_{7,8} + a_{4,1}\,a_{1,2}\,a_{8,4}\,a_{5,5}\,a_{2,6}\,a_{6,8}\,a_{7,7} - a_{4,1}\,a_{1,2}\,a_{8,4}\,a_{5,5}\,a_{6,6}\,a_{2,7}\,a_{7,8}\\
&\quad + a_{5,1}\,a_{1,2}\,a_{6,4}\,a_{4,5}\,a_{2,6}\,a_{7,8}\,a_{8,7} + a_{7,1}\,a_{1,2}\,a_{6,3}\,a_{3,5}\,a_{2,6}\,a_{5,8}\,a_{8,7} + a_{7,1}\,a_{1,2}\,a_{6,3}\,a_{5,5}\,a_{2,6}\,a_{3,8}\,a_{8,7}\\
&\quad + a_{8,1}\,a_{1,2}\,a_{6,3}\,a_{3,5}\,a_{2,6}\,a_{5,8}\,a_{7,7} + a_{8,1}\,a_{1,2}\,a_{6,3}\,a_{3,5}\,a_{5,6}\,a_{2,7}\,a_{7,8} + a_{8,1}\,a_{1,2}\,a_{6,3}\,a_{5,5}\,a_{2,6}\,a_{3,8}\,a_{7,7}\\
&\quad + a_{3,1}\,a_{1,2}\,a_{6,3}\,a_{5,5}\,a_{2,6}\,a_{7,8}\,a_{8,7} - a_{5,1}\,a_{1,2}\,a_{6,3}\,a_{3,5}\,a_{2,6}\,a_{7,8}\,a_{8,7} + a_{7,1}\,a_{1,2}\,a_{6,3}\,a_{8,4}\,a_{4,6}\,a_{2,7}\,a_{3,8}\\
&\quad + a_{3,1}\,a_{1,2}\,a_{6,3}\,a_{8,4}\,a_{4,6}\,a_{2,7}\,a_{7,8} + a_{4,1}\,a_{1,2}\,a_{6,3}\,a_{8,4}\,a_{2,6}\,a_{3,8}\,a_{7,7} - a_{8,1}\,a_{1,2}\,a_{6,3}\,a_{5,4}\,a_{4,5}\,a_{2,6}\,a_{3,8}\\
&\quad + a_{4,1}\,a_{1,2}\,a_{6,3}\,a_{8,4}\,a_{3,5}\,a_{2,6}\,a_{5,8} + a_{4,1}\,a_{1,2}\,a_{6,3}\,a_{8,4}\,a_{5,5}\,a_{2,6}\,a_{3,8} + a_{3,1}\,a_{1,2}\,a_{6,3}\,a_{8,4}\,a_{4,5}\,a_{2,6}\,a_{5,8}\\
&\quad + a_{5,1}\,a_{1,2}\,a_{6,3}\,a_{8,4}\,a_{4,5}\,a_{2,6}\,a_{3,8} - a_{7,1}\,a_{1,2}\,a_{6,3}\,a_{5,4}\,a_{3,5}\,a_{2,7}\,a_{4,6} - a_{3,1}\,a_{1,2}\,a_{6,3}\,a_{5,4}\,a_{4,5}\,a_{2,6}\,a_{7,7}\\
&\quad - a_{4,1}\,a_{1,2}\,a_{6,3}\,a_{5,4}\,a_{3,5}\,a_{2,6}\,a_{7,7}
\end{aligned}
$$

$$F_6$$

$$
\begin{aligned}
&-a_{7,8}\,a_{8,7}\,a_{6,4}\,a_{4,1}\,a_{1,2}\,a_{2,6} - a_{7,8}\,a_{8,7}\,a_{6,3}\,a_{3,5}\,a_{5,4}\,a_{4,6} - a_{7,8}\,a_{8,7}\,a_{6,3}\,a_{3,1}\,a_{1,2}\,a_{2,6} - a_{8,7}\,a_{7,1}\,a_{1,2}\,a_{2,6}\,a_{6,8}\,a_{5,5}\\
&\quad + a_{8,4}\,a_{4,5}\,a_{5,6}\,a_{6,3}\,a_{3,8}\,a_{7,7} + a_{8,4}\,a_{4,5}\,a_{5,1}\,a_{1,7}\,a_{7,8}\,a_{6,6} - a_{8,4}\,a_{4,1}\,a_{1,2}\,a_{2,7}\,a_{7,8}\,a_{5,5} + a_{8,4}\,a_{4,1}\,a_{1,2}\,a_{2,7}\,a_{7,8}\,a_{6,6}\\
&\quad - a_{8,4}\,a_{4,1}\,a_{1,2}\,a_{2,6}\,a_{6,8}\,a_{5,5} - a_{8,4}\,a_{4,1}\,a_{1,2}\,a_{2,6}\,a_{6,8}\,a_{7,7} + a_{8,4}\,a_{4,6}\,a_{6,3}\,a_{3,5}\,a_{5,8}\,a_{7,7} + a_{8,1}\,a_{1,2}\,a_{2,6}\,a_{6,7}\,a_{7,8}\,a_{5,5}\\
&\quad - a_{8,1}\,a_{1,2}\,a_{2,6}\,a_{6,3}\,a_{3,8}\,a_{5,5} - a_{8,1}\,a_{1,2}\,a_{2,6}\,a_{6,3}\,a_{3,8}\,a_{7,7} - a_{8,4}\,a_{4,6}\,a_{6,8}\,a_{7,1}\,a_{1,7}\,a_{5,5} - a_{2,7}\,a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{4,6}\,a_{5,5}\\
&\quad - a_{8,4}\,a_{4,6}\,a_{6,8}\,a_{2,7}\,a_{7,1}\,a_{1,2} - a_{8,4}\,a_{4,5}\,a_{5,8}\,a_{7,1}\,a_{1,7}\,a_{6,6} + a_{8,4}\,a_{4,5}\,a_{5,8}\,a_{2,7}\,a_{7,1}\,a_{1,2} - a_{8,1}\,a_{1,7}\,a_{7,8}\,a_{6,6}\,a_{5,4}\,a_{4,5}\\
&\quad + a_{8,1}\,a_{1,7}\,a_{7,8}\,a_{6,4}\,a_{4,6}\,a_{5,5} + a_{8,1}\,a_{1,7}\,a_{7,8}\,a_{6,4}\,a_{4,5}\,a_{5,6} - a_{8,1}\,a_{1,7}\,a_{7,8}\,a_{6,3}\,a_{3,5}\,a_{5,6} + a_{8,1}\,a_{1,2}\,a_{2,7}\,a_{7,8}\,a_{6,4}\,a_{4,6}\\
&\quad + a_{8,1}\,a_{1,2}\,a_{2,6}\,a_{6,8}\,a_{5,4}\,a_{4,5} - a_{8,1}\,a_{1,2}\,a_{2,6}\,a_{6,8}\,a_{7,7}\,a_{5,5} - a_{8,4}\,a_{4,5}\,a_{5,6}\,a_{6,8}\,a_{7,1}\,a_{1,7} + a_{8,4}\,a_{4,1}\,a_{1,7}\,a_{7,8}\,a_{6,6}\,a_{5,5}\\
&\quad + a_{8,1}\,a_{1,2}\,a_{2,7}\,a_{7,8}\,a_{5,4}\,a_{4,5} + a_{8,1}\,a_{1,2}\,a_{2,7}\,a_{7,8}\,a_{6,6}\,a_{5,5} + a_{8,4}\,a_{4,6}\,a_{6,3}\,a_{3,8}\,a_{7,7}\,a_{5,5} - a_{8,4}\,a_{4,6}\,a_{6,3}\,a_{3,8}\,a_{7,1}\,a_{1,7}\\
&\quad + a_{6,3}\,a_{3,5}\,a_{5,4}\,a_{4,1}\,a_{1,2}\,a_{2,6} + a_{5,4}\,a_{4,5}\,a_{6,3}\,a_{3,1}\,a_{1,2}\,a_{2,6} + a_{7,7}\,a_{6,3}\,a_{3,5}\,a_{5,1}\,a_{1,2}\,a_{2,6} - a_{7,7}\,a_{6,4}\,a_{4,5}\,a_{5,1}\,a_{1,2}\,a_{2,6}\\
&\quad - a_{7,7}\,a_{5,5}\,a_{6,3}\,a_{3,1}\,a_{1,2}\,a_{2,6} - a_{7,7}\,a_{5,5}\,a_{6,4}\,a_{4,1}\,a_{1,2}\,a_{2,6} + a_{7,1}\,a_{1,2}\,a_{2,6}\,a_{6,7}\,a_{5,4}\,a_{4,5} + a_{2,7}\,a_{7,1}\,a_{1,2}\,a_{6,3}\,a_{3,5}\,a_{5,6}\\
&\quad - a_{2,7}\,a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{4,5}\,a_{5,6} + a_{2,7}\,a_{7,1}\,a_{1,2}\,a_{6,6}\,a_{5,4}\,a_{4,5} + a_{7,1}\,a_{1,7}\,a_{6,3}\,a_{3,5}\,a_{5,4}\,a_{4,6} + a_{8,4}\,a_{4,1}\,a_{1,2}\,a_{2,6}\,a_{6,7}\,a_{7,8}\\
&\quad - a_{8,7}\,a_{7,1}\,a_{1,2}\,a_{2,6}\,a_{6,3}\,a_{3,8} - a_{8,4}\,a_{4,5}\,a_{5,1}\,a_{1,2}\,a_{2,7}\,a_{7,8} - a_{8,4}\,a_{4,6}\,a_{6,3}\,a_{3,1}\,a_{1,7}\,a_{7,8} - a_{8,4}\,a_{4,5}\,a_{5,1}\,a_{1,2}\,a_{2,6}\,a_{6,8}\\
&\quad - a_{8,4}\,a_{4,1}\,a_{1,2}\,a_{2,6}\,a_{6,3}\,a_{3,8} - a_{8,1}\,a_{1,2}\,a_{2,6}\,a_{6,3}\,a_{3,5}\,a_{5,8} + a_{8,1}\,a_{1,2}\,a_{2,6}\,a_{6,4}\,a_{4,5}\,a_{5,8}
\end{aligned}
$$

$$F_5$$

$$
\begin{aligned}
&-a_{8,1}\,a_{1,2}\,a_{2,7}\,a_{7,8}\,a_{6,6} + a_{8,1}\,a_{1,2}\,a_{2,7}\,a_{7,8}\,a_{5,5} + a_{8,1}\,a_{1,2}\,a_{2,6}\,a_{6,8}\,a_{5,5} + a_{8,1}\,a_{1,2}\,a_{2,6}\,a_{6,8}\,a_{7,7} - a_{8,4}\,a_{4,1}\,a_{1,7}\,a_{7,8}\,a_{6,6}\\
&\quad + a_{8,4}\,a_{4,1}\,a_{1,7}\,a_{7,8}\,a_{5,5} - a_{8,4}\,a_{4,6}\,a_{6,3}\,a_{3,8}\,a_{5,5} + a_{8,4}\,a_{4,6}\,a_{6,8}\,a_{7,1}\,a_{1,7} - a_{8,4}\,a_{4,5}\,a_{5,8}\,a_{7,1}\,a_{1,7} - a_{8,1}\,a_{1,7}\,a_{7,8}\,a_{5,4}\,a_{4,5}\\
&\quad - a_{8,1}\,a_{1,7}\,a_{7,8}\,a_{6,4}\,a_{4,6} - a_{8,1}\,a_{1,7}\,a_{7,8}\,a_{6,6}\,a_{5,5} + a_{7,1}\,a_{1,7}\,a_{6,4}\,a_{4,6}\,a_{5,5} + a_{2,7}\,a_{7,1}\,a_{1,2}\,a_{6,4}\,a_{4,6} - a_{6,3}\,a_{3,5}\,a_{5,1}\,a_{1,2}\,a_{2,6}\\
&\quad + a_{6,4}\,a_{4,5}\,a_{5,1}\,a_{1,2}\,a_{2,6} + a_{5,5}\,a_{6,3}\,a_{3,1}\,a_{1,2}\,a_{2,6} + a_{5,5}\,a_{6,4}\,a_{4,1}\,a_{1,2}\,a_{2,6} + a_{7,7}\,a_{6,3}\,a_{3,1}\,a_{1,2}\,a_{2,6} + a_{7,7}\,a_{6,4}\,a_{4,1}\,a_{1,2}\,a_{2,6}\\
&\quad + a_{7,1}\,a_{1,2}\,a_{2,6}\,a_{6,7}\,a_{5,5} + a_{2,7}\,a_{7,1}\,a_{1,2}\,a_{6,6}\,a_{5,5} + a_{2,7}\,a_{7,1}\,a_{1,2}\,a_{5,4}\,a_{4,5} - a_{7,1}\,a_{1,7}\,a_{6,3}\,a_{3,5}\,a_{5,6} + a_{7,1}\,a_{1,7}\,a_{6,4}\,a_{4,5}\,a_{5,6}\\
&\quad - a_{7,1}\,a_{1,7}\,a_{6,6}\,a_{5,4}\,a_{4,5} + a_{8,7}\,a_{7,1}\,a_{1,2}\,a_{2,6}\,a_{6,8} + a_{8,4}\,a_{4,1}\,a_{1,2}\,a_{2,7}\,a_{7,8} + a_{8,4}\,a_{4,5}\,a_{5,1}\,a_{1,7}\,a_{7,8} - a_{8,4}\,a_{4,5}\,a_{5,6}\,a_{6,3}\,a_{3,8}\\
&\quad - a_{8,4}\,a_{4,6}\,a_{6,3}\,a_{3,5}\,a_{5,8} + a_{8,4}\,a_{4,1}\,a_{1,2}\,a_{2,6}\,a_{6,8} + a_{8,1}\,a_{1,2}\,a_{2,6}\,a_{6,3}\,a_{3,8} - a_{8,1}\,a_{1,2}\,a_{2,6}\,a_{6,7}\,a_{7,8} + a_{5,4}\,a_{4,5}\,a_{6,6}\,a_{7,8}\,a_{8,7}\\
&\quad - a_{6,4}\,a_{4,5}\,a_{5,6}\,a_{7,8}\,a_{8,7} - a_{6,4}\,a_{5,5}\,a_{4,6}\,a_{7,8}\,a_{8,7} + a_{8,4}\,a_{4,5}\,a_{5,6}\,a_{6,7}\,a_{7,8} - a_{8,4}\,a_{4,5}\,a_{5,6}\,a_{6,8}\,a_{7,7} - a_{8,4}\,a_{4,5}\,a_{6,6}\,a_{5,8}\,a_{7,7}\\
&\quad + a_{8,4}\,a_{5,5}\,a_{4,6}\,a_{6,7}\,a_{7,8} - a_{8,4}\,a_{5,5}\,a_{4,6}\,a_{6,8}\,a_{7,7} + a_{6,3}\,a_{3,5}\,a_{5,6}\,a_{7,8}\,a_{8,7} - a_{6,3}\,a_{8,4}\,a_{4,6}\,a_{3,8}\,a_{7,7} + a_{6,3}\,a_{5,4}\,a_{3,5}\,a_{4,6}\,a_{7,7}
\end{aligned}
$$

$$F_4$$

$$
\begin{aligned}
&a_{7,8}\,a_{8,7}\,a_{6,6}\,a_{5,5} + a_{7,8}\,a_{8,7}\,a_{6,4}\,a_{4,6} + a_{8,4}\,a_{4,6}\,a_{6,8}\,a_{5,5} + a_{8,4}\,a_{4,6}\,a_{6,8}\,a_{7,7} + a_{8,4}\,a_{4,5}\,a_{5,8}\,a_{6,6} - a_{8,1}\,a_{1,7}\,a_{7,8}\,a_{5,5}\\
&\quad + a_{8,1}\,a_{1,7}\,a_{7,8}\,a_{6,6} - a_{7,1}\,a_{1,7}\,a_{6,4}\,a_{4,6} + a_{5,4}\,a_{4,5}\,a_{7,8}\,a_{8,7} - a_{6,3}\,a_{3,1}\,a_{1,2}\,a_{2,6} - a_{6,3}\,a_{3,5}\,a_{5,4}\,a_{4,6} - a_{6,4}\,a_{4,1}\,a_{1,2}\,a_{2,6}\\
&\quad - a_{7,1}\,a_{1,2}\,a_{2,6}\,a_{6,7} - a_{2,7}\,a_{7,1}\,a_{1,2}\,a_{6,6} - a_{7,1}\,a_{1,7}\,a_{6,6}\,a_{5,5} - a_{7,1}\,a_{1,7}\,a_{5,4}\,a_{4,5} - a_{7,7}\,a_{6,3}\,a_{3,5}\,a_{5,6} + a_{7,7}\,a_{6,4}\,a_{4,5}\,a_{5,6}\\
&\quad + a_{7,7}\,a_{6,4}\,a_{4,6}\,a_{5,5} - a_{7,7}\,a_{6,6}\,a_{5,4}\,a_{4,5} - a_{8,1}\,a_{1,2}\,a_{2,6}\,a_{6,8} - a_{8,1}\,a_{1,2}\,a_{2,7}\,a_{7,8} - a_{8,4}\,a_{4,1}\,a_{1,7}\,a_{7,8} + a_{8,4}\,a_{4,5}\,a_{5,6}\,a_{6,8}
\end{aligned}
$$

$$+ a_{8,4}\, a_{4,6}\, a_{6,3}\, a_{3,8} - a_{8,4}\, a_{4,6}\, a_{6,7}\, a_{7,8} - a_{8,4}\, a_{4,5}\, a_{5,8}\, a_{7,7} + a_{2,7}\, a_{7,1}\, a_{1,2}\, a_{5,5}$$

$$F_3$$

$$a_{8,1}\, a_{1,7}\, a_{7,8} + a_{8,4}\, a_{4,5}\, a_{5,8} - a_{8,4}\, a_{4,6}\, a_{6,8} - a_{6,6}\, a_{7,8}\, a_{8,7} + a_{7,8}\, a_{8,7}\, a_{5,5} - a_{2,7}\, a_{7,1}\, a_{1,2} + a_{7,1}\, a_{1,7}\, a_{6,6} - a_{7,1}\, a_{1,7}\, a_{5,5}$$
$$- a_{7,7}\, a_{6,4}\, a_{4,6} - a_{7,7}\, a_{6,6}\, a_{5,5} - a_{7,7}\, a_{5,4}\, a_{4,5} + a_{6,3}\, a_{3,5}\, a_{5,6} - a_{6,4}\, a_{4,5}\, a_{5,6} - a_{6,4}\, a_{4,6}\, a_{5,5} + a_{6,6}\, a_{5,4}\, a_{4,5}$$

$$F_2$$

$$-a_{7,8}\, a_{8,7} + a_{7,1}\, a_{1,7} + a_{7,7}\, a_{6,6} - a_{7,7}\, a_{5,5} + a_{6,4}\, a_{4,6} + a_{6,6}\, a_{5,5} + a_{5,4}\, a_{4,5}$$

$$F_1$$

$$a_{7,7} - a_{6,6} + a_{5,5}$$

$$F_0$$

-1

"adjoint (-A)"

$$[0\,,\ a_{1,2}\, a_{6,3}\, (-a_{5,4}\, a_{3,5}\, a_{4,6}\, a_{7,8}\, a_{8,7} + a_{8,4}\, a_{3,5}\, a_{4,6}\, a_{5,8}\, a_{7,7} + a_{8,4}\, a_{4,5}\, a_{5,6}\, a_{3,8}\, a_{7,7} + a_{8,4}\, a_{5,5}\, a_{4,6}\, a_{3,8}\, a_{7,7})\,,$$
$$-a_{1,2}\, a_{6,3}\, (-a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{7,8}\, a_{8,7} + a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{5,8}\, a_{7,7} + a_{8,4}\, a_{4,5}\, a_{5,6}\, a_{2,7}\, a_{7,8} + a_{8,4}\, a_{5,5}\, a_{4,6}\, a_{2,7}\, a_{7,8})\,,$$
$$a_{1,2}\, a_{6,3}\, (a_{5,4}\, a_{3,5}\, a_{2,6}\, a_{7,8}\, a_{8,7} - a_{8,4}\, a_{3,5}\, a_{2,6}\, a_{5,8}\, a_{7,7} - a_{8,4}\, a_{3,5}\, a_{5,6}\, a_{2,7}\, a_{7,8} - a_{8,4}\, a_{5,5}\, a_{2,6}\, a_{3,8}\, a_{7,7})\,,$$
$$a_{1,2}\, a_{6,3}\, a_{8,4}\, (-a_{3,5}\, a_{4,6}\, a_{2,7}\, a_{7,8} + a_{4,5}\, a_{2,6}\, a_{3,8}\, a_{7,7})\,,\ 0\,,$$
$$a_{1,2}\, a_{6,3}\, (a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{3,8}\, a_{8,7} - a_{8,4}\, a_{3,5}\, a_{4,6}\, a_{2,7}\, a_{5,8} - a_{8,4}\, a_{4,5}\, a_{5,6}\, a_{2,7}\, a_{3,8} - a_{8,4}\, a_{5,5}\, a_{4,6}\, a_{2,7}\, a_{3,8})\,,$$
$$-a_{1,2}\, a_{6,3}\, a_{5,4}\, (-a_{3,5}\, a_{4,6}\, a_{2,7}\, a_{7,8} + a_{4,5}\, a_{2,6}\, a_{3,8}\, a_{7,7})]$$

$$[-a_{3,1}\, a_{6,3}\, a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{7,8}\, a_{8,7} + a_{3,1}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{5,8}\, a_{7,7} + a_{3,1}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{5,6}\, a_{2,7}\, a_{7,8}$$
$$+ a_{3,1}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{4,6}\, a_{2,7}\, a_{7,8} - a_{4,1}\, a_{6,3}\, a_{5,4}\, a_{3,5}\, a_{2,6}\, a_{7,8}\, a_{8,7} + a_{4,1}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{2,6}\, a_{5,8}\, a_{7,7}$$
$$+ a_{4,1}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{5,6}\, a_{2,7}\, a_{7,8} + a_{4,1}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{2,6}\, a_{3,8}\, a_{7,7} - a_{5,1}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{4,6}\, a_{2,7}\, a_{7,8}$$
$$+ a_{5,1}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{3,8}\, a_{7,7} - a_{7,1}\, a_{6,3}\, a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{3,8}\, a_{8,7} + a_{7,1}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{4,6}\, a_{2,7}\, a_{5,8}$$
$$+ a_{7,1}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{5,6}\, a_{2,7}\, a_{3,8} + a_{8,1}\, a_{6,3}\, a_{5,4}\, a_{3,5}\, a_{4,6}\, a_{2,7}\, a_{7,8}$$
$$- a_{8,1}\, a_{6,3}\, a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{3,8}\, a_{7,7}\,, -a_{3,1}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{5,6}\, a_{1,7}\, a_{7,8} - a_{3,1}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{4,6}\, a_{1,7}\, a_{7,8}$$
$$- a_{4,1}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{5,6}\, a_{1,7}\, a_{7,8} + a_{5,1}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{4,6}\, a_{1,7}\, a_{7,8} - a_{7,1}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{4,6}\, a_{1,7}\, a_{5,8}$$
$$- a_{7,1}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{5,6}\, a_{1,7}\, a_{3,8} - a_{7,1}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{4,6}\, a_{1,7}\, a_{3,8} - a_{8,1}\, a_{6,3}\, a_{5,4}\, a_{3,5}\, a_{4,6}\, a_{1,7}\, a_{7,8}\,,$$
$$-a_{4,1}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{2,6}\, a_{1,7}\, a_{7,8} - a_{5,1}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{1,7}\, a_{7,8} + a_{7,1}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{1,7}\, a_{5,8}$$
$$+ a_{8,1}\, a_{6,3}\, a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{1,7}\, a_{7,8}\,, a_{3,1}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{2,6}\, a_{1,7}\, a_{7,8} - a_{5,1}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{2,6}\, a_{1,7}\, a_{7,8}$$
$$+ a_{7,1}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{2,6}\, a_{1,7}\, a_{5,8} + a_{7,1}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{2,6}\, a_{1,7}\, a_{3,8} + a_{8,1}\, a_{6,3}\, a_{5,4}\, a_{3,5}\, a_{2,6}\, a_{1,7}\, a_{7,8}\,,$$
$$-a_{3,1}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{1,7}\, a_{7,8} - a_{4,1}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{2,6}\, a_{1,7}\, a_{7,8} - a_{7,1}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{1,7}\, a_{3,8}\,,\ 0\,,$$
$$-a_{3,1}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{1,7}\, a_{5,8} - a_{4,1}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{2,6}\, a_{1,7}\, a_{5,8} - a_{4,1}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{2,6}\, a_{1,7}\, a_{3,8}$$
$$- a_{5,1}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{1,7}\, a_{3,8} + a_{8,1}\, a_{6,3}\, a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{1,7}\, a_{3,8}\,,$$
$$a_{3,1}\, a_{6,3}\, a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{1,7}\, a_{7,8} + a_{4,1}\, a_{6,3}\, a_{5,4}\, a_{3,5}\, a_{2,6}\, a_{1,7}\, a_{7,8} + a_{7,1}\, a_{6,3}\, a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{1,7}\, a_{3,8}]$$

$$[0\,, -a_{8,1}\, a_{1,2}\, a_{6,4}\, a_{5,5}\, a_{4,6}\, a_{3,8}\, a_{7,7} - a_{8,1}\, a_{1,2}\, a_{6,4}\, a_{4,5}\, a_{5,6}\, a_{3,8}\, a_{7,7} + a_{7,1}\, a_{1,2}\, a_{5,4}\, a_{4,5}\, a_{6,6}\, a_{3,8}\, a_{8,7}$$
$$- a_{7,1}\, a_{1,2}\, a_{6,4}\, a_{3,5}\, a_{4,6}\, a_{5,8}\, a_{8,7} - a_{7,1}\, a_{1,2}\, a_{6,4}\, a_{4,5}\, a_{5,6}\, a_{3,8}\, a_{8,7} - a_{7,1}\, a_{1,2}\, a_{6,4}\, a_{5,5}\, a_{4,6}\, a_{3,8}\, a_{8,7}$$
$$+ a_{7,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{4,6}\, a_{5,8}\, a_{6,7} + a_{7,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{5,6}\, a_{3,8}\, a_{6,7} + a_{7,1}\, a_{1,2}\, a_{8,4}\, a_{5,5}\, a_{4,6}\, a_{3,8}\, a_{6,7}$$
$$+ a_{8,1}\, a_{1,2}\, a_{5,4}\, a_{3,5}\, a_{4,6}\, a_{6,7}\, a_{7,8} - a_{8,1}\, a_{1,2}\, a_{5,4}\, a_{3,5}\, a_{4,6}\, a_{6,8}\, a_{7,7} + a_{8,1}\, a_{1,2}\, a_{5,4}\, a_{4,5}\, a_{6,6}\, a_{3,8}\, a_{7,7}$$
$$- a_{8,1}\, a_{1,2}\, a_{6,4}\, a_{3,5}\, a_{4,6}\, a_{5,8}\, a_{7,7} - a_{4,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{5,6}\, a_{6,8}\, a_{7,7} - a_{4,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{6,6}\, a_{5,8}\, a_{7,7}$$
$$- a_{4,1}\, a_{1,2}\, a_{8,4}\, a_{5,5}\, a_{6,6}\, a_{3,8}\, a_{7,7} + a_{5,1}\, a_{1,2}\, a_{6,4}\, a_{3,5}\, a_{4,6}\, a_{7,8}\, a_{8,7} - a_{5,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{4,6}\, a_{6,7}\, a_{7,8}$$
$$+ a_{5,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{4,6}\, a_{6,8}\, a_{7,7} - a_{5,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{6,6}\, a_{3,8}\, a_{7,7} - a_{7,1}\, a_{1,2}\, a_{5,4}\, a_{3,5}\, a_{4,6}\, a_{6,8}\, a_{8,7}$$
$$+ a_{3,1}\, a_{1,2}\, a_{5,4}\, a_{4,5}\, a_{6,6}\, a_{7,8}\, a_{8,7} - a_{3,1}\, a_{1,2}\, a_{6,4}\, a_{4,5}\, a_{5,6}\, a_{7,8}\, a_{8,7} - a_{3,1}\, a_{1,2}\, a_{6,4}\, a_{5,5}\, a_{4,6}\, a_{7,8}\, a_{8,7}$$
$$+ a_{3,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{5,6}\, a_{6,7}\, a_{7,8} - a_{3,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{5,6}\, a_{6,8}\, a_{7,7} - a_{3,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{6,6}\, a_{5,8}\, a_{7,7}$$
$$+ a_{3,1}\, a_{1,2}\, a_{8,4}\, a_{5,5}\, a_{4,6}\, a_{6,7}\, a_{7,8} - a_{3,1}\, a_{1,2}\, a_{8,4}\, a_{5,5}\, a_{4,6}\, a_{6,8}\, a_{7,7} + a_{4,1}\, a_{1,2}\, a_{5,4}\, a_{3,5}\, a_{6,6}\, a_{7,8}\, a_{8,7}$$
$$- a_{4,1}\, a_{1,2}\, a_{6,4}\, a_{3,5}\, a_{5,6}\, a_{7,8}\, a_{8,7} + a_{4,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{5,6}\, a_{6,7}\, a_{7,8}\,, -a_{4,1}\, a_{1,2}\, a_{6,4}\, a_{5,5}\, a_{2,6}\, a_{7,8}\, a_{8,7}$$
$$+ a_{4,1}\, a_{1,2}\, a_{8,4}\, a_{5,5}\, a_{2,6}\, a_{6,7}\, a_{7,8} - a_{4,1}\, a_{1,2}\, a_{8,4}\, a_{5,5}\, a_{2,6}\, a_{6,8}\, a_{7,7} + a_{4,1}\, a_{1,2}\, a_{8,4}\, a_{5,5}\, a_{6,6}\, a_{2,7}\, a_{7,8}$$
$$- a_{5,1}\, a_{1,2}\, a_{6,4}\, a_{4,5}\, a_{2,6}\, a_{7,8}\, a_{8,7} + a_{5,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{6,7}\, a_{7,8} - a_{5,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{6,8}\, a_{7,7}$$
$$+ a_{5,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{6,6}\, a_{2,7}\, a_{7,8} + a_{7,1}\, a_{1,2}\, a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{6,8}\, a_{8,7} + a_{7,1}\, a_{1,2}\, a_{6,4}\, a_{4,5}\, a_{2,6}\, a_{5,8}\, a_{8,7}$$

$$
\begin{aligned}
&- a_{7,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{5,8}\, a_{6,7} - a_{7,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{5,6}\, a_{2,7}\, a_{6,8} - a_{7,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{6,6}\, a_{2,7}\, a_{5,8} \\
&- a_{7,1}\, a_{1,2}\, a_{8,4}\, a_{5,5}\, a_{4,6}\, a_{2,7}\, a_{6,8} - a_{8,1}\, a_{1,2}\, a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{6,7}\, a_{7,8} + a_{8,1}\, a_{1,2}\, a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{6,8}\, a_{7,7} \\
&- a_{8,1}\, a_{1,2}\, a_{5,4}\, a_{4,5}\, a_{6,6}\, a_{2,7}\, a_{7,8} + a_{8,1}\, a_{1,2}\, a_{6,4}\, a_{4,5}\, a_{2,6}\, a_{5,8}\, a_{7,7} + a_{8,1}\, a_{1,2}\, a_{6,4}\, a_{4,5}\, a_{5,6}\, a_{2,7}\, a_{7,8} \\
&+ a_{8,1}\, a_{1,2}\, a_{6,4}\, a_{5,5}\, a_{4,6}\, a_{2,7}\, a_{7,8}\,,\, a_{3,1}\, a_{1,2}\, a_{6,4}\, a_{5,5}\, a_{2,6}\, a_{7,8}\, a_{8,7} - a_{3,1}\, a_{1,2}\, a_{8,4}\, a_{5,5}\, a_{2,6}\, a_{6,7}\, a_{7,8} \\
&+ a_{3,1}\, a_{1,2}\, a_{8,4}\, a_{5,5}\, a_{2,6}\, a_{6,8}\, a_{7,7} - a_{3,1}\, a_{1,2}\, a_{8,4}\, a_{5,5}\, a_{6,6}\, a_{2,7}\, a_{7,8} - a_{5,1}\, a_{1,2}\, a_{6,4}\, a_{3,5}\, a_{2,6}\, a_{7,8}\, a_{8,7} \\
&+ a_{5,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{2,6}\, a_{6,7}\, a_{7,8} - a_{5,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{2,6}\, a_{6,8}\, a_{7,7} + a_{5,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{6,6}\, a_{2,7}\, a_{7,8} \\
&+ a_{7,1}\, a_{1,2}\, a_{5,4}\, a_{3,5}\, a_{2,6}\, a_{6,8}\, a_{8,7} + a_{7,1}\, a_{1,2}\, a_{6,4}\, a_{3,5}\, a_{2,6}\, a_{5,8}\, a_{8,7} + a_{7,1}\, a_{1,2}\, a_{6,4}\, a_{5,5}\, a_{2,6}\, a_{3,8}\, a_{8,7} \\
&- a_{7,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{2,6}\, a_{5,8}\, a_{6,7} - a_{7,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{5,6}\, a_{2,7}\, a_{6,8} - a_{7,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{6,6}\, a_{2,7}\, a_{5,8} \\
&- a_{7,1}\, a_{1,2}\, a_{8,4}\, a_{5,5}\, a_{2,6}\, a_{3,8}\, a_{6,7} - a_{7,1}\, a_{1,2}\, a_{8,4}\, a_{5,5}\, a_{6,6}\, a_{2,7}\, a_{3,8} - a_{8,1}\, a_{1,2}\, a_{5,4}\, a_{3,5}\, a_{2,6}\, a_{6,7}\, a_{7,8} \\
&+ a_{8,1}\, a_{1,2}\, a_{5,4}\, a_{3,5}\, a_{2,6}\, a_{6,8}\, a_{7,7} - a_{8,1}\, a_{1,2}\, a_{5,4}\, a_{3,5}\, a_{6,6}\, a_{2,7}\, a_{7,8} + a_{8,1}\, a_{1,2}\, a_{6,4}\, a_{3,5}\, a_{2,6}\, a_{5,8}\, a_{7,7} \\
&+ a_{8,1}\, a_{1,2}\, a_{6,4}\, a_{3,5}\, a_{5,6}\, a_{2,7}\, a_{7,8} + a_{8,1}\, a_{1,2}\, a_{6,4}\, a_{4,5}\, a_{2,6}\, a_{3,8}\, a_{7,7}\,,\, -a_{3,1}\, a_{1,2}\, a_{6,4}\, a_{4,5}\, a_{2,6}\, a_{7,8}\, a_{8,7} \\
&+ a_{3,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{6,7}\, a_{7,8} - a_{3,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{6,8}\, a_{7,7} + a_{3,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{6,6}\, a_{2,7}\, a_{7,8} \\
&- a_{4,1}\, a_{1,2}\, a_{6,4}\, a_{3,5}\, a_{2,6}\, a_{7,8}\, a_{8,7} + a_{4,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{2,6}\, a_{6,7}\, a_{7,8} - a_{4,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{2,6}\, a_{6,8}\, a_{7,7} \\
&+ a_{4,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{6,6}\, a_{2,7}\, a_{7,8} - a_{7,1}\, a_{1,2}\, a_{6,4}\, a_{4,5}\, a_{2,6}\, a_{3,8}\, a_{8,7} - a_{7,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{4,6}\, a_{2,7}\, a_{6,8} \\
&+ a_{7,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{3,8}\, a_{6,7} + a_{7,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{6,6}\, a_{2,7}\, a_{3,8} + a_{8,1}\, a_{1,2}\, a_{6,4}\, a_{3,5}\, a_{4,6}\, a_{2,7}\, a_{7,8} \\
&- a_{8,1}\, a_{1,2}\, a_{6,4}\, a_{4,5}\, a_{2,6}\, a_{3,8}\, a_{7,7}\,,\, -a_{3,1}\, a_{1,2}\, a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{7,8}\, a_{8,7} + a_{3,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{5,8}\, a_{7,7} \\
&+ a_{3,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{5,6}\, a_{2,7}\, a_{7,8} + a_{3,1}\, a_{1,2}\, a_{8,4}\, a_{5,5}\, a_{4,6}\, a_{2,7}\, a_{7,8} - a_{4,1}\, a_{1,2}\, a_{5,4}\, a_{3,5}\, a_{2,6}\, a_{7,8}\, a_{8,7} \\
&+ a_{4,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{2,6}\, a_{5,8}\, a_{7,7} + a_{4,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{5,6}\, a_{2,7}\, a_{7,8} + a_{4,1}\, a_{1,2}\, a_{8,4}\, a_{5,5}\, a_{2,6}\, a_{3,8}\, a_{7,7} \\
&- a_{5,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{4,6}\, a_{2,7}\, a_{7,8} + a_{5,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{3,8}\, a_{7,7} - a_{7,1}\, a_{1,2}\, a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{3,8}\, a_{8,7} \\
&+ a_{7,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{4,6}\, a_{2,7}\, a_{5,8} + a_{7,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{5,6}\, a_{2,7}\, a_{3,8} + a_{7,1}\, a_{1,2}\, a_{8,4}\, a_{5,5}\, a_{4,6}\, a_{2,7}\, a_{3,8} \\
&+ a_{8,1}\, a_{1,2}\, a_{5,4}\, a_{3,5}\, a_{4,6}\, a_{2,7}\, a_{7,8} - a_{8,1}\, a_{1,2}\, a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{3,8}\, a_{7,7}\,,\, -a_{3,1}\, a_{1,2}\, a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{6,8}\, a_{8,7} \\
&- a_{3,1}\, a_{1,2}\, a_{6,4}\, a_{4,5}\, a_{2,6}\, a_{5,8}\, a_{8,7} + a_{3,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{5,8}\, a_{6,7} + a_{3,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{5,6}\, a_{2,7}\, a_{6,8} \\
&+ a_{3,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{6,6}\, a_{2,7}\, a_{5,8} + a_{3,1}\, a_{1,2}\, a_{8,4}\, a_{5,5}\, a_{4,6}\, a_{2,7}\, a_{6,8} - a_{4,1}\, a_{1,2}\, a_{5,4}\, a_{3,5}\, a_{2,6}\, a_{6,8}\, a_{8,7} \\
&- a_{4,1}\, a_{1,2}\, a_{6,4}\, a_{3,5}\, a_{2,6}\, a_{5,8}\, a_{8,7} - a_{4,1}\, a_{1,2}\, a_{6,4}\, a_{5,5}\, a_{2,6}\, a_{3,8}\, a_{8,7} + a_{4,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{2,6}\, a_{5,8}\, a_{6,7} \\
&+ a_{4,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{5,6}\, a_{2,7}\, a_{6,8} + a_{4,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{6,6}\, a_{2,7}\, a_{5,8} + a_{4,1}\, a_{1,2}\, a_{8,4}\, a_{5,5}\, a_{2,6}\, a_{3,8}\, a_{6,7} \\
&+ a_{4,1}\, a_{1,2}\, a_{8,4}\, a_{5,5}\, a_{6,6}\, a_{2,7}\, a_{3,8} - a_{5,1}\, a_{1,2}\, a_{6,4}\, a_{4,5}\, a_{2,6}\, a_{3,8}\, a_{8,7} - a_{5,1}\, a_{1,2}\, a_{8,4}\, a_{3,5}\, a_{4,6}\, a_{2,7}\, a_{6,8} \\
&+ a_{5,1}\, a_{1,2}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{3,8}\, a_{6,7} + a_{7,1}\, a_{1,2}\, a_{5,4}\, a_{3,5}\, a_{4,6}\, a_{2,7}\, a_{6,8} - a_{7,1}\, a_{1,2}\, a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{3,8}\, a_{6,7} \\
&- a_{7,1}\, a_{1,2}\, a_{5,4}\, a_{4,5}\, a_{6,6}\, a_{2,7}\, a_{3,8} + a_{7,1}\, a_{1,2}\, a_{6,4}\, a_{3,5}\, a_{4,6}\, a_{2,7}\, a_{5,8} + a_{7,1}\, a_{1,2}\, a_{6,4}\, a_{4,5}\, a_{5,6}\, a_{2,7}\, a_{3,8} \\
&+ a_{7,1}\, a_{1,2}\, a_{6,4}\, a_{5,5}\, a_{4,6}\, a_{2,7}\, a_{3,8}\,]
\end{aligned}
$$

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\begin{aligned}
&[0\,,\, a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{4,5}\, a_{5,6}\, a_{7,8}\, a_{8,7} + a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{5,5}\, a_{4,6}\, a_{7,8}\, a_{8,7} + a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{3,5}\, a_{5,6}\, a_{7,8}\, a_{8,7} \\
&- a_{5,1}\, a_{1,2}\, a_{6,3}\, a_{3,5}\, a_{4,6}\, a_{7,8}\, a_{8,7} + a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{3,5}\, a_{4,6}\, a_{5,8}\, a_{8,7} + a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{4,5}\, a_{5,6}\, a_{3,8}\, a_{8,7} \\
&+ a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{3,5}\, a_{5,5}\, a_{4,6}\, a_{3,8}\, a_{8,7} + a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{3,5}\, a_{4,6}\, a_{5,8}\, a_{7,7} + a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{4,5}\, a_{5,6}\, a_{3,8}\, a_{7,7} \\
&+ a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{3,5}\, a_{5,5}\, a_{4,6}\, a_{3,8}\, a_{7,7}\,,\, a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{3,5}\, a_{5,5}\, a_{2,6}\, a_{7,8}\, a_{8,7} + a_{5,1}\, a_{1,2}\, a_{6,3}\, a_{4,5}\, a_{2,6}\, a_{7,8}\, a_{8,7} \\
&- a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{4,5}\, a_{2,6}\, a_{5,8}\, a_{8,7} - a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{4,5}\, a_{2,6}\, a_{5,8}\, a_{7,7} - a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{4,5}\, a_{5,6}\, a_{2,7}\, a_{7,8} \\
&- a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{5,5}\, a_{4,6}\, a_{2,7}\, a_{7,8}\,,\, -a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{5,5}\, a_{2,6}\, a_{7,8}\, a_{8,7} + a_{5,1}\, a_{1,2}\, a_{6,3}\, a_{3,5}\, a_{2,6}\, a_{7,8}\, a_{8,7} \\
&- a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{3,5}\, a_{2,6}\, a_{5,8}\, a_{8,7} - a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{5,5}\, a_{2,6}\, a_{3,8}\, a_{8,7} - a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{3,5}\, a_{2,6}\, a_{5,8}\, a_{7,7} \\
&- a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{3,5}\, a_{5,6}\, a_{2,7}\, a_{7,8} - a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{5,5}\, a_{2,6}\, a_{3,8}\, a_{7,7}\,,\, a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{4,5}\, a_{2,6}\, a_{7,8}\, a_{8,7} \\
&+ a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{3,5}\, a_{2,6}\, a_{7,8}\, a_{8,7} + a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{4,5}\, a_{2,6}\, a_{3,8}\, a_{8,7} - a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{3,5}\, a_{4,6}\, a_{2,7}\, a_{7,8}
\end{aligned}
$$

$$+ a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{4,5}\, a_{2,6}\, a_{3,8}\, a_{7,7}\,,\ 0\,,\ a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{4,5}\, a_{2,6}\, a_{5,8}\, a_{8,7} + a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{3,5}\, a_{2,6}\, a_{5,8}\, a_{8,7}$$
$$+ a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{5,5}\, a_{2,6}\, a_{3,8}\, a_{8,7} + a_{5,1}\, a_{1,2}\, a_{6,3}\, a_{4,5}\, a_{2,6}\, a_{3,8}\, a_{8,7} - a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{3,5}\, a_{4,6}\, a_{2,7}\, a_{5,8}$$
$$- a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{4,5}\, a_{5,6}\, a_{2,7}\, a_{3,8} - a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{5,5}\, a_{4,6}\, a_{2,7}\, a_{3,8}\,,\ -a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{4,5}\, a_{2,6}\, a_{5,8}\, a_{7,7}$$
$$- a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{4,5}\, a_{5,6}\, a_{2,7}\, a_{7,8} - a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{5,5}\, a_{4,6}\, a_{2,7}\, a_{7,8} - a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{3,5}\, a_{2,6}\, a_{5,8}\, a_{7,7}$$
$$- a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{3,5}\, a_{5,6}\, a_{2,7}\, a_{7,8} - a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{5,5}\, a_{2,6}\, a_{3,8}\, a_{7,7} + a_{5,1}\, a_{1,2}\, a_{6,3}\, a_{3,5}\, a_{4,6}\, a_{2,7}\, a_{7,8}$$
$$- a_{5,1}\, a_{1,2}\, a_{6,3}\, a_{4,5}\, a_{2,6}\, a_{3,8}\, a_{7,7} - a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{3,5}\, a_{4,6}\, a_{2,7}\, a_{5,8} - a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{4,5}\, a_{5,6}\, a_{2,7}\, a_{3,8}$$
$$- a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{5,5}\, a_{4,6}\, a_{2,7}\, a_{3,8}\,]$$
$$[0\,,\ -a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{4,6}\, a_{7,8}\, a_{8,7} + a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,6}\, a_{5,8}\, a_{7,7} - a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{5,6}\, a_{3,8}\, a_{7,7}$$
$$+ a_{5,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,6}\, a_{3,8}\, a_{7,7} - a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{4,6}\, a_{3,8}\, a_{8,7} - a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{4,6}\, a_{3,8}\, a_{7,7}\,,$$
$$-a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{2,6}\, a_{7,8}\, a_{8,7} + a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{2,6}\, a_{5,8}\, a_{7,7} + a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{5,6}\, a_{2,7}\, a_{7,8}$$
$$- a_{5,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,6}\, a_{2,7}\, a_{7,8} + a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,6}\, a_{2,7}\, a_{5,8} + a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{4,6}\, a_{2,7}\, a_{7,8}\,,$$
$$a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{2,6}\, a_{7,8}\, a_{8,7} - a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{2,6}\, a_{5,8}\, a_{7,7} - a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{5,6}\, a_{2,7}\, a_{7,8}$$
$$- a_{5,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{2,6}\, a_{3,8}\, a_{7,7} + a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{2,6}\, a_{3,8}\, a_{8,7} - a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{5,6}\, a_{2,7}\, a_{3,8}$$
$$+ a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{2,6}\, a_{3,8}\, a_{7,7}\,,$$
$$-a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,6}\, a_{2,7}\, a_{7,8} - a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{2,6}\, a_{3,8}\, a_{7,7} - a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,6}\, a_{2,7}\, a_{3,8}\,,\ 0\,,$$
$$-a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,6}\, a_{2,7}\, a_{5,8} - a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{2,6}\, a_{3,8}\, a_{8,7} + a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{5,6}\, a_{2,7}\, a_{3,8}$$
$$- a_{5,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,6}\, a_{2,7}\, a_{3,8} + a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{4,6}\, a_{2,7}\, a_{3,8}\,,$$
$$a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{4,6}\, a_{2,7}\, a_{7,8} + a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{2,6}\, a_{3,8}\, a_{7,7} + a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{4,6}\, a_{2,7}\, a_{3,8}\,]$$
$$[0\,,\ -a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{4,5}\, a_{7,8}\, a_{8,7} + a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{5,8}\, a_{7,7} - a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{3,5}\, a_{7,8}\, a_{8,7}$$
$$+ a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{5,8}\, a_{7,7} + a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{3,8}\, a_{7,7} + a_{5,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{3,8}\, a_{7,7}$$
$$- a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{4,5}\, a_{3,8}\, a_{8,7} - a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{4,5}\, a_{3,8}\, a_{7,7}\,,\ -a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{2,7}\, a_{7,8}$$
$$- a_{5,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,7}\, a_{7,8} + a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,7}\, a_{5,8} + a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{4,5}\, a_{2,7}\, a_{7,8}\,,$$
$$a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{2,7}\, a_{7,8} - a_{5,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{2,7}\, a_{7,8} + a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{2,7}\, a_{5,8}$$
$$+ a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{2,7}\, a_{3,8} + a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{3,5}\, a_{2,7}\, a_{7,8}\,,$$
$$-a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,7}\, a_{7,8} - a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{2,7}\, a_{7,8} - a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,7}\, a_{3,8}\,,\ 0\,,$$
$$-a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,7}\, a_{5,8} - a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{2,7}\, a_{5,8} - a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{2,7}\, a_{3,8}$$
$$- a_{5,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,7}\, a_{3,8} + a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{3,5}\, a_{2,7}\, a_{7,8}\,,$$
$$a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{4,5}\, a_{2,7}\, a_{7,8} + a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{3,5}\, a_{2,7}\, a_{7,8} + a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{4,5}\, a_{2,7}\, a_{3,8}\,]$$
$$[0\,,\ -a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{5,6}\, a_{7,8} - a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{4,6}\, a_{7,8} - a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{5,6}\, a_{7,8}$$
$$+ a_{5,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{4,6}\, a_{7,8} - a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{4,6}\, a_{5,8} - a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{3,8}\, a_{5,6}$$
$$- a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{3,8}\, a_{4,6} - a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{3,5}\, a_{4,6}\, a_{7,8}\,,\ -a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{2,6}\, a_{7,8}$$
$$- a_{5,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{7,8} + a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{5,8} + a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{7,8}\,,$$
$$a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{2,6}\, a_{7,8} - a_{5,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{2,6}\, a_{7,8} + a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{2,6}\, a_{5,8}$$
$$+ a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{2,6}\, a_{3,8} + a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{3,5}\, a_{2,6}\, a_{7,8}\,,$$
$$-a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{7,8} - a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{2,6}\, a_{7,8} - a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{3,8}\,,\ 0\,,$$
$$-a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{5,8} - a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{2,6}\, a_{5,8} - a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{2,6}\, a_{3,8}$$
$$- a_{5,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{3,8} + a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{3,8}\,,$$
$$a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{7,8} + a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{3,5}\, a_{2,6}\, a_{7,8} + a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{3,8}\,]$$
$$[0\,,\ -a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{5,6}\, a_{7,7} - a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{4,6}\, a_{7,7} - a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{5,6}\, a_{7,7}$$
$$+ a_{5,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{4,6}\, a_{7,7} - a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{3,5}\, a_{4,6}\, a_{8,7} - a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{3,5}\, a_{4,6}\, a_{7,7}\,,$$
$$-a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{2,6}\, a_{7,7} - a_{5,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{7,7} + a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{8,7}$$
$$- a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,7}\, a_{5,6} - a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{2,7}\, a_{4,6} + a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{4,5}\, a_{2,6}\, a_{7,7}\,,$$
$$a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{5,5}\, a_{2,6}\, a_{7,7} - a_{5,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{2,6}\, a_{7,7} + a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{3,5}\, a_{2,6}\, a_{8,7}$$
$$- a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{2,7}\, a_{5,6} + a_{8,1}\, a_{1,2}\, a_{6,3}\, a_{5,4}\, a_{3,5}\, a_{2,6}\, a_{7,7}\,,$$
$$-a_{3,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{4,5}\, a_{2,6}\, a_{7,7} - a_{4,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{2,6}\, a_{7,7} - a_{7,1}\, a_{1,2}\, a_{6,3}\, a_{8,4}\, a_{3,5}\, a_{2,7}\, a_{4,6}\,,\ 0\,,$$

$$-a_{3,1}\,a_{1,2}\,a_{6,3}\,a_{5,4}\,a_{4,5}\,a_{2,6}\,a_{8,7}+a_{3,1}\,a_{1,2}\,a_{6,3}\,a_{8,4}\,a_{4,5}\,a_{2,7}\,a_{5,6}+a_{3,1}\,a_{1,2}\,a_{6,3}\,a_{8,4}\,a_{5,5}\,a_{2,7}\,a_{4,6}$$
$$-a_{4,1}\,a_{1,2}\,a_{6,3}\,a_{5,4}\,a_{3,5}\,a_{2,6}\,a_{8,7}+a_{4,1}\,a_{1,2}\,a_{6,3}\,a_{8,4}\,a_{3,5}\,a_{2,7}\,a_{5,6}-a_{5,1}\,a_{1,2}\,a_{6,3}\,a_{8,4}\,a_{3,5}\,a_{2,7}\,a_{4,6}$$
$$+a_{8,1}\,a_{1,2}\,a_{6,3}\,a_{5,4}\,a_{3,5}\,a_{2,7}\,a_{4,6}\,,$$
$$a_{3,1}\,a_{1,2}\,a_{6,3}\,a_{5,4}\,a_{4,5}\,a_{2,6}\,a_{7,7}+a_{4,1}\,a_{1,2}\,a_{6,3}\,a_{5,4}\,a_{3,5}\,a_{2,6}\,a_{7,7}+a_{7,1}\,a_{1,2}\,a_{6,3}\,a_{5,4}\,a_{3,5}\,a_{2,7}\,a_{4,6}\,]$$

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$$A := \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & -1 & 0 & 0 & 1 & -1 & 1 & 0 \\ -1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & -1 & -1 \\ 0 & 0 & -1 & -1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

poly_coef_F0_to_Fn = [-1, 0, 4, -3, -2, 8, -13, 0, 5]

positive_feedback = [0, 2, 7, 10, 21, 38, 33, 32, 15]

negative_feedback = [-1, -2, -3, -13, -23, -30, -46, -32, -10]

absolute_feedback = [1, 4, 10, 23, 44, 68, 79, 64, 25]

wFn = [-1., 0., 0.40, -0.13, -0.045, 0.12, -0.16, 0., 0.20]

"Criterion ii"

$wD_7 = -0.36\ 10^{-7}$

ratio_to_model_C = -0.11

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 1 & 1 & 1 & 2 & -4 & -1 & -1 & 1 \\ -2 & -2 & 3 & 1 & -2 & -3 & -3 & 3 \\ 3 & 3 & 3 & 1 & -2 & -3 & 2 & -2 \\ -2 & -2 & -2 & 1 & 3 & 2 & -3 & -2 \\ 1 & 1 & 1 & 2 & 1 & -1 & -1 & 1 \\ 2 & -3 & 2 & 4 & 2 & 3 & -2 & 2 \\ -1 & -1 & 4 & 3 & -1 & 1 & -4 & 4 \\ 2 & 2 & -3 & -6 & -3 & -2 & 3 & -3 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 3 & 3 & 3 & 4 & 18 & 3 & 3 & 3 \\ 8 & 8 & 5 & 9 & 12 & 17 & 5 & 5 \\ 7 & 7 & 15 & 11 & 20 & 7 & 10 & 10 \\ 8 & 8 & 12 & 13 & 19 & 8 & 13 & 12 \\ 3 & 3 & 3 & 4 & 7 & 3 & 3 & 3 \\ 16 & 9 & 6 & 14 & 16 & 9 & 6 & 6 \\ 5 & 5 & 4 & 7 & 9 & 5 & 4 & 4 \\ 6 & 6 & 5 & 14 & 15 & 6 & 5 & 5 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 0.33 & 0.33 & 0.33 & 0.50 & 0.22 & 0.33 & 0.33 & 0.33 \\ 0.25 & 0.25 & 0.60 & 0.11 & 0.17 & 0.18 & 0.60 & 0.60 \\ 0.43 & 0.43 & 0.20 & 0.091 & 0.10 & 0.43 & 0.20 & 0.20 \\ 0.25 & 0.25 & 0.17 & 0.077 & 0.16 & 0.25 & 0.23 & 0.17 \\ 0.33 & 0.33 & 0.33 & 0.50 & 0.14 & 0.33 & 0.33 & 0.33 \\ 0.12 & 0.33 & 0.33 & 0.29 & 0.12 & 0.33 & 0.33 & 0.33 \\ 0.20 & 0.20 & 1. & 0.43 & 0.11 & 0.20 & 1. & 1. \\ 0.33 & 0.33 & 0.60 & 0.43 & 0.20 & 0.33 & 0.60 & 0.60 \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & -5 & -5 & 0 & 0 & 5 & -5 \\ -2 & -2 & -2 & -4 & 3 & 2 & 2 & -2 \\ -1 & -1 & -1 & 3 & 4 & 1 & 1 & -1 \\ -1 & -1 & -1 & -2 & 4 & 1 & 1 & -1 \\ 1 & 1 & -4 & -3 & 1 & 4 & 4 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 \\ -1 & -1 & -1 & -2 & -1 & 1 & 1 & 4 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 17 & 5 & 9 & 12 & 8 & 5 & 5 \\ 6 & 6 & 6 & 8 & 19 & 6 & 6 & 6 \\ 3 & 3 & 3 & 21 & 18 & 3 & 3 & 3 \\ 3 & 3 & 3 & 4 & 18 & 3 & 3 & 3 \\ 5 & 5 & 4 & 7 & 9 & 20 & 4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 25 & 0 \\ 3 & 3 & 3 & 4 & 7 & 3 & 3 & 22 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 0. & 0.29 & 1. & 0.56 & 0. & 0. & 1. & 1. \\ 0.33 & 0.33 & 0.33 & 0.50 & 0.16 & 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 & 0.14 & 0.22 & 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 & 0.50 & 0.22 & 0.33 & 0.33 & 0.33 \\ 0.20 & 0.20 & 1. & 0.43 & 0.11 & 0.20 & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 0.20 & 1. \\ 0.33 & 0.33 & 0.33 & 0.50 & 0.14 & 0.33 & 0.33 & 0.18 \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -5 & -5 & 0 & 0 & 5 & -5 \\ -2 & -2 & -7 & -4 & 3 & 2 & 2 & -2 \\ -1 & -1 & -1 & -2 & 4 & 1 & 1 & -1 \\ -1 & -1 & -1 & -2 & -1 & 1 & 1 & -1 \\ 1 & 1 & -4 & -3 & 1 & -1 & 4 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & -2 & -1 & 1 & 1 & -1 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 8 & 5 & 9 & 12 & 8 & 5 & 5 \\ 6 & 6 & 19 & 8 & 19 & 6 & 6 & 6 \\ 3 & 3 & 3 & 4 & 18 & 3 & 3 & 3 \\ 3 & 3 & 3 & 4 & 7 & 3 & 3 & 3 \\ 5 & 5 & 4 & 7 & 9 & 5 & 4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 3 & 3 & 4 & 7 & 3 & 3 & 3 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 0.20 & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 0. & 0. & 1. & 0.56 & 0. & 0. & 1. & 1. \\ 0.33 & 0.33 & 0.37 & 0.50 & 0.16 & 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 & 0.50 & 0.22 & 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 & 0.50 & 0.14 & 0.33 & 0.33 & 0.33 \\ 0.20 & 0.20 & 1. & 0.43 & 0.11 & 0.20 & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 0.33 & 0.33 & 0.33 & 0.50 & 0.14 & 0.33 & 0.33 & 0.33 \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & a_{1,5} & a_{1,6} & 0 & a_{1,8} \\ 0 & -a_{2,2} & 0 & 0 & a_{2,5} & -a_{2,6} & a_{2,7} & 0 \\ -a_{3,1} & 0 & a_{3,3} & 0 & -a_{3,5} & 0 & a_{3,7} & 0 \\ a_{4,1} & 0 & 0 & 0 & 0 & 0 & -a_{4,7} & -a_{4,8} \\ -a_{5,1} & 0 & 0 & 0 & a_{5,5} & 0 & 0 & 0 \\ 0 & -a_{6,2} & 0 & 0 & -a_{6,5} & 0 & a_{6,7} & 0 \\ 0 & 0 & 0 & -a_{7,4} & -a_{7,5} & 0 & -a_{7,7} & -a_{7,8} \\ 0 & 0 & -a_{8,3} & -a_{8,4} & a_{8,5} & 0 & 0 & 0 \end{bmatrix}$$

"system feedback"

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$$\begin{aligned} &-a_{7,8}\,a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{1,5}\,a_{2,6}\,a_{4,7} - a_{7,8}\,a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{2,5}\,a_{1,6}\,a_{4,7} - a_{7,8}\,a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{6,5}\,a_{1,6}\,a_{4,7} \\ &\quad - a_{7,4}\,a_{3,1}\,a_{2,2}\,a_{8,3}\,a_{5,5}\,a_{1,6}\,a_{4,8}\,a_{6,7} + a_{7,4}\,a_{3,1}\,a_{6,2}\,a_{8,3}\,a_{5,5}\,a_{1,6}\,a_{2,7}\,a_{4,8} - a_{7,4}\,a_{3,1}\,a_{6,2}\,a_{8,3}\,a_{5,5}\,a_{2,6}\,a_{1,8}\,a_{4,7} \\ &\quad + a_{7,4}\,a_{4,1}\,a_{6,2}\,a_{8,3}\,a_{5,5}\,a_{2,6}\,a_{1,8}\,a_{3,7} + a_{7,4}\,a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{8,5}\,a_{1,6}\,a_{4,8}\,a_{6,7} - a_{7,4}\,a_{5,1}\,a_{2,2}\,a_{8,3}\,a_{3,5}\,a_{1,6}\,a_{4,8}\,a_{6,7} \\ &\quad + a_{7,4}\,a_{5,1}\,a_{2,2}\,a_{8,3}\,a_{6,5}\,a_{1,6}\,a_{3,7}\,a_{4,8} - a_{7,4}\,a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{8,5}\,a_{1,6}\,a_{2,7}\,a_{4,8} + a_{7,4}\,a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{8,5}\,a_{2,6}\,a_{1,8}\,a_{4,7} \\ &\quad + a_{7,4}\,a_{5,1}\,a_{6,2}\,a_{8,3}\,a_{1,5}\,a_{2,6}\,a_{3,7}\,a_{4,8} + a_{7,4}\,a_{5,1}\,a_{6,2}\,a_{8,3}\,a_{2,5}\,a_{1,6}\,a_{3,7}\,a_{4,8} + a_{7,4}\,a_{5,1}\,a_{6,2}\,a_{8,3}\,a_{3,5}\,a_{1,6}\,a_{2,7}\,a_{4,8} \\ &\quad - a_{7,4}\,a_{5,1}\,a_{6,2}\,a_{8,3}\,a_{3,5}\,a_{2,6}\,a_{1,8}\,a_{4,7} + a_{7,5}\,a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{1,6}\,a_{4,8}\,a_{6,7} - a_{7,5}\,a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{1,6}\,a_{2,7}\,a_{4,8} \\ &\quad + a_{7,5}\,a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{2,6}\,a_{1,8}\,a_{4,7} + a_{7,7}\,a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{1,8}\,a_{2,6} + a_{7,7}\,a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{6,5}\,a_{1,6}\,a_{4,8} \\ &\quad + a_{7,7}\,a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{1,5}\,a_{2,6}\,a_{4,8} + a_{7,7}\,a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{2,5}\,a_{1,6}\,a_{4,8} + a_{7,8}\,a_{4,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{1,6}\,a_{6,7} \\ &\quad - a_{7,8}\,a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{1,6}\,a_{2,7} \end{aligned}$$

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$$\begin{aligned} &-a_{8,4}\,a_{4,8}\,a_{7,7}\,a_{6,2}\,a_{2,6}\,a_{5,5}\,a_{3,3} + a_{8,4}\,a_{4,7}\,a_{7,8}\,a_{6,2}\,a_{2,6}\,a_{5,5}\,a_{3,3} - a_{8,3}\,a_{7,4}\,a_{3,7}\,a_{4,8}\,a_{6,2}\,a_{2,6}\,a_{5,5} \\ &\quad + a_{4,1}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{1,6}\,a_{6,7}\,a_{7,8} - a_{3,1}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{1,6}\,a_{4,8}\,a_{6,7} - a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{6,5}\,a_{1,6}\,a_{4,7}\,a_{7,8} \\ &\quad + a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{6,5}\,a_{1,6}\,a_{4,8}\,a_{7,7} + a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{7,5}\,a_{1,6}\,a_{4,8}\,a_{6,7} - a_{5,1}\,a_{8,3}\,a_{7,4}\,a_{3,5}\,a_{1,6}\,a_{4,8}\,a_{6,7} \\ &\quad + a_{5,1}\,a_{8,3}\,a_{7,4}\,a_{6,5}\,a_{1,6}\,a_{3,7}\,a_{4,8} + a_{5,1}\,a_{3,3}\,a_{7,4}\,a_{8,5}\,a_{1,6}\,a_{4,8}\,a_{6,7} - a_{5,1}\,a_{6,2}\,a_{8,4}\,a_{7,5}\,a_{2,6}\,a_{1,8}\,a_{4,7} \\ &\quad + a_{5,1}\,a_{6,2}\,a_{8,4}\,a_{2,5}\,a_{1,6}\,a_{4,7}\,a_{7,8} - a_{5,1}\,a_{6,2}\,a_{8,4}\,a_{2,5}\,a_{1,6}\,a_{4,8}\,a_{7,7} + a_{5,1}\,a_{6,2}\,a_{8,4}\,a_{7,5}\,a_{1,6}\,a_{2,7}\,a_{4,8} \\ &\quad - a_{5,1}\,a_{6,2}\,a_{7,4}\,a_{8,5}\,a_{2,6}\,a_{1,8}\,a_{4,7} + a_{5,1}\,a_{6,2}\,a_{8,4}\,a_{1,5}\,a_{2,6}\,a_{4,7}\,a_{7,8} - a_{5,1}\,a_{6,2}\,a_{8,4}\,a_{1,5}\,a_{2,6}\,a_{4,8}\,a_{7,7} \\ &\quad - a_{5,1}\,a_{2,2}\,a_{8,4}\,a_{7,5}\,a_{1,6}\,a_{4,8}\,a_{6,7} + a_{5,1}\,a_{6,2}\,a_{7,4}\,a_{8,5}\,a_{1,6}\,a_{2,7}\,a_{4,8} - a_{5,1}\,a_{2,2}\,a_{7,4}\,a_{8,5}\,a_{1,6}\,a_{4,8}\,a_{6,7} \\ &\quad + a_{5,1}\,a_{2,2}\,a_{8,4}\,a_{6,5}\,a_{1,6}\,a_{4,7}\,a_{7,8} - a_{5,1}\,a_{2,2}\,a_{8,4}\,a_{6,5}\,a_{1,6}\,a_{4,8}\,a_{7,7} + a_{4,1}\,a_{6,2}\,a_{8,4}\,a_{5,5}\,a_{1,6}\,a_{2,7}\,a_{7,8} \\ &\quad - a_{4,1}\,a_{6,2}\,a_{8,4}\,a_{5,5}\,a_{2,6}\,a_{1,8}\,a_{7,7} - a_{4,1}\,a_{2,2}\,a_{8,4}\,a_{5,5}\,a_{1,6}\,a_{6,7}\,a_{7,8} + a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{8,5}\,a_{1,6}\,a_{2,7}\,a_{7,8} \\ &\quad - a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{8,5}\,a_{2,6}\,a_{1,8}\,a_{7,7} - a_{5,1}\,a_{6,2}\,a_{8,3}\,a_{3,5}\,a_{1,6}\,a_{2,7}\,a_{7,8} - a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{8,5}\,a_{1,6}\,a_{6,7}\,a_{7,8} \\ &\quad + a_{5,1}\,a_{2,2}\,a_{8,3}\,a_{3,5}\,a_{1,6}\,a_{6,7}\,a_{7,8} - a_{5,1}\,a_{2,2}\,a_{8,3}\,a_{6,5}\,a_{1,6}\,a_{3,7}\,a_{7,8} + a_{3,1}\,a_{6,2}\,a_{8,3}\,a_{5,5}\,a_{2,6}\,a_{1,8}\,a_{7,7} \\ &\quad + a_{3,1}\,a_{2,2}\,a_{8,3}\,a_{5,5}\,a_{1,6}\,a_{6,7}\,a_{7,8} - a_{3,1}\,a_{6,2}\,a_{8,3}\,a_{5,5}\,a_{1,6}\,a_{2,7}\,a_{7,8} + a_{5,1}\,a_{6,2}\,a_{8,3}\,a_{3,5}\,a_{2,6}\,a_{1,8}\,a_{7,7} \\ &\quad + a_{5,1}\,a_{6,2}\,a_{8,3}\,a_{7,5}\,a_{2,6}\,a_{1,8}\,a_{3,7} - a_{5,1}\,a_{6,2}\,a_{8,3}\,a_{2,5}\,a_{1,6}\,a_{3,7}\,a_{7,8} - a_{5,1}\,a_{6,2}\,a_{8,3}\,a_{3,5}\,a_{1,6}\,a_{2,7}\,a_{7,8} \\ &\quad + a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{1,6}\,a_{2,7}\,a_{7,8} - a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{2,6}\,a_{1,8}\,a_{7,7} - a_{4,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{2,6}\,a_{1,8}\,a_{3,7} \\ &\quad - a_{4,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{1,6}\,a_{6,7}\,a_{7,8} + a_{3,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{1,6}\,a_{4,8}\,a_{6,7} - a_{3,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{1,6}\,a_{2,7}\,a_{4,8} \\ &\quad + a_{3,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{2,6}\,a_{1,8}\,a_{4,7} + a_{5,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{3,5}\,a_{1,8}\,a_{4,7} - a_{5,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{1,5}\,a_{3,7}\,a_{4,8} \\ &\quad - a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{7,4}\,a_{8,5}\,a_{1,8}\,a_{4,7} + a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{1,5}\,a_{4,7}\,a_{7,8} - a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{1,5}\,a_{4,8}\,a_{7,7} \\ &\quad - a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{7,5}\,a_{1,8}\,a_{4,7} - a_{4,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{1,8}\,a_{3,7} - a_{4,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{1,8}\,a_{7,7} \\ &\quad + a_{3,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{1,8}\,a_{4,7} + a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{1,8}\,a_{2,6} + a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{6,5}\,a_{1,6}\,a_{4,8} \\ &\quad + a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{1,5}\,a_{2,6}\,a_{4,8} + a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{2,5}\,a_{1,6}\,a_{4,8} + a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{7,4}\,a_{6,5}\,a_{1,6}\,a_{4,7} \\ &\quad + a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{7,4}\,a_{1,5}\,a_{2,6}\,a_{4,7} + a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{7,4}\,a_{2,5}\,a_{1,6}\,a_{4,7} - a_{4,1}\,a_{2,2}\,a_{3,3}\,a_{7,4}\,a_{5,5}\,a_{1,6}\,a_{6,7} \\ &\quad + a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{7,4}\,a_{5,5}\,a_{1,6}\,a_{2,7} \end{aligned}$$

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$$\begin{aligned} &-a_{7,4}\,a_{4,1}\,a_{1,6}\,a_{6,2}\,a_{2,7}\,a_{5,5} - a_{7,4}\,a_{4,1}\,a_{1,6}\,a_{6,2}\,a_{2,7}\,a_{3,3} + a_{7,5}\,a_{5,1}\,a_{1,6}\,a_{6,2}\,a_{2,7}\,a_{3,3} - a_{7,4}\,a_{4,1}\,a_{1,6}\,a_{6,7}\,a_{5,5}\,a_{3,3} \\ &\quad + a_{7,4}\,a_{4,1}\,a_{1,6}\,a_{6,7}\,a_{5,5}\,a_{2,2} + a_{7,4}\,a_{4,1}\,a_{1,6}\,a_{6,7}\,a_{3,3}\,a_{2,2} - a_{7,5}\,a_{5,1}\,a_{1,6}\,a_{6,7}\,a_{3,3}\,a_{2,2} - a_{7,4}\,a_{4,7}\,a_{6,2}\,a_{2,5}\,a_{5,1}\,a_{1,6} \\ &\quad + a_{7,4}\,a_{4,7}\,a_{6,5}\,a_{5,1}\,a_{1,6}\,a_{3,3} - a_{7,4}\,a_{4,7}\,a_{6,5}\,a_{5,1}\,a_{1,6}\,a_{2,2} - a_{7,4}\,a_{4,7}\,a_{6,2}\,a_{2,6}\,a_{1,5}\,a_{5,1} - a_{7,4}\,a_{4,7}\,a_{6,2}\,a_{2,6}\,a_{5,5}\,a_{3,3} \end{aligned}$$

$$
\begin{aligned}
&- a_{7,4}\,a_{4,7}\,a_{1,5}\,a_{5,1}\,a_{3,3}\,a_{2,2} - a_{7,7}\,a_{6,2}\,a_{2,6}\,a_{1,5}\,a_{5,1}\,a_{3,3} - a_{7,7}\,a_{6,2}\,a_{2,5}\,a_{5,1}\,a_{1,6}\,a_{3,3} - a_{7,7}\,a_{6,5}\,a_{5,1}\,a_{1,6}\,a_{3,3}\,a_{2,2}\\
&- a_{8,4}\,a_{4,1}\,a_{1,6}\,a_{6,2}\,a_{2,7}\,a_{7,8} - a_{8,5}\,a_{5,1}\,a_{1,6}\,a_{6,2}\,a_{2,7}\,a_{7,8} - a_{8,5}\,a_{5,1}\,a_{7,4}\,a_{1,6}\,a_{4,8}\,a_{6,7} + a_{8,3}\,a_{3,5}\,a_{5,1}\,a_{1,6}\,a_{6,7}\,a_{7,8}\\
&+ a_{8,3}\,a_{3,1}\,a_{1,6}\,a_{6,2}\,a_{2,7}\,a_{7,8} + a_{8,3}\,a_{3,1}\,a_{7,4}\,a_{1,6}\,a_{4,8}\,a_{6,7} + a_{6,2}\,a_{8,3}\,a_{5,5}\,a_{2,6}\,a_{3,7}\,a_{7,8} + a_{8,5}\,a_{5,1}\,a_{1,8}\,a_{7,7}\,a_{3,3}\,a_{2,2}\\
&- a_{8,5}\,a_{5,1}\,a_{1,8}\,a_{6,2}\,a_{2,6}\,a_{3,3} - a_{8,3}\,a_{3,1}\,a_{1,8}\,a_{7,7}\,a_{5,5}\,a_{2,2} + a_{8,3}\,a_{3,1}\,a_{1,8}\,a_{6,2}\,a_{2,6}\,a_{5,5} - a_{8,3}\,a_{3,1}\,a_{1,8}\,a_{7,7}\,a_{6,2}\,a_{2,6}\\
&- a_{8,3}\,a_{3,7}\,a_{7,8}\,a_{6,5}\,a_{5,1}\,a_{1,6} + a_{8,3}\,a_{3,7}\,a_{7,8}\,a_{1,5}\,a_{5,1}\,a_{2,2} + a_{8,4}\,a_{4,1}\,a_{1,8}\,a_{7,7}\,a_{6,2}\,a_{2,6} - a_{8,4}\,a_{4,1}\,a_{1,8}\,a_{7,7}\,a_{5,5}\,a_{3,3}\\
&+ a_{8,4}\,a_{4,1}\,a_{1,8}\,a_{7,7}\,a_{5,5}\,a_{2,2} + a_{8,4}\,a_{4,1}\,a_{1,8}\,a_{7,7}\,a_{3,3}\,a_{2,2} - a_{8,4}\,a_{4,1}\,a_{1,8}\,a_{6,2}\,a_{2,6}\,a_{5,5} - a_{8,4}\,a_{4,1}\,a_{1,8}\,a_{6,2}\,a_{2,6}\,a_{3,3}\\
&- a_{8,4}\,a_{4,1}\,a_{1,8}\,a_{5,5}\,a_{3,3}\,a_{2,2} + a_{8,4}\,a_{4,7}\,a_{7,8}\,a_{6,5}\,a_{5,1}\,a_{1,6} - a_{8,4}\,a_{4,7}\,a_{7,8}\,a_{6,2}\,a_{2,6}\,a_{5,5} - a_{8,4}\,a_{4,7}\,a_{7,8}\,a_{6,2}\,a_{2,6}\,a_{3,3}\\
&+ a_{8,4}\,a_{4,7}\,a_{7,8}\,a_{1,5}\,a_{5,1}\,a_{3,3} - a_{8,4}\,a_{4,7}\,a_{7,8}\,a_{1,5}\,a_{5,1}\,a_{2,2} - a_{8,4}\,a_{4,7}\,a_{7,8}\,a_{5,5}\,a_{3,3}\,a_{2,2} - a_{8,5}\,a_{5,1}\,a_{1,8}\,a_{7,4}\,a_{4,7}\,a_{3,3}\\
&+ a_{8,5}\,a_{5,1}\,a_{1,8}\,a_{7,4}\,a_{4,7}\,a_{2,2} + a_{8,5}\,a_{5,1}\,a_{1,8}\,a_{7,7}\,a_{6,2}\,a_{2,6} + a_{8,3}\,a_{3,5}\,a_{5,1}\,a_{1,8}\,a_{7,4}\,a_{4,7} - a_{8,3}\,a_{3,5}\,a_{5,1}\,a_{1,8}\,a_{7,7}\,a_{2,2}\\
&+ a_{8,3}\,a_{3,5}\,a_{5,1}\,a_{1,8}\,a_{6,2}\,a_{2,6} + a_{8,3}\,a_{7,4}\,a_{3,7}\,a_{4,8}\,a_{6,2}\,a_{2,6} - a_{8,3}\,a_{7,4}\,a_{3,7}\,a_{4,8}\,a_{1,5}\,a_{5,1} - a_{8,5}\,a_{5,1}\,a_{1,6}\,a_{6,7}\,a_{7,8}\,a_{3,3}\\
&+ a_{8,5}\,a_{5,1}\,a_{1,6}\,a_{6,7}\,a_{7,8}\,a_{2,2} + a_{8,4}\,a_{4,1}\,a_{1,6}\,a_{6,7}\,a_{7,8}\,a_{2,2} - a_{8,4}\,a_{4,7}\,a_{7,5}\,a_{5,1}\,a_{1,8}\,a_{3,3} + a_{8,4}\,a_{4,7}\,a_{7,5}\,a_{5,1}\,a_{1,8}\,a_{2,2}\\
&- a_{8,3}\,a_{3,7}\,a_{7,4}\,a_{4,1}\,a_{1,8}\,a_{5,5} + a_{8,3}\,a_{3,7}\,a_{7,4}\,a_{4,1}\,a_{1,8}\,a_{2,2} - a_{8,3}\,a_{3,7}\,a_{7,5}\,a_{5,1}\,a_{1,8}\,a_{2,2} - a_{8,4}\,a_{4,1}\,a_{1,6}\,a_{6,7}\,a_{7,8}\,a_{5,5}\\
&+ a_{8,3}\,a_{3,1}\,a_{1,6}\,a_{6,7}\,a_{7,8}\,a_{5,5} - a_{8,3}\,a_{3,1}\,a_{1,6}\,a_{6,7}\,a_{7,8}\,a_{2,2} - a_{8,4}\,a_{4,8}\,a_{6,2}\,a_{2,5}\,a_{5,1}\,a_{1,6} + a_{8,4}\,a_{4,8}\,a_{6,5}\,a_{5,1}\,a_{1,6}\,a_{3,3}\\
&- a_{8,4}\,a_{4,8}\,a_{6,5}\,a_{5,1}\,a_{1,6}\,a_{2,2} - a_{8,4}\,a_{4,8}\,a_{6,2}\,a_{2,6}\,a_{1,5}\,a_{5,1} - a_{8,4}\,a_{4,8}\,a_{6,2}\,a_{2,6}\,a_{5,5}\,a_{3,3} - a_{8,4}\,a_{4,8}\,a_{1,5}\,a_{5,1}\,a_{3,3}\,a_{2,2}\\
&- a_{8,4}\,a_{4,8}\,a_{7,5}\,a_{5,1}\,a_{1,6}\,a_{6,7} - a_{8,4}\,a_{4,8}\,a_{7,7}\,a_{6,5}\,a_{5,1}\,a_{1,6} + a_{8,4}\,a_{4,8}\,a_{7,7}\,a_{6,2}\,a_{2,6}\,a_{5,5} + a_{8,4}\,a_{4,8}\,a_{7,7}\,a_{6,2}\,a_{2,6}\,a_{3,3}\\
&- a_{8,4}\,a_{4,8}\,a_{7,7}\,a_{1,5}\,a_{5,1}\,a_{3,3} + a_{8,4}\,a_{4,8}\,a_{7,7}\,a_{1,5}\,a_{5,1}\,a_{2,2} + a_{8,4}\,a_{4,8}\,a_{7,7}\,a_{5,5}\,a_{3,3}\,a_{2,2} - a_{8,4}\,a_{4,1}\,a_{1,6}\,a_{6,7}\,a_{7,8}\,a_{3,3}\\
&+ a_{8,3}\,a_{7,4}\,a_{3,7}\,a_{4,8}\,a_{5,5}\,a_{2,2} + a_{8,3}\,a_{3,1}\,a_{1,8}\,a_{7,4}\,a_{4,7}\,a_{5,5} - a_{8,3}\,a_{3,1}\,a_{1,8}\,a_{7,4}\,a_{4,7}\,a_{2,2}
\end{aligned}
$$

$$F_5$$

$$
\begin{aligned}
&-a_{6,2}\,a_{2,6}\,a_{1,5}\,a_{5,1}\,a_{3,3} - a_{6,2}\,a_{2,5}\,a_{5,1}\,a_{1,6}\,a_{3,3} - a_{6,5}\,a_{5,1}\,a_{1,6}\,a_{3,3}\,a_{2,2} + a_{7,4}\,a_{4,1}\,a_{1,6}\,a_{6,2}\,a_{2,7} - a_{7,5}\,a_{5,1}\,a_{1,6}\,a_{6,2}\,a_{2,7}\\
&+ a_{7,4}\,a_{4,7}\,a_{6,2}\,a_{2,6}\,a_{5,5} + a_{7,4}\,a_{4,7}\,a_{6,2}\,a_{2,6}\,a_{3,3} - a_{7,4}\,a_{4,7}\,a_{1,5}\,a_{5,1}\,a_{3,3} + a_{7,4}\,a_{4,7}\,a_{1,5}\,a_{5,1}\,a_{2,2} + a_{7,4}\,a_{4,7}\,a_{5,5}\,a_{3,3}\,a_{2,2}\\
&+ a_{7,4}\,a_{4,1}\,a_{1,6}\,a_{6,7}\,a_{3,3} - a_{7,4}\,a_{4,1}\,a_{1,6}\,a_{6,7}\,a_{2,2} - a_{7,5}\,a_{5,1}\,a_{1,6}\,a_{6,7}\,a_{3,3} + a_{7,5}\,a_{5,1}\,a_{1,6}\,a_{6,7}\,a_{2,2} + a_{7,4}\,a_{4,1}\,a_{1,6}\,a_{6,7}\,a_{5,5}\\
&- a_{7,4}\,a_{4,7}\,a_{6,5}\,a_{5,1}\,a_{1,6} + a_{7,7}\,a_{6,2}\,a_{2,5}\,a_{5,1}\,a_{1,6} - a_{7,7}\,a_{6,5}\,a_{5,1}\,a_{1,6}\,a_{3,3} + a_{7,7}\,a_{6,5}\,a_{5,1}\,a_{1,6}\,a_{2,2} + a_{7,7}\,a_{6,2}\,a_{2,6}\,a_{1,5}\,a_{5,1}\\
&+ a_{7,7}\,a_{6,2}\,a_{2,6}\,a_{5,5}\,a_{3,3} + a_{7,7}\,a_{1,5}\,a_{5,1}\,a_{3,3}\,a_{2,2} - a_{8,3}\,a_{3,7}\,a_{7,5}\,a_{5,1}\,a_{1,8} + a_{8,4}\,a_{4,1}\,a_{1,6}\,a_{6,7}\,a_{7,8} + a_{8,4}\,a_{4,7}\,a_{7,5}\,a_{5,1}\,a_{1,8}\\
&+ a_{8,5}\,a_{5,1}\,a_{1,6}\,a_{6,7}\,a_{7,8} - a_{8,3}\,a_{3,1}\,a_{1,6}\,a_{6,7}\,a_{7,8} + a_{8,3}\,a_{3,7}\,a_{7,4}\,a_{4,1}\,a_{1,8} + a_{8,4}\,a_{4,8}\,a_{7,7}\,a_{5,5}\,a_{3,3} - a_{8,4}\,a_{4,8}\,a_{7,7}\,a_{5,5}\,a_{2,2}\\
&- a_{8,4}\,a_{4,8}\,a_{7,7}\,a_{3,3}\,a_{2,2} - a_{8,4}\,a_{4,8}\,a_{6,5}\,a_{5,1}\,a_{1,6} + a_{8,4}\,a_{4,8}\,a_{6,2}\,a_{2,6}\,a_{5,5} + a_{8,4}\,a_{4,8}\,a_{6,2}\,a_{2,6}\,a_{3,3} - a_{8,4}\,a_{4,8}\,a_{1,5}\,a_{5,1}\,a_{3,3}\\
&+ a_{8,4}\,a_{4,8}\,a_{1,5}\,a_{5,1}\,a_{2,2} + a_{8,4}\,a_{4,8}\,a_{5,5}\,a_{3,3}\,a_{2,2} + a_{8,5}\,a_{5,1}\,a_{1,8}\,a_{7,7}\,a_{3,3} - a_{8,5}\,a_{5,1}\,a_{1,8}\,a_{7,7}\,a_{2,2} + a_{8,5}\,a_{5,1}\,a_{1,8}\,a_{6,2}\,a_{2,6}\\
&+ a_{8,5}\,a_{5,1}\,a_{1,8}\,a_{3,3}\,a_{2,2} - a_{8,4}\,a_{4,8}\,a_{7,7}\,a_{6,2}\,a_{2,6} + a_{8,5}\,a_{5,1}\,a_{1,8}\,a_{7,4}\,a_{4,7} + a_{8,4}\,a_{4,8}\,a_{7,7}\,a_{1,5}\,a_{5,1} - a_{8,3}\,a_{3,1}\,a_{1,8}\,a_{7,4}\,a_{4,7}\\
&- a_{8,3}\,a_{3,1}\,a_{1,8}\,a_{7,7}\,a_{5,5} + a_{8,3}\,a_{3,1}\,a_{1,8}\,a_{7,7}\,a_{2,2} - a_{8,3}\,a_{3,1}\,a_{1,8}\,a_{6,2}\,a_{2,6} - a_{8,3}\,a_{3,1}\,a_{1,8}\,a_{5,5}\,a_{2,2} - a_{8,3}\,a_{3,7}\,a_{7,8}\,a_{6,2}\,a_{2,6}\\
&+ a_{8,3}\,a_{3,7}\,a_{7,8}\,a_{1,5}\,a_{5,1} - a_{8,3}\,a_{3,7}\,a_{7,8}\,a_{5,5}\,a_{2,2} + a_{8,4}\,a_{4,1}\,a_{1,8}\,a_{7,7}\,a_{5,5} + a_{8,4}\,a_{4,1}\,a_{1,8}\,a_{7,7}\,a_{3,3} - a_{8,4}\,a_{4,1}\,a_{1,8}\,a_{7,7}\,a_{2,2}\\
&+ a_{8,4}\,a_{4,1}\,a_{1,8}\,a_{6,2}\,a_{2,6} - a_{8,4}\,a_{4,1}\,a_{1,8}\,a_{5,5}\,a_{3,3} + a_{8,4}\,a_{4,1}\,a_{1,8}\,a_{5,5}\,a_{2,2} + a_{8,4}\,a_{4,1}\,a_{1,8}\,a_{3,3}\,a_{2,2} + a_{8,4}\,a_{4,7}\,a_{7,8}\,a_{6,2}\,a_{2,6}\\
&- a_{8,4}\,a_{4,7}\,a_{7,8}\,a_{1,5}\,a_{5,1} - a_{8,4}\,a_{4,7}\,a_{7,8}\,a_{5,5}\,a_{3,3} + a_{8,4}\,a_{4,7}\,a_{7,8}\,a_{5,5}\,a_{2,2} + a_{8,4}\,a_{4,7}\,a_{7,8}\,a_{3,3}\,a_{2,2} - a_{8,3}\,a_{3,5}\,a_{5,1}\,a_{1,8}\,a_{7,7}\\
&- a_{8,3}\,a_{3,5}\,a_{5,1}\,a_{1,8}\,a_{2,2} + a_{8,3}\,a_{7,4}\,a_{3,7}\,a_{4,8}\,a_{5,5} - a_{8,3}\,a_{7,4}\,a_{3,7}\,a_{4,8}\,a_{2,2}
\end{aligned}
$$

$$F_4$$

$$
\begin{aligned}
&a_{7,5}\,a_{5,1}\,a_{1,6}\,a_{6,7} - a_{8,4}\,a_{4,8}\,a_{7,7}\,a_{5,5} + a_{8,4}\,a_{4,8}\,a_{7,7}\,a_{2,2} - a_{8,4}\,a_{4,8}\,a_{6,2}\,a_{2,6} + a_{8,4}\,a_{4,8}\,a_{1,5}\,a_{5,1} + a_{8,4}\,a_{4,8}\,a_{5,5}\,a_{3,3}\\
&- a_{8,4}\,a_{4,8}\,a_{5,5}\,a_{2,2} - a_{8,4}\,a_{4,8}\,a_{3,3}\,a_{2,2} + a_{8,3}\,a_{3,1}\,a_{1,8}\,a_{7,7} - a_{8,3}\,a_{3,1}\,a_{1,8}\,a_{5,5} + a_{8,3}\,a_{3,1}\,a_{1,8}\,a_{2,2} + a_{6,2}\,a_{2,5}\,a_{5,1}\,a_{1,6}\\
&- a_{8,3}\,a_{3,7}\,a_{7,8}\,a_{5,5} + a_{8,3}\,a_{3,7}\,a_{7,8}\,a_{2,2} - a_{8,4}\,a_{4,1}\,a_{1,8}\,a_{7,7} + a_{8,4}\,a_{4,1}\,a_{1,8}\,a_{5,5} + a_{8,4}\,a_{4,1}\,a_{1,8}\,a_{3,3} - a_{8,4}\,a_{4,1}\,a_{1,8}\,a_{2,2}\\
&+ a_{8,4}\,a_{4,7}\,a_{7,8}\,a_{5,5} - a_{8,4}\,a_{4,7}\,a_{7,8}\,a_{2,2} - a_{8,5}\,a_{5,1}\,a_{1,8}\,a_{7,7} + a_{8,5}\,a_{5,1}\,a_{1,8}\,a_{3,3} - a_{8,5}\,a_{5,1}\,a_{1,8}\,a_{2,2} + a_{6,2}\,a_{2,6}\,a_{5,5}\,a_{3,3}\\
&+ a_{6,2}\,a_{2,6}\,a_{1,5}\,a_{5,1} + a_{6,5}\,a_{5,1}\,a_{1,6}\,a_{2,2} - a_{6,5}\,a_{5,1}\,a_{1,6}\,a_{3,3} - a_{7,4}\,a_{4,1}\,a_{1,6}\,a_{6,7} - a_{7,7}\,a_{6,2}\,a_{2,6}\,a_{5,5} + a_{7,7}\,a_{6,5}\,a_{5,1}\,a_{1,6}\\
&- a_{7,4}\,a_{4,7}\,a_{3,3}\,a_{2,2} - a_{7,4}\,a_{4,7}\,a_{5,5}\,a_{2,2} + a_{7,4}\,a_{4,7}\,a_{5,5}\,a_{3,3} + a_{7,4}\,a_{4,7}\,a_{1,5}\,a_{5,1} - a_{7,4}\,a_{4,7}\,a_{6,2}\,a_{2,6} - a_{7,7}\,a_{5,5}\,a_{3,3}\,a_{2,2}\\
&- a_{7,7}\,a_{1,5}\,a_{5,1}\,a_{2,2} + a_{7,7}\,a_{1,5}\,a_{5,1}\,a_{3,3} - a_{7,7}\,a_{6,2}\,a_{2,6}\,a_{3,3} + a_{3,3}\,a_{8,4}\,a_{4,7}\,a_{7,8} - a_{3,3}\,a_{8,4}\,a_{4,8}\,a_{7,7} - a_{8,3}\,a_{7,4}\,a_{3,7}\,a_{4,8}\\
&- a_{8,3}\,a_{3,5}\,a_{5,1}\,a_{1,8} + a_{1,5}\,a_{5,1}\,a_{3,3}\,a_{2,2}
\end{aligned}
$$

$$F_3$$

$$
\begin{aligned}
&a_{8,3}\,a_{3,1}\,a_{1,8} + a_{8,3}\,a_{3,7}\,a_{7,8} - a_{8,4}\,a_{4,1}\,a_{1,8} - a_{8,4}\,a_{4,7}\,a_{7,8} - a_{8,5}\,a_{5,1}\,a_{1,8} + a_{8,4}\,a_{4,8}\,a_{7,7} - a_{8,4}\,a_{4,8}\,a_{5,5} - a_{8,4}\,a_{4,8}\,a_{3,3}\\
&+ a_{8,4}\,a_{4,8}\,a_{2,2} - a_{7,4}\,a_{4,7}\,a_{5,5} - a_{7,4}\,a_{4,7}\,a_{3,3} + a_{7,4}\,a_{4,7}\,a_{2,2} + a_{7,7}\,a_{6,2}\,a_{2,6} - a_{7,7}\,a_{1,5}\,a_{5,1} - a_{7,7}\,a_{5,5}\,a_{3,3} + a_{7,7}\,a_{5,5}\,a_{2,2}\\
&+ a_{7,7}\,a_{3,3}\,a_{2,2} + a_{6,5}\,a_{5,1}\,a_{1,6} - a_{6,2}\,a_{2,6}\,a_{5,5} - a_{6,2}\,a_{2,6}\,a_{3,3} + a_{1,5}\,a_{5,1}\,a_{3,3} - a_{1,5}\,a_{5,1}\,a_{2,2} - a_{5,5}\,a_{3,3}\,a_{2,2}
\end{aligned}
$$

$$F_2$$

$$a_{8,4}\,a_{4,8}+a_{7,4}\,a_{4,7}+a_{7,7}\,a_{5,5}+a_{7,7}\,a_{3,3}-a_{7,7}\,a_{2,2}+a_{6,2}\,a_{2,6}-a_{1,5}\,a_{5,1}-a_{5,5}\,a_{3,3}+a_{5,5}\,a_{2,2}+a_{3,3}\,a_{2,2}$$

$$F_1$$

$$-a_{7,7}+a_{5,5}+a_{3,3}-a_{2,2}$$

$$F_0$$

-1

"adjoint (-A)"

$$\begin{bmatrix}
a_{6,2}\,(-a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{2,6}\,a_{4,7}\,a_{7,8}+a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{2,6}\,a_{4,8}\,a_{7,7}+a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{2,6}\,a_{3,7}\,a_{4,8}), \\
-a_{6,2}\,(a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{1,6}\,a_{4,7}\,a_{7,8}-a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{1,6}\,a_{4,8}\,a_{7,7}-a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{1,6}\,a_{3,7}\,a_{4,8}), \\
a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{1,6}\,a_{4,8}\,a_{6,7}-a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{1,6}\,a_{2,7}\,a_{4,8}+a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{2,6}\,a_{1,8}\,a_{4,7}, a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{1,6}\,a_{6,7}\,a_{7,8} \\
-a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{1,6}\,a_{2,7}\,a_{7,8}+a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{2,6}\,a_{1,8}\,a_{7,7}+a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{2,6}\,a_{1,8}\,a_{3,7}, \\
-a_{2,2}\,a_{3,3}\,a_{7,4}\,a_{8,5}\,a_{1,6}\,a_{4,8}\,a_{6,7}+a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{6,5}\,a_{1,6}\,a_{4,7}\,a_{7,8}-a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{6,5}\,a_{1,6}\,a_{4,8}\,a_{7,7} \\
-a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{7,5}\,a_{1,6}\,a_{4,8}\,a_{6,7}+a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{3,5}\,a_{1,6}\,a_{4,8}\,a_{6,7}-a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{6,5}\,a_{1,6}\,a_{3,7}\,a_{4,8} \\
+a_{6,2}\,a_{3,3}\,a_{7,4}\,a_{8,5}\,a_{1,6}\,a_{2,7}\,a_{4,8}-a_{6,2}\,a_{3,3}\,a_{7,4}\,a_{8,5}\,a_{2,6}\,a_{1,8}\,a_{4,7}+a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{1,5}\,a_{2,6}\,a_{4,7}\,a_{7,8} \\
-a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{1,5}\,a_{2,6}\,a_{4,8}\,a_{7,7}+a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{2,5}\,a_{1,6}\,a_{4,7}\,a_{7,8}-a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{2,5}\,a_{1,6}\,a_{4,8}\,a_{7,7} \\
+a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{7,5}\,a_{1,6}\,a_{2,7}\,a_{4,8}-a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{7,5}\,a_{2,6}\,a_{1,8}\,a_{4,7}-a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{1,5}\,a_{2,6}\,a_{3,7}\,a_{4,8} \\
-a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{2,5}\,a_{1,6}\,a_{3,7}\,a_{4,8}-a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{3,5}\,a_{1,6}\,a_{2,7}\,a_{4,8}+a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{3,5}\,a_{2,6}\,a_{1,8}\,a_{4,7}, \\
a_{2,2}\,(a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{1,6}\,a_{4,7}\,a_{7,8}-a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{1,6}\,a_{4,8}\,a_{7,7}-a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{1,6}\,a_{3,7}\,a_{4,8}), \\
-a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{1,6}\,a_{4,8}\,a_{6,7}+a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{1,6}\,a_{2,7}\,a_{4,8}-a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{2,6}\,a_{1,8}\,a_{4,7}, \\
a_{2,2}\,a_{3,3}\,a_{7,4}\,a_{5,5}\,a_{1,6}\,a_{4,8}\,a_{6,7}-a_{6,2}\,a_{3,3}\,a_{7,4}\,a_{5,5}\,a_{1,6}\,a_{2,7}\,a_{4,8}+a_{6,2}\,a_{3,3}\,a_{7,4}\,a_{5,5}\,a_{2,6}\,a_{1,8}\,a_{4,7}
\end{bmatrix}$$

$$\begin{bmatrix}
a_{3,1}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{2,6}\,a_{4,8}\,a_{6,7}-a_{4,1}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{2,6}\,a_{6,7}\,a_{7,8}-a_{5,1}\,a_{3,3}\,a_{7,4}\,a_{8,5}\,a_{2,6}\,a_{4,8}\,a_{6,7} \\
+a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{6,5}\,a_{2,6}\,a_{4,7}\,a_{7,8}-a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{6,5}\,a_{2,6}\,a_{4,8}\,a_{7,7}-a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{7,5}\,a_{2,6}\,a_{4,8}\,a_{6,7} \\
+a_{5,1}\,a_{8,3}\,a_{7,4}\,a_{3,5}\,a_{2,6}\,a_{4,8}\,a_{6,7}-a_{5,1}\,a_{8,3}\,a_{7,4}\,a_{6,5}\,a_{2,6}\,a_{3,7}\,a_{4,8}, a_{3,1}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{1,6}\,a_{4,8}\,a_{6,7} \\
-a_{4,1}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{1,6}\,a_{6,7}\,a_{7,8}-a_{5,1}\,a_{3,3}\,a_{7,4}\,a_{8,5}\,a_{1,6}\,a_{4,8}\,a_{6,7}+a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{6,5}\,a_{1,6}\,a_{4,7}\,a_{7,8} \\
-a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{6,5}\,a_{1,6}\,a_{4,8}\,a_{7,7}-a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{7,5}\,a_{1,6}\,a_{4,8}\,a_{6,7}+a_{5,1}\,a_{8,3}\,a_{7,4}\,a_{3,5}\,a_{1,6}\,a_{4,8}\,a_{6,7} \\
-a_{5,1}\,a_{8,3}\,a_{7,4}\,a_{6,5}\,a_{1,6}\,a_{3,7}\,a_{4,8}, a_{4,1}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{2,6}\,a_{1,8}\,a_{6,7}+a_{5,1}\,a_{8,3}\,a_{7,4}\,a_{1,5}\,a_{2,6}\,a_{4,8}\,a_{6,7} \\
+a_{5,1}\,a_{8,3}\,a_{7,4}\,a_{2,5}\,a_{1,6}\,a_{4,8}\,a_{6,7}+a_{5,1}\,a_{8,3}\,a_{7,4}\,a_{6,5}\,a_{1,6}\,a_{2,7}\,a_{4,8}-a_{5,1}\,a_{8,3}\,a_{7,4}\,a_{6,5}\,a_{2,6}\,a_{1,8}\,a_{4,7}, \\
a_{3,1}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{2,6}\,a_{1,8}\,a_{6,7}-a_{5,1}\,a_{3,3}\,a_{7,4}\,a_{8,5}\,a_{2,6}\,a_{1,8}\,a_{6,7}+a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{1,5}\,a_{2,6}\,a_{6,7}\,a_{7,8} \\
+a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{2,5}\,a_{1,6}\,a_{6,7}\,a_{7,8}+a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{6,5}\,a_{1,6}\,a_{2,7}\,a_{7,8}-a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{6,5}\,a_{2,6}\,a_{1,8}\,a_{7,7} \\
-a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{7,5}\,a_{2,6}\,a_{1,8}\,a_{6,7}+a_{5,1}\,a_{8,3}\,a_{7,4}\,a_{3,5}\,a_{2,6}\,a_{1,8}\,a_{6,7}-a_{5,1}\,a_{8,3}\,a_{7,4}\,a_{6,5}\,a_{2,6}\,a_{1,8}\,a_{3,7}, \\
-a_{3,1}\,a_{8,3}\,a_{7,4}\,a_{1,5}\,a_{2,6}\,a_{4,8}\,a_{6,7}-a_{3,1}\,a_{8,3}\,a_{7,4}\,a_{2,5}\,a_{1,6}\,a_{4,8}\,a_{6,7}-a_{3,1}\,a_{8,3}\,a_{7,4}\,a_{6,5}\,a_{1,6}\,a_{2,7}\,a_{4,8} \\
+a_{3,1}\,a_{8,3}\,a_{7,4}\,a_{6,5}\,a_{2,6}\,a_{1,8}\,a_{4,7}-a_{4,1}\,a_{3,3}\,a_{7,4}\,a_{8,5}\,a_{2,6}\,a_{1,8}\,a_{6,7}+a_{4,1}\,a_{3,3}\,a_{8,4}\,a_{1,5}\,a_{2,6}\,a_{6,7}\,a_{7,8} \\
+a_{4,1}\,a_{3,3}\,a_{8,4}\,a_{2,5}\,a_{1,6}\,a_{6,7}\,a_{7,8}+a_{4,1}\,a_{3,3}\,a_{8,4}\,a_{6,5}\,a_{1,6}\,a_{2,7}\,a_{7,8}-a_{4,1}\,a_{3,3}\,a_{8,4}\,a_{6,5}\,a_{2,6}\,a_{1,8}\,a_{7,7} \\
-a_{4,1}\,a_{3,3}\,a_{8,4}\,a_{7,5}\,a_{2,6}\,a_{1,8}\,a_{6,7}+a_{4,1}\,a_{8,3}\,a_{7,4}\,a_{3,5}\,a_{2,6}\,a_{1,8}\,a_{6,7}-a_{4,1}\,a_{8,3}\,a_{7,4}\,a_{6,5}\,a_{2,6}\,a_{1,8}\,a_{3,7}, \\
-a_{3,1}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{1,6}\,a_{2,7}\,a_{4,8}+a_{3,1}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{2,6}\,a_{1,8}\,a_{4,7}+a_{4,1}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{1,6}\,a_{2,7}\,a_{7,8} \\
-a_{4,1}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{2,6}\,a_{1,8}\,a_{7,7}-a_{4,1}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{2,6}\,a_{1,8}\,a_{3,7}+a_{5,1}\,a_{3,3}\,a_{7,4}\,a_{8,5}\,a_{1,6}\,a_{2,7}\,a_{4,8} \\
-a_{5,1}\,a_{3,3}\,a_{7,4}\,a_{8,5}\,a_{2,6}\,a_{1,8}\,a_{4,7}+a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{1,5}\,a_{2,6}\,a_{4,7}\,a_{7,8}-a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{1,5}\,a_{2,6}\,a_{4,8}\,a_{7,7} \\
+a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{2,5}\,a_{1,6}\,a_{4,7}\,a_{7,8}-a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{2,5}\,a_{1,6}\,a_{4,8}\,a_{7,7}+a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{7,5}\,a_{1,6}\,a_{2,7}\,a_{4,8} \\
-a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{7,5}\,a_{2,6}\,a_{1,8}\,a_{4,7}-a_{5,1}\,a_{8,3}\,a_{7,4}\,a_{1,5}\,a_{2,6}\,a_{3,7}\,a_{4,8}-a_{5,1}\,a_{8,3}\,a_{7,4}\,a_{2,5}\,a_{1,6}\,a_{3,7}\,a_{4,8} \\
-a_{5,1}\,a_{8,3}\,a_{7,4}\,a_{3,5}\,a_{1,6}\,a_{2,7}\,a_{4,8}+a_{5,1}\,a_{8,3}\,a_{7,4}\,a_{3,5}\,a_{2,6}\,a_{1,8}\,a_{4,7}, -a_{4,1}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{2,6}\,a_{1,8}\,a_{6,7} \\
-a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{1,5}\,a_{2,6}\,a_{4,8}\,a_{6,7}-a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{2,5}\,a_{1,6}\,a_{4,8}\,a_{6,7}-a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{6,5}\,a_{1,6}\,a_{2,7}\,a_{4,8} \\
+a_{5,1}\,a_{3,3}\,a_{8,4}\,a_{6,5}\,a_{2,6}\,a_{1,8}\,a_{4,7}, a_{4,1}\,a_{3,3}\,a_{7,4}\,a_{5,5}\,a_{2,6}\,a_{1,8}\,a_{6,7}+a_{5,1}\,a_{3,3}\,a_{7,4}\,a_{1,5}\,a_{2,6}\,a_{4,8}\,a_{6,7} \\
+a_{5,1}\,a_{3,3}\,a_{7,4}\,a_{2,5}\,a_{1,6}\,a_{4,8}\,a_{6,7}+a_{5,1}\,a_{3,3}\,a_{7,4}\,a_{6,5}\,a_{1,6}\,a_{2,7}\,a_{4,8}-a_{5,1}\,a_{3,3}\,a_{7,4}\,a_{6,5}\,a_{2,6}\,a_{1,8}\,a_{4,7}
\end{bmatrix}$$

$$\begin{bmatrix}
-a_{3,1}\,a_{6,2}\,a_{8,4}\,a_{5,5}\,a_{2,6}\,a_{4,7}\,a_{7,8}+a_{3,1}\,a_{6,2}\,a_{8,4}\,a_{5,5}\,a_{2,6}\,a_{4,8}\,a_{7,7}+a_{4,1}\,a_{6,2}\,a_{8,4}\,a_{5,5}\,a_{2,6}\,a_{3,7}\,a_{7,8} \\
+a_{5,1}\,a_{6,2}\,a_{7,4}\,a_{8,5}\,a_{2,6}\,a_{3,7}\,a_{4,8}-a_{5,1}\,a_{6,2}\,a_{8,4}\,a_{3,5}\,a_{2,6}\,a_{4,7}\,a_{7,8}+a_{5,1}\,a_{6,2}\,a_{8,4}\,a_{3,5}\,a_{2,6}\,a_{4,8}\,a_{7,7} \\
+a_{5,1}\,a_{6,2}\,a_{8,4}\,a_{7,5}\,a_{2,6}\,a_{3,7}\,a_{4,8}, -a_{3,1}\,a_{6,2}\,a_{8,4}\,a_{5,5}\,a_{1,6}\,a_{4,7}\,a_{7,8}+a_{3,1}\,a_{6,2}\,a_{8,4}\,a_{5,5}\,a_{1,6}\,a_{4,8}\,a_{7,7}
\end{bmatrix}$$

$$+a_{4,1}a_{6,2}a_{8,4}a_{5,5}a_{1,6}a_{3,7}a_{7,8}+a_{5,1}a_{6,2}a_{7,4}a_{8,5}a_{1,6}a_{3,7}a_{4,8}-a_{5,1}a_{6,2}a_{8,4}a_{3,5}a_{1,6}a_{4,7}a_{7,8}$$
$$+a_{5,1}a_{6,2}a_{8,4}a_{3,5}a_{1,6}a_{4,8}a_{7,7}+a_{5,1}a_{6,2}a_{8,4}a_{7,5}a_{1,6}a_{3,7}a_{4,8},a_{4,1}a_{2,2}a_{8,4}a_{5,5}a_{1,6}a_{6,7}a_{7,8}$$
$$-a_{4,1}a_{6,2}a_{8,4}a_{5,5}a_{1,6}a_{2,7}a_{7,8}+a_{4,1}a_{6,2}a_{8,4}a_{5,5}a_{2,6}a_{1,8}a_{7,7}+a_{5,1}a_{2,2}a_{7,4}a_{8,5}a_{1,6}a_{4,8}a_{6,7}$$
$$-a_{5,1}a_{2,2}a_{8,4}a_{6,5}a_{1,6}a_{4,7}a_{7,8}+a_{5,1}a_{2,2}a_{8,4}a_{6,5}a_{1,6}a_{4,8}a_{7,7}+a_{5,1}a_{2,2}a_{8,4}a_{7,5}a_{1,6}a_{4,8}a_{6,7}$$
$$-a_{5,1}a_{6,2}a_{7,4}a_{8,5}a_{1,6}a_{2,7}a_{4,8}+a_{5,1}a_{6,2}a_{7,4}a_{8,5}a_{2,6}a_{1,8}a_{4,7}-a_{5,1}a_{6,2}a_{8,4}a_{1,5}a_{2,6}a_{4,7}a_{7,8}$$
$$+a_{5,1}a_{6,2}a_{8,4}a_{1,5}a_{2,6}a_{4,8}a_{7,7}-a_{5,1}a_{6,2}a_{8,4}a_{2,5}a_{1,6}a_{4,7}a_{7,8}+a_{5,1}a_{6,2}a_{8,4}a_{2,5}a_{1,6}a_{4,8}a_{7,7}$$
$$-a_{5,1}a_{6,2}a_{8,4}a_{7,5}a_{1,6}a_{2,7}a_{4,8}+a_{5,1}a_{6,2}a_{8,4}a_{7,5}a_{2,6}a_{1,8}a_{4,7},a_{3,1}a_{2,2}a_{8,4}a_{5,5}a_{1,6}a_{6,7}a_{7,8}$$
$$-a_{3,1}a_{6,2}a_{8,4}a_{5,5}a_{1,6}a_{2,7}a_{7,8}+a_{3,1}a_{6,2}a_{8,4}a_{5,5}a_{2,6}a_{1,8}a_{7,7}+a_{5,1}a_{2,2}a_{8,4}a_{3,5}a_{1,6}a_{6,7}a_{7,8}$$
$$-a_{5,1}a_{2,2}a_{8,4}a_{6,5}a_{1,6}a_{3,7}a_{7,8}+a_{5,1}a_{6,2}a_{7,4}a_{8,5}a_{2,6}a_{1,8}a_{3,7}-a_{5,1}a_{6,2}a_{8,4}a_{1,5}a_{2,6}a_{3,7}a_{7,8}$$
$$-a_{5,1}a_{6,2}a_{8,4}a_{2,5}a_{1,6}a_{3,7}a_{7,8}-a_{5,1}a_{6,2}a_{8,4}a_{3,5}a_{1,6}a_{2,7}a_{7,8}+a_{5,1}a_{6,2}a_{8,4}a_{3,5}a_{2,6}a_{1,8}a_{7,7}$$
$$+a_{5,1}a_{6,2}a_{8,4}a_{7,5}a_{2,6}a_{1,8}a_{3,7},-a_{3,1}a_{2,2}a_{7,4}a_{8,5}a_{1,6}a_{4,8}a_{6,7}+a_{3,1}a_{2,2}a_{8,4}a_{6,5}a_{1,6}a_{4,7}a_{7,8}$$
$$-a_{3,1}a_{2,2}a_{8,4}a_{6,5}a_{1,6}a_{4,8}a_{7,7}-a_{3,1}a_{2,2}a_{8,4}a_{7,5}a_{1,6}a_{4,8}a_{6,7}+a_{3,1}a_{6,2}a_{7,4}a_{8,5}a_{1,6}a_{2,7}a_{4,8}$$
$$-a_{3,1}a_{6,2}a_{7,4}a_{8,5}a_{2,6}a_{1,8}a_{4,7}+a_{3,1}a_{6,2}a_{8,4}a_{1,5}a_{2,6}a_{4,7}a_{7,8}-a_{3,1}a_{6,2}a_{8,4}a_{1,5}a_{2,6}a_{4,8}a_{7,7}$$
$$+a_{3,1}a_{6,2}a_{8,4}a_{2,5}a_{1,6}a_{4,7}a_{7,8}-a_{3,1}a_{6,2}a_{8,4}a_{2,5}a_{1,6}a_{4,8}a_{7,7}+a_{3,1}a_{6,2}a_{8,4}a_{7,5}a_{1,6}a_{2,7}a_{4,8}$$
$$-a_{3,1}a_{6,2}a_{8,4}a_{7,5}a_{2,6}a_{1,8}a_{4,7}+a_{4,1}a_{2,2}a_{8,4}a_{3,5}a_{1,6}a_{6,7}a_{7,8}-a_{4,1}a_{2,2}a_{8,4}a_{6,5}a_{1,6}a_{3,7}a_{7,8}$$
$$+a_{4,1}a_{6,2}a_{7,4}a_{8,5}a_{2,6}a_{1,8}a_{3,7}-a_{4,1}a_{6,2}a_{8,4}a_{1,5}a_{2,6}a_{3,7}a_{7,8}-a_{4,1}a_{6,2}a_{8,4}a_{2,5}a_{1,6}a_{3,7}a_{7,8}$$
$$-a_{4,1}a_{6,2}a_{8,4}a_{3,5}a_{1,6}a_{2,7}a_{7,8}+a_{4,1}a_{6,2}a_{8,4}a_{3,5}a_{2,6}a_{1,8}a_{7,7}+a_{4,1}a_{6,2}a_{8,4}a_{7,5}a_{2,6}a_{1,8}a_{3,7},$$
$$a_{3,1}a_{2,2}a_{8,4}a_{5,5}a_{1,6}a_{4,7}a_{7,8}-a_{3,1}a_{2,2}a_{8,4}a_{5,5}a_{1,6}a_{4,8}a_{7,7}-a_{4,1}a_{2,2}a_{8,4}a_{5,5}a_{1,6}a_{3,7}a_{7,8}$$
$$-a_{5,1}a_{2,2}a_{7,4}a_{8,5}a_{1,6}a_{3,7}a_{4,8}+a_{5,1}a_{2,2}a_{8,4}a_{3,5}a_{1,6}a_{4,7}a_{7,8}-a_{5,1}a_{2,2}a_{8,4}a_{3,5}a_{1,6}a_{4,8}a_{7,7}$$
$$-a_{5,1}a_{2,2}a_{8,4}a_{7,5}a_{1,6}a_{3,7}a_{4,8},-a_{3,1}a_{2,2}a_{8,4}a_{5,5}a_{1,6}a_{4,8}a_{6,7}+a_{3,1}a_{6,2}a_{8,4}a_{5,5}a_{1,6}a_{2,7}a_{4,8}$$
$$-a_{3,1}a_{6,2}a_{8,4}a_{5,5}a_{2,6}a_{1,8}a_{4,7}+a_{4,1}a_{6,2}a_{8,4}a_{5,5}a_{2,6}a_{1,8}a_{3,7}-a_{5,1}a_{2,2}a_{8,4}a_{3,5}a_{1,6}a_{4,8}a_{6,7}$$
$$+a_{5,1}a_{2,2}a_{8,4}a_{6,5}a_{1,6}a_{3,7}a_{4,8}+a_{5,1}a_{6,2}a_{8,4}a_{1,5}a_{2,6}a_{3,7}a_{4,8}+a_{5,1}a_{6,2}a_{8,4}a_{2,5}a_{1,6}a_{3,7}a_{4,8}$$
$$+a_{5,1}a_{6,2}a_{8,4}a_{3,5}a_{1,6}a_{2,7}a_{4,8}-a_{5,1}a_{6,2}a_{8,4}a_{3,5}a_{2,6}a_{1,8}a_{4,7},a_{3,1}a_{2,2}a_{7,4}a_{5,5}a_{1,6}a_{4,8}a_{6,7}$$
$$-a_{3,1}a_{6,2}a_{7,4}a_{5,5}a_{1,6}a_{2,7}a_{4,8}+a_{3,1}a_{6,2}a_{7,4}a_{5,5}a_{2,6}a_{1,8}a_{4,7}-a_{4,1}a_{6,2}a_{7,4}a_{5,5}a_{2,6}a_{1,8}a_{3,7}$$
$$+a_{5,1}a_{2,2}a_{7,4}a_{3,5}a_{1,6}a_{4,8}a_{6,7}-a_{5,1}a_{2,2}a_{7,4}a_{6,5}a_{1,6}a_{3,7}a_{4,8}-a_{5,1}a_{6,2}a_{7,4}a_{1,5}a_{2,6}a_{3,7}a_{4,8}$$
$$-a_{5,1}a_{6,2}a_{7,4}a_{2,5}a_{1,6}a_{3,7}a_{4,8}-a_{5,1}a_{6,2}a_{7,4}a_{3,5}a_{1,6}a_{2,7}a_{4,8}+a_{5,1}a_{6,2}a_{7,4}a_{3,5}a_{2,6}a_{1,8}a_{4,7}]$$
$$[a_{3,1}a_{6,2}a_{8,3}a_{5,5}a_{2,6}a_{4,7}a_{7,8}-a_{3,1}a_{6,2}a_{8,3}a_{5,5}a_{2,6}a_{4,8}a_{7,7}-a_{4,1}a_{6,2}a_{8,3}a_{5,5}a_{2,6}a_{3,7}a_{7,8}$$
$$-a_{5,1}a_{6,2}a_{3,3}a_{8,5}a_{2,6}a_{4,7}a_{7,8}+a_{5,1}a_{6,2}a_{3,3}a_{8,5}a_{2,6}a_{4,8}a_{7,7}+a_{5,1}a_{6,2}a_{8,3}a_{3,5}a_{2,6}a_{4,7}a_{7,8}$$
$$-a_{5,1}a_{6,2}a_{8,3}a_{3,5}a_{2,6}a_{4,8}a_{7,7}-a_{5,1}a_{6,2}a_{8,3}a_{7,5}a_{2,6}a_{3,7}a_{4,8},a_{3,1}a_{6,2}a_{8,3}a_{5,5}a_{1,6}a_{4,7}a_{7,8}$$
$$-a_{3,1}a_{6,2}a_{8,3}a_{5,5}a_{1,6}a_{4,8}a_{7,7}-a_{4,1}a_{6,2}a_{8,3}a_{5,5}a_{1,6}a_{3,7}a_{7,8}-a_{5,1}a_{6,2}a_{3,3}a_{8,5}a_{1,6}a_{4,7}a_{7,8}$$
$$+a_{5,1}a_{6,2}a_{3,3}a_{8,5}a_{1,6}a_{4,8}a_{7,7}+a_{5,1}a_{6,2}a_{8,3}a_{3,5}a_{1,6}a_{4,7}a_{7,8}-a_{5,1}a_{6,2}a_{8,3}a_{3,5}a_{1,6}a_{4,8}a_{7,7}$$
$$-a_{5,1}a_{6,2}a_{8,3}a_{7,5}a_{1,6}a_{3,7}a_{4,8},-a_{4,1}a_{2,2}a_{8,3}a_{5,5}a_{1,6}a_{6,7}a_{7,8}+a_{4,1}a_{6,2}a_{8,3}a_{5,5}a_{1,6}a_{2,7}a_{7,8}$$
$$-a_{4,1}a_{6,2}a_{8,3}a_{5,5}a_{1,6}a_{2,7}a_{7,8}+a_{5,1}a_{2,2}a_{8,3}a_{6,5}a_{1,6}a_{4,7}a_{7,8}-a_{5,1}a_{2,2}a_{8,3}a_{6,5}a_{1,6}a_{4,8}a_{7,7}$$
$$-a_{5,1}a_{2,2}a_{8,3}a_{7,5}a_{1,6}a_{4,8}a_{6,7}+a_{5,1}a_{6,2}a_{8,3}a_{1,5}a_{2,6}a_{4,7}a_{7,8}-a_{5,1}a_{6,2}a_{8,3}a_{1,5}a_{2,6}a_{4,8}a_{7,7}$$
$$+a_{5,1}a_{6,2}a_{8,3}a_{2,5}a_{1,6}a_{4,7}a_{7,8}-a_{5,1}a_{6,2}a_{8,3}a_{2,5}a_{1,6}a_{4,8}a_{7,7}+a_{5,1}a_{6,2}a_{8,3}a_{7,5}a_{1,6}a_{2,7}a_{4,8}$$
$$-a_{5,1}a_{6,2}a_{8,3}a_{7,5}a_{2,6}a_{1,8}a_{4,7},-a_{3,1}a_{2,2}a_{8,3}a_{5,5}a_{1,6}a_{6,7}a_{7,8}+a_{3,1}a_{6,2}a_{8,3}a_{5,5}a_{1,6}a_{2,7}a_{7,8}$$
$$-a_{3,1}a_{6,2}a_{8,3}a_{5,5}a_{1,6}a_{2,7}a_{7,8}+a_{3,1}a_{2,2}a_{8,3}a_{5,5}a_{1,6}a_{6,7}a_{7,8}-a_{5,1}a_{2,2}a_{8,3}a_{3,5}a_{1,6}a_{6,7}a_{7,8}$$
$$+a_{5,1}a_{2,2}a_{8,3}a_{6,5}a_{1,6}a_{3,7}a_{7,8}-a_{5,1}a_{6,2}a_{3,3}a_{8,5}a_{1,6}a_{2,7}a_{7,8}+a_{5,1}a_{6,2}a_{3,3}a_{8,5}a_{2,6}a_{1,8}a_{7,7}$$
$$+a_{5,1}a_{6,2}a_{8,3}a_{1,5}a_{2,6}a_{3,7}a_{7,8}+a_{5,1}a_{6,2}a_{8,3}a_{2,5}a_{1,6}a_{3,7}a_{7,8}+a_{5,1}a_{6,2}a_{8,3}a_{3,5}a_{1,6}a_{2,7}a_{7,8}$$
$$-a_{5,1}a_{6,2}a_{8,3}a_{3,5}a_{2,6}a_{1,8}a_{7,7}-a_{5,1}a_{6,2}a_{8,3}a_{7,5}a_{2,6}a_{1,8}a_{3,7},-a_{3,1}a_{2,2}a_{8,3}a_{6,5}a_{1,6}a_{4,7}a_{7,8}$$
$$+a_{3,1}a_{2,2}a_{8,3}a_{6,5}a_{1,6}a_{4,8}a_{7,7}+a_{3,1}a_{6,2}a_{8,3}a_{7,5}a_{1,6}a_{4,8}a_{6,7}-a_{3,1}a_{6,2}a_{8,3}a_{1,5}a_{2,6}a_{4,7}a_{7,8}$$
$$+a_{3,1}a_{6,2}a_{8,3}a_{1,5}a_{2,6}a_{4,8}a_{7,7}-a_{3,1}a_{6,2}a_{8,3}a_{2,5}a_{1,6}a_{4,7}a_{7,8}+a_{3,1}a_{6,2}a_{8,3}a_{2,5}a_{1,6}a_{4,8}a_{7,7}$$
$$-a_{3,1}a_{6,2}a_{8,3}a_{7,5}a_{1,6}a_{2,7}a_{4,8}+a_{3,1}a_{6,2}a_{8,3}a_{7,5}a_{2,6}a_{1,8}a_{4,7}+a_{4,1}a_{2,2}a_{3,3}a_{8,5}a_{1,6}a_{6,7}a_{7,8}$$
$$-a_{4,1}a_{2,2}a_{8,3}a_{3,5}a_{1,6}a_{6,7}a_{7,8}+a_{4,1}a_{6,2}a_{3,3}a_{8,5}a_{1,6}a_{2,7}a_{7,8}$$
$$+a_{4,1}a_{6,2}a_{3,3}a_{8,5}a_{2,6}a_{1,8}a_{7,7}+a_{4,1}a_{6,2}a_{8,3}a_{1,5}a_{2,6}a_{3,7}a_{7,8}+a_{4,1}a_{6,2}a_{8,3}a_{2,5}a_{1,6}a_{3,7}a_{7,8}$$

$$
\begin{aligned}
&+ a_{4,1}\,a_{6,2}\,a_{8,3}\,a_{3,5}\,a_{1,6}\,a_{2,7}\,a_{7,8} - a_{4,1}\,a_{6,2}\,a_{8,3}\,a_{3,5}\,a_{2,6}\,a_{1,8}\,a_{7,7} - a_{4,1}\,a_{6,2}\,a_{8,3}\,a_{7,5}\,a_{2,6}\,a_{1,8}\,a_{3,7},\\
&-a_{3,1}\,a_{2,2}\,a_{8,3}\,a_{5,5}\,a_{1,6}\,a_{4,7}\,a_{7,8} + a_{3,1}\,a_{2,2}\,a_{8,3}\,a_{5,5}\,a_{1,6}\,a_{4,8}\,a_{7,7} + a_{4,1}\,a_{2,2}\,a_{8,3}\,a_{5,5}\,a_{1,6}\,a_{3,7}\,a_{7,8}\\
&+ a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{8,5}\,a_{1,6}\,a_{4,7}\,a_{7,8} - a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{8,5}\,a_{1,6}\,a_{4,8}\,a_{7,7} - a_{5,1}\,a_{2,2}\,a_{8,3}\,a_{3,5}\,a_{1,6}\,a_{4,7}\,a_{7,8}\\
&+ a_{5,1}\,a_{2,2}\,a_{8,3}\,a_{3,5}\,a_{1,6}\,a_{4,8}\,a_{7,7} + a_{5,1}\,a_{2,2}\,a_{8,3}\,a_{7,5}\,a_{1,6}\,a_{3,7}\,a_{4,8},\ a_{3,1}\,a_{2,2}\,a_{8,3}\,a_{5,5}\,a_{1,6}\,a_{4,8}\,a_{6,7}\\
&- a_{3,1}\,a_{6,2}\,a_{8,3}\,a_{5,5}\,a_{1,6}\,a_{2,7}\,a_{4,8} + a_{3,1}\,a_{6,2}\,a_{8,3}\,a_{5,5}\,a_{2,6}\,a_{1,8}\,a_{4,7} - a_{4,1}\,a_{6,2}\,a_{8,3}\,a_{5,5}\,a_{2,6}\,a_{1,8}\,a_{3,7}\\
&- a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{8,5}\,a_{1,6}\,a_{4,8}\,a_{6,7} + a_{5,1}\,a_{2,2}\,a_{8,3}\,a_{3,5}\,a_{1,6}\,a_{4,8}\,a_{6,7} - a_{5,1}\,a_{2,2}\,a_{8,3}\,a_{6,5}\,a_{1,6}\,a_{3,7}\,a_{4,8}\\
&+ a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{8,5}\,a_{1,6}\,a_{2,7}\,a_{4,8} - a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{8,5}\,a_{2,6}\,a_{1,8}\,a_{4,7} - a_{5,1}\,a_{6,2}\,a_{8,3}\,a_{1,5}\,a_{2,6}\,a_{3,7}\,a_{4,8}\\
&- a_{5,1}\,a_{6,2}\,a_{8,3}\,a_{2,5}\,a_{1,6}\,a_{3,7}\,a_{4,8} - a_{5,1}\,a_{6,2}\,a_{8,3}\,a_{3,5}\,a_{1,6}\,a_{2,7}\,a_{4,8} + a_{5,1}\,a_{6,2}\,a_{8,3}\,a_{3,5}\,a_{2,6}\,a_{1,8}\,a_{4,7},\\
&-a_{4,1}\,a_{2,2}\,a_{3,3}\,a_{5,5}\,a_{1,6}\,a_{6,7}\,a_{7,8} + a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{5,5}\,a_{1,6}\,a_{2,7}\,a_{7,8} - a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{5,5}\,a_{2,6}\,a_{1,8}\,a_{7,7}\\
&+ a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{6,5}\,a_{1,6}\,a_{4,7}\,a_{7,8} - a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{6,5}\,a_{1,6}\,a_{4,8}\,a_{7,7} - a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{7,5}\,a_{1,6}\,a_{4,8}\,a_{6,7}\\
&+ a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{1,5}\,a_{2,6}\,a_{4,7}\,a_{7,8} - a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{1,5}\,a_{2,6}\,a_{4,8}\,a_{7,7} + a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{2,5}\,a_{1,6}\,a_{4,7}\,a_{7,8}\\
&- a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{2,5}\,a_{1,6}\,a_{4,8}\,a_{7,7} + a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{7,5}\,a_{1,6}\,a_{2,7}\,a_{4,8} - a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{7,5}\,a_{2,6}\,a_{1,8}\,a_{4,7}]\\
&[a_{5,1}\,a_{6,2}\,(-a_{3,3}\,a_{8,4}\,a_{2,6}\,a_{4,7}\,a_{7,8} + a_{3,3}\,a_{8,4}\,a_{2,6}\,a_{4,8}\,a_{7,7} + a_{8,3}\,a_{7,4}\,a_{2,6}\,a_{3,7}\,a_{4,8}),\\
&-a_{5,1}\,a_{6,2}\,(a_{3,3}\,a_{8,4}\,a_{1,6}\,a_{4,7}\,a_{7,8} - a_{3,3}\,a_{8,4}\,a_{1,6}\,a_{4,8}\,a_{7,7} - a_{8,3}\,a_{7,4}\,a_{1,6}\,a_{3,7}\,a_{4,8}),\\
&-a_{5,1}\,(-a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{1,6}\,a_{4,8}\,a_{6,7} + a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{1,6}\,a_{2,7}\,a_{4,8} - a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{2,6}\,a_{1,8}\,a_{4,7}),\\
&a_{5,1}\,(a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{1,6}\,a_{6,7}\,a_{7,8} - a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{1,6}\,a_{2,7}\,a_{7,8} + a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{2,6}\,a_{1,8}\,a_{7,7} + a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{2,6}\,a_{1,8}\,a_{3,7}),\\
&-a_{3,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{1,6}\,a_{4,8}\,a_{6,7} + a_{3,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{1,6}\,a_{2,7}\,a_{4,8} - a_{3,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{2,6}\,a_{1,8}\,a_{4,7}\\
&+ a_{4,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{1,6}\,a_{6,7}\,a_{7,8} - a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{1,6}\,a_{2,7}\,a_{7,8} + a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{2,6}\,a_{1,8}\,a_{7,7}\\
&+ a_{4,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{2,6}\,a_{1,8}\,a_{3,7},\ a_{5,1}\,a_{2,2}\,(a_{3,3}\,a_{8,4}\,a_{1,6}\,a_{4,7}\,a_{7,8} - a_{3,3}\,a_{8,4}\,a_{1,6}\,a_{4,8}\,a_{7,7} - a_{8,3}\,a_{7,4}\,a_{1,6}\,a_{3,7}\,a_{4,8}),\\
&a_{5,1}\,(-a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{1,6}\,a_{4,8}\,a_{6,7} + a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{1,6}\,a_{2,7}\,a_{4,8} - a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{2,6}\,a_{1,8}\,a_{4,7}),\\
&-a_{5,1}\,(-a_{2,2}\,a_{3,3}\,a_{7,4}\,a_{1,6}\,a_{4,8}\,a_{6,7} + a_{6,2}\,a_{3,3}\,a_{7,4}\,a_{1,6}\,a_{2,7}\,a_{4,8} - a_{6,2}\,a_{3,3}\,a_{7,4}\,a_{2,6}\,a_{1,8}\,a_{4,7})]\\
&[-a_{3,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{4,8}\,a_{6,7} + a_{3,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{2,7}\,a_{4,8} + a_{4,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{6,7}\,a_{7,8}\\
&- a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{2,7}\,a_{7,8} + a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{7,4}\,a_{8,5}\,a_{4,8}\,a_{6,7} - a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{6,5}\,a_{4,7}\,a_{7,8}\\
&+ a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{6,5}\,a_{4,8}\,a_{7,7} + a_{5,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{3,5}\,a_{4,8}\,a_{6,7} - a_{5,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{4,5}\,a_{4,8}\,a_{6,7}\\
&+ a_{5,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{6,5}\,a_{3,7}\,a_{4,8} - a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{7,4}\,a_{8,5}\,a_{2,7}\,a_{4,8} - a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{2,5}\,a_{4,7}\,a_{7,8}\\
&+ a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{2,5}\,a_{4,8}\,a_{7,7} - a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{7,5}\,a_{2,7}\,a_{4,8} + a_{5,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{2,5}\,a_{3,7}\,a_{4,8}\\
&+ a_{5,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{3,5}\,a_{2,7}\,a_{4,8},\ a_{3,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{1,8}\,a_{4,7} - a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{1,8}\,a_{7,7}\\
&- a_{4,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{1,8}\,a_{3,7} - a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{7,4}\,a_{8,5}\,a_{1,8}\,a_{4,7} + a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{1,5}\,a_{4,7}\,a_{7,8}\\
&- a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{1,5}\,a_{4,8}\,a_{7,7} - a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{7,5}\,a_{1,8}\,a_{4,7} - a_{5,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{1,5}\,a_{3,7}\,a_{4,8}\\
&+ a_{5,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{3,5}\,a_{1,8}\,a_{4,7},\ -a_{4,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{1,8}\,a_{6,7} + a_{4,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{1,8}\,a_{2,7}\\
&- a_{5,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{1,5}\,a_{4,8}\,a_{6,7} + a_{5,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{6,5}\,a_{1,8}\,a_{4,7} + a_{5,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{1,5}\,a_{2,7}\,a_{4,8}\\
&+ a_{5,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{2,5}\,a_{1,8}\,a_{4,7},\ -a_{3,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{1,8}\,a_{6,7} + a_{3,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{1,8}\,a_{2,7}\\
&+ a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{7,4}\,a_{8,5}\,a_{1,8}\,a_{6,7} - a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{1,5}\,a_{6,7}\,a_{7,8} + a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{6,5}\,a_{1,8}\,a_{7,7}\\
&+ a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{7,5}\,a_{1,8}\,a_{6,7} - a_{5,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{3,5}\,a_{1,8}\,a_{6,7} + a_{5,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{6,5}\,a_{1,8}\,a_{3,7}\\
&- a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{7,4}\,a_{8,5}\,a_{1,8}\,a_{2,7} + a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{1,5}\,a_{4,8}\,a_{7,7} + a_{5,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{2,5}\,a_{1,8}\,a_{3,7}\\
&- a_{5,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{7,5}\,a_{1,8}\,a_{2,7} + a_{5,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{3,5}\,a_{1,8}\,a_{2,7},\\
&a_{3,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{1,5}\,a_{4,8}\,a_{6,7} - a_{3,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{6,5}\,a_{1,8}\,a_{4,7} - a_{3,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{1,5}\,a_{2,7}\,a_{4,8}\\
&- a_{3,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{2,5}\,a_{1,8}\,a_{4,7} + a_{4,1}\,a_{2,2}\,a_{3,3}\,a_{7,4}\,a_{8,5}\,a_{1,8}\,a_{6,7} - a_{4,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{1,5}\,a_{6,7}\,a_{7,8}\\
&+ a_{4,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{6,5}\,a_{1,8}\,a_{7,7} + a_{4,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{7,5}\,a_{1,8}\,a_{6,7} - a_{4,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{3,5}\,a_{1,8}\,a_{6,7}\\
&+ a_{4,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{6,5}\,a_{1,8}\,a_{3,7} - a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{7,4}\,a_{8,5}\,a_{1,8}\,a_{2,7} + a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{1,5}\,a_{2,7}\,a_{4,8}\\
&+ a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{2,5}\,a_{1,8}\,a_{3,7} - a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{7,5}\,a_{1,8}\,a_{2,7} + a_{4,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{2,5}\,a_{1,8}\,a_{4,7}\\
&+ a_{4,1}\,a_{6,2}\,a_{8,3}\,a_{7,4}\,a_{3,5}\,a_{1,8}\,a_{2,7},\ -a_{3,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{1,8}\,a_{4,7} + a_{4,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{1,8}\,a_{7,7}\\
&+ a_{4,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{5,5}\,a_{1,8}\,a_{3,7} + a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{7,4}\,a_{8,5}\,a_{1,8}\,a_{4,7} - a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{1,5}\,a_{4,7}\,a_{7,8}\\
&+ a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{1,5}\,a_{4,8}\,a_{7,7} + a_{5,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{7,5}\,a_{1,8}\,a_{4,7} + a_{5,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{1,5}\,a_{3,7}\,a_{4,8}\\
&- a_{5,1}\,a_{2,2}\,a_{8,3}\,a_{7,4}\,a_{3,5}\,a_{1,8}\,a_{4,7},\ a_{4,1}\,a_{2,2}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{1,8}\,a_{6,7} - a_{4,1}\,a_{6,2}\,a_{3,3}\,a_{8,4}\,a_{5,5}\,a_{1,8}\,a_{2,7}
\end{aligned}
$$

$$+a_{5,1}a_{2,2}a_{3,3}a_{8,4}a_{1,5}a_{4,8}a_{6,7}-a_{5,1}a_{2,2}a_{3,3}a_{8,4}a_{6,5}a_{1,8}a_{4,7}-a_{5,1}a_{6,2}a_{3,3}a_{8,4}a_{1,5}a_{2,7}a_{4,8}$$
$$-a_{5,1}a_{6,2}a_{3,3}a_{8,4}a_{2,5}a_{1,8}a_{4,7},-a_{4,1}a_{2,2}a_{3,3}a_{7,4}a_{5,5}a_{1,8}a_{6,7}+a_{4,1}a_{6,2}a_{3,3}a_{7,4}a_{5,5}a_{1,8}a_{2,7}$$
$$-a_{5,1}a_{2,2}a_{3,3}a_{7,4}a_{1,5}a_{4,8}a_{6,7}+a_{5,1}a_{2,2}a_{3,3}a_{7,4}a_{6,5}a_{1,8}a_{4,7}+a_{5,1}a_{6,2}a_{3,3}a_{7,4}a_{1,5}a_{2,7}a_{4,8}$$
$$+a_{5,1}a_{6,2}a_{3,3}a_{7,4}a_{2,5}a_{1,8}a_{4,7}]$$
$$[a_{3,1}a_{6,2}a_{8,3}a_{7,4}a_{5,5}a_{2,6}a_{4,8}-a_{4,1}a_{6,2}a_{3,3}a_{8,4}a_{5,5}a_{2,6}a_{7,8}-a_{5,1}a_{6,2}a_{3,3}a_{7,4}a_{8,5}a_{2,6}a_{4,8}$$
$$-a_{5,1}a_{6,2}a_{3,3}a_{8,4}a_{7,5}a_{2,6}a_{4,8}+a_{5,1}a_{6,2}a_{8,3}a_{7,4}a_{3,5}a_{2,6}a_{4,8},a_{3,1}a_{6,2}a_{8,3}a_{7,4}a_{5,5}a_{1,6}a_{4,8}$$
$$-a_{4,1}a_{6,2}a_{3,3}a_{8,4}a_{5,5}a_{1,6}a_{7,8}-a_{5,1}a_{6,2}a_{3,3}a_{7,4}a_{8,5}a_{1,6}a_{4,8}-a_{5,1}a_{6,2}a_{3,3}a_{8,4}a_{7,5}a_{1,6}a_{4,8}$$
$$+a_{5,1}a_{6,2}a_{8,3}a_{7,4}a_{3,5}a_{1,6}a_{4,8},a_{4,1}a_{6,2}a_{8,3}a_{7,4}a_{5,5}a_{1,8}a_{2,6}+a_{5,1}a_{2,2}a_{8,3}a_{7,4}a_{6,5}a_{1,6}a_{4,8}$$
$$+a_{5,1}a_{6,2}a_{8,3}a_{7,4}a_{1,5}a_{2,6}a_{4,8}+a_{5,1}a_{6,2}a_{8,3}a_{7,4}a_{2,5}a_{1,6}a_{4,8},a_{3,1}a_{6,2}a_{8,3}a_{7,4}a_{5,5}a_{1,8}a_{2,6}$$
$$+a_{5,1}a_{2,2}a_{3,3}a_{8,4}a_{6,5}a_{1,6}a_{7,8}-a_{5,1}a_{6,2}a_{3,3}a_{7,4}a_{8,5}a_{1,8}a_{2,6}+a_{5,1}a_{6,2}a_{3,3}a_{8,4}a_{1,5}a_{2,6}a_{7,8}$$
$$+a_{5,1}a_{6,2}a_{3,3}a_{8,4}a_{2,5}a_{1,6}a_{7,8}-a_{5,1}a_{6,2}a_{3,3}a_{8,4}a_{7,5}a_{1,8}a_{2,6}+a_{5,1}a_{6,2}a_{8,3}a_{7,4}a_{3,5}a_{1,8}a_{2,6},$$
$$-a_{3,1}a_{2,2}a_{8,3}a_{7,4}a_{6,5}a_{1,6}a_{4,8}-a_{3,1}a_{6,2}a_{8,3}a_{7,4}a_{1,5}a_{2,6}a_{4,8}-a_{3,1}a_{6,2}a_{8,3}a_{7,4}a_{2,5}a_{1,6}a_{4,8}$$
$$+a_{4,1}a_{2,2}a_{3,3}a_{8,4}a_{6,5}a_{1,6}a_{7,8}-a_{4,1}a_{6,2}a_{3,3}a_{7,4}a_{8,5}a_{1,8}a_{2,6}+a_{4,1}a_{6,2}a_{3,3}a_{8,4}a_{1,5}a_{2,6}a_{7,8}$$
$$+a_{4,1}a_{6,2}a_{3,3}a_{8,4}a_{2,5}a_{1,6}a_{7,8}-a_{4,1}a_{6,2}a_{3,3}a_{8,4}a_{7,5}a_{1,8}a_{2,6}+a_{4,1}a_{6,2}a_{8,3}a_{7,4}a_{3,5}a_{1,8}a_{2,6},$$
$$-a_{3,1}a_{2,2}a_{8,3}a_{7,4}a_{5,5}a_{1,6}a_{4,8}+a_{4,1}a_{2,2}a_{3,3}a_{8,4}a_{5,5}a_{1,6}a_{7,8}+a_{5,1}a_{2,2}a_{3,3}a_{7,4}a_{8,5}a_{1,6}a_{4,8}$$
$$+a_{5,1}a_{2,2}a_{3,3}a_{8,4}a_{7,5}a_{1,6}a_{4,8}-a_{5,1}a_{2,2}a_{8,3}a_{7,4}a_{3,5}a_{1,6}a_{4,8},-a_{4,1}a_{6,2}a_{3,3}a_{8,4}a_{5,5}a_{1,8}a_{2,6}$$
$$-a_{5,1}a_{2,2}a_{3,3}a_{8,4}a_{6,5}a_{1,6}a_{4,8}-a_{5,1}a_{6,2}a_{3,3}a_{8,4}a_{1,5}a_{2,6}a_{4,8}-a_{5,1}a_{6,2}a_{3,3}a_{8,4}a_{2,5}a_{1,6}a_{4,8},$$
$$a_{4,1}a_{6,2}a_{3,3}a_{7,4}a_{5,5}a_{1,8}a_{2,6}+a_{5,1}a_{2,2}a_{3,3}a_{7,4}a_{6,5}a_{1,6}a_{4,8}+a_{5,1}a_{6,2}a_{3,3}a_{7,4}a_{1,5}a_{2,6}a_{4,8}$$
$$+a_{5,1}a_{6,2}a_{3,3}a_{7,4}a_{2,5}a_{1,6}a_{4,8}]$$
$$[-a_{3,1}a_{6,2}a_{8,3}a_{7,4}a_{5,5}a_{2,6}a_{4,7}+a_{4,1}a_{6,2}a_{3,3}a_{8,4}a_{5,5}a_{2,6}a_{7,7}+a_{4,1}a_{6,2}a_{8,3}a_{7,4}a_{5,5}a_{2,6}a_{3,7}$$
$$+a_{5,1}a_{6,2}a_{3,3}a_{7,4}a_{8,5}a_{2,6}a_{4,7}+a_{5,1}a_{6,2}a_{3,3}a_{8,4}a_{7,5}a_{2,6}a_{4,7}-a_{5,1}a_{6,2}a_{8,3}a_{7,4}a_{3,5}a_{2,6}a_{4,7},$$
$$-a_{3,1}a_{6,2}a_{8,3}a_{7,4}a_{5,5}a_{1,6}a_{4,7}+a_{4,1}a_{6,2}a_{3,3}a_{8,4}a_{5,5}a_{1,6}a_{7,7}+a_{4,1}a_{6,2}a_{8,3}a_{7,4}a_{5,5}a_{1,6}a_{3,7}$$
$$+a_{5,1}a_{6,2}a_{3,3}a_{7,4}a_{8,5}a_{1,6}a_{4,7}+a_{5,1}a_{6,2}a_{3,3}a_{8,4}a_{7,5}a_{1,6}a_{4,7}-a_{5,1}a_{6,2}a_{8,3}a_{7,4}a_{3,5}a_{1,6}a_{4,7},$$
$$a_{4,1}a_{2,2}a_{8,3}a_{7,4}a_{5,5}a_{1,6}a_{6,7}-a_{4,1}a_{6,2}a_{8,3}a_{7,4}a_{5,5}a_{1,6}a_{2,7}-a_{5,1}a_{2,2}a_{8,3}a_{7,4}a_{6,5}a_{1,6}a_{4,7}$$
$$-a_{5,1}a_{6,2}a_{8,3}a_{7,4}a_{1,5}a_{2,6}a_{4,7}-a_{5,1}a_{6,2}a_{8,3}a_{7,4}a_{2,5}a_{1,6}a_{4,7},a_{3,1}a_{2,2}a_{8,3}a_{7,4}a_{5,5}a_{1,6}a_{6,7}$$
$$-a_{3,1}a_{6,2}a_{8,3}a_{7,4}a_{5,5}a_{1,6}a_{2,7}-a_{5,1}a_{2,2}a_{3,3}a_{7,4}a_{8,5}a_{1,6}a_{6,7}-a_{5,1}a_{2,2}a_{3,3}a_{8,4}a_{6,5}a_{1,6}a_{7,7}$$
$$-a_{5,1}a_{2,2}a_{3,3}a_{8,4}a_{7,5}a_{1,6}a_{6,7}+a_{5,1}a_{2,2}a_{8,3}a_{7,4}a_{3,5}a_{1,6}a_{6,7}-a_{5,1}a_{2,2}a_{8,3}a_{7,4}a_{6,5}a_{1,6}a_{3,7}$$
$$+a_{5,1}a_{6,2}a_{3,3}a_{7,4}a_{8,5}a_{1,6}a_{2,7}-a_{5,1}a_{6,2}a_{3,3}a_{8,4}a_{1,5}a_{2,6}a_{7,7}-a_{5,1}a_{6,2}a_{3,3}a_{8,4}a_{2,5}a_{1,6}a_{7,7}$$
$$+a_{5,1}a_{6,2}a_{3,3}a_{8,4}a_{7,5}a_{1,6}a_{2,7}-a_{5,1}a_{6,2}a_{8,3}a_{7,4}a_{1,5}a_{2,6}a_{3,7}-a_{5,1}a_{6,2}a_{8,3}a_{7,4}a_{2,5}a_{1,6}a_{3,7}$$
$$-a_{5,1}a_{6,2}a_{8,3}a_{7,4}a_{3,5}a_{1,6}a_{2,7},a_{3,1}a_{2,2}a_{8,3}a_{7,4}a_{6,5}a_{1,6}a_{4,7}+a_{3,1}a_{6,2}a_{8,3}a_{7,4}a_{1,5}a_{2,6}a_{4,7}$$
$$+a_{3,1}a_{6,2}a_{8,3}a_{7,4}a_{2,5}a_{1,6}a_{4,7}-a_{4,1}a_{2,2}a_{3,3}a_{7,4}a_{8,5}a_{1,6}a_{6,7}-a_{4,1}a_{2,2}a_{3,3}a_{8,4}a_{6,5}a_{1,6}a_{7,7}$$
$$-a_{4,1}a_{2,2}a_{3,3}a_{8,4}a_{7,5}a_{1,6}a_{6,7}+a_{4,1}a_{2,2}a_{8,3}a_{7,4}a_{3,5}a_{1,6}a_{6,7}-a_{4,1}a_{2,2}a_{8,3}a_{7,4}a_{6,5}a_{1,6}a_{3,7}$$
$$+a_{4,1}a_{6,2}a_{3,3}a_{7,4}a_{8,5}a_{1,6}a_{2,7}-a_{4,1}a_{6,2}a_{3,3}a_{8,4}a_{1,5}a_{2,6}a_{7,7}-a_{4,1}a_{6,2}a_{3,3}a_{8,4}a_{2,5}a_{1,6}a_{7,7}$$
$$+a_{4,1}a_{6,2}a_{3,3}a_{8,4}a_{7,5}a_{1,6}a_{2,7}-a_{4,1}a_{6,2}a_{8,3}a_{7,4}a_{1,5}a_{2,6}a_{3,7}-a_{4,1}a_{6,2}a_{8,3}a_{7,4}a_{2,5}a_{1,6}a_{3,7}$$
$$-a_{4,1}a_{6,2}a_{8,3}a_{7,4}a_{3,5}a_{1,6}a_{2,7},a_{3,1}a_{2,2}a_{8,3}a_{7,4}a_{5,5}a_{1,6}a_{4,7}-a_{4,1}a_{2,2}a_{3,3}a_{8,4}a_{5,5}a_{1,6}a_{7,7}$$
$$-a_{4,1}a_{2,2}a_{8,3}a_{7,4}a_{5,5}a_{1,6}a_{3,7}-a_{5,1}a_{2,2}a_{3,3}a_{7,4}a_{8,5}a_{1,6}a_{4,7}-a_{5,1}a_{2,2}a_{3,3}a_{8,4}a_{7,5}a_{1,6}a_{4,7}$$
$$+a_{5,1}a_{2,2}a_{8,3}a_{7,4}a_{3,5}a_{1,6}a_{4,7},-a_{4,1}a_{2,2}a_{3,3}a_{8,4}a_{5,5}a_{1,6}a_{6,7}+a_{4,1}a_{6,2}a_{3,3}a_{8,4}a_{5,5}a_{1,6}a_{2,7}$$
$$+a_{5,1}a_{2,2}a_{3,3}a_{8,4}a_{6,5}a_{1,6}a_{4,7}+a_{5,1}a_{6,2}a_{3,3}a_{8,4}a_{1,5}a_{2,6}a_{4,7}+a_{5,1}a_{6,2}a_{3,3}a_{8,4}a_{2,5}a_{1,6}a_{4,7},$$
$$a_{4,1}a_{2,2}a_{3,3}a_{7,4}a_{5,5}a_{1,6}a_{6,7}-a_{4,1}a_{6,2}a_{3,3}a_{7,4}a_{5,5}a_{1,6}a_{2,7}-a_{5,1}a_{2,2}a_{3,3}a_{7,4}a_{6,5}a_{1,6}a_{4,7}$$
$$-a_{5,1}a_{6,2}a_{3,3}a_{7,4}a_{1,5}a_{2,6}a_{4,7}-a_{5,1}a_{6,2}a_{3,3}a_{7,4}a_{2,5}a_{1,6}a_{4,7}]$$

$$A := \begin{bmatrix} 0 & -1 & 0 & -1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

poly_coef_F0_to_Fn = [-1, 0, 0, -1, 1, 6, -7, -2, 3]

positive_feedback = [0, 0, 2, 2, 4, 10, 2, 4, 3]

negative_feedback = [-1, 0, -2, -3, -3, -4, -9, -6, 0]

absolute_feedback = [1, 0, 4, 5, 7, 14, 11, 10, 3]

wFn = [-1., 1., 0., -0.20, 0.14, 0.43, -0.64, -0.20, 1.]

"Note: absence of feedback at one or more levels in system."

"Criterion ii"

wD_7 = -0.00021

ratio_to_model_C = -650.

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 1 & -1 & -3 & -3 & -1 & -1 & 1 & 2 \\ 0 & 0 & 0 & -3 & 0 & 0 & 0 & 0 \\ -1 & 1 & 3 & 3 & -2 & 1 & -1 & -2 \\ -1 & -2 & 0 & 0 & 1 & 1 & 2 & 1 \\ -1 & 1 & 0 & 0 & 1 & 1 & -1 & 1 \\ 1 & -1 & 0 & 0 & -1 & -1 & -2 & -1 \\ 0 & 0 & -3 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 3 & -2 & -2 & -1 & -2 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 1 & 1 & 3 & 3 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 1 & 1 & 3 & 3 & 2 & 1 & 1 & 2 \\ 1 & 2 & 2 & 2 & 1 & 1 & 2 & 1 \\ 1 & 1 & 2 & 2 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 & 1 & 1 & 2 & 1 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 4 & 3 & 2 & 2 & 1 & 2 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 0. & 0. & 1. & 1. & 1. & 1. \\ 1. & 1. & 0. & 0. & 1. & 1. & 1. & 1. \\ 1. & 1. & 0. & 0. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 0. & 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 1 & -1 & 0 & -3 & 2 & 2 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 1 & 1 & 4 & 3 & 2 & 2 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 0. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & -3 & 2 & -1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 3 & 0 & 0 & 0 & -3 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 4 & 3 & 2 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 3 & 0 & 0 & 0 & 3 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 0. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & -a_{1,2} & 0 & -a_{1,4} & -a_{1,5} & 0 & 0 & -a_{1,8} \\ 0 & 0 & 0 & -a_{2,4} & 0 & -a_{2,6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -a_{3,7} & 0 \\ 0 & -a_{4,2} & 0 & 0 & 0 & 0 & 0 & 0 \\ -a_{5,1} & 0 & -a_{5,3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{6,3} & 0 & 0 & 0 & a_{6,7} & -a_{6,8} \\ 0 & 0 & 0 & 0 & -a_{7,5} & -a_{7,6} & 0 & 0 \\ a_{8,1} & -a_{8,2} & 0 & 0 & a_{8,5} & 0 & -a_{8,7} & 0 \end{bmatrix}$$

"system feedback"

F_8

$$a_{1,4}\,a_{8,1}\,a_{4,2}\,a_{5,3}\,a_{7,5}\,a_{2,6}\,a_{3,7}\,a_{6,8} + a_{1,5}\,a_{8,1}\,a_{4,2}\,a_{5,3}\,a_{2,4}\,a_{7,6}\,a_{3,7}\,a_{6,8} + a_{1,8}\,a_{5,1}\,a_{4,2}\,a_{6,3}\,a_{2,4}\,a_{8,5}\,a_{3,7}\,a_{7,6}$$

F_7

$$\begin{aligned} &-a_{4,2}\,a_{5,3}\,a_{2,4}\,a_{8,5}\,a_{7,6}\,a_{3,7}\,a_{6,8} - a_{8,1}\,a_{1,2}\,a_{5,3}\,a_{7,5}\,a_{2,6}\,a_{3,7}\,a_{6,8} + a_{5,1}\,a_{8,2}\,a_{6,3}\,a_{7,5}\,a_{2,6}\,a_{1,8}\,a_{3,7} \\ &\quad - a_{8,1}\,a_{4,2}\,a_{6,3}\,a_{2,4}\,a_{7,6}\,a_{1,8}\,a_{3,7} + a_{8,1}\,a_{4,2}\,a_{5,3}\,a_{2,4}\,a_{7,5}\,a_{1,8}\,a_{3,7} + a_{1,5}\,a_{5,1}\,a_{4,2}\,a_{2,4}\,a_{7,6}\,a_{6,3}\,a_{3,7} \\ &\quad + a_{7,5}\,a_{5,1}\,a_{1,4}\,a_{4,2}\,a_{2,6}\,a_{6,3}\,a_{3,7} - a_{8,7}\,a_{7,5}\,a_{5,1}\,a_{1,4}\,a_{4,2}\,a_{2,6}\,a_{6,8} - a_{8,5}\,a_{5,1}\,a_{1,8}\,a_{6,7}\,a_{7,6}\,a_{4,2}\,a_{2,4} \\ &\quad - a_{7,6}\,a_{6,8}\,a_{8,7}\,a_{1,5}\,a_{5,1}\,a_{4,2}\,a_{2,4} \end{aligned}$$

F_6

$$\begin{aligned} &-a_{7,5}\,a_{5,1}\,a_{2,6}\,a_{1,2}\,a_{6,3}\,a_{3,7} - a_{7,5}\,a_{5,1}\,a_{2,6}\,a_{1,4}\,a_{4,2}\,a_{6,7} - a_{6,7}\,a_{7,6}\,a_{1,5}\,a_{5,1}\,a_{4,2}\,a_{2,4} - a_{8,2}\,a_{2,6}\,a_{6,7}\,a_{7,5}\,a_{5,1}\,a_{1,8} \\ &\quad - a_{8,5}\,a_{5,1}\,a_{1,4}\,a_{4,2}\,a_{2,6}\,a_{6,8} + a_{8,7}\,a_{7,5}\,a_{5,1}\,a_{1,2}\,a_{2,6}\,a_{6,8} - a_{8,1}\,a_{5,3}\,a_{1,5}\,a_{7,6}\,a_{3,7}\,a_{6,8} + a_{8,1}\,a_{1,8}\,a_{6,7}\,a_{7,6}\,a_{4,2}\,a_{2,4} \\ &\quad - a_{8,2}\,a_{2,6}\,a_{6,8}\,a_{7,5}\,a_{5,3}\,a_{3,7} - a_{8,5}\,a_{5,1}\,a_{1,8}\,a_{7,6}\,a_{6,3}\,a_{3,7} - a_{8,7}\,a_{7,5}\,a_{5,1}\,a_{1,8}\,a_{4,2}\,a_{2,4} \end{aligned}$$

F_5

$$\begin{aligned} &a_{8,1}\,a_{1,4}\,a_{4,2}\,a_{2,6}\,a_{6,8} + a_{8,5}\,a_{5,1}\,a_{1,2}\,a_{2,6}\,a_{6,8} + a_{8,5}\,a_{5,3}\,a_{7,6}\,a_{3,7}\,a_{6,8} + a_{8,2}\,a_{2,6}\,a_{6,8}\,a_{1,5}\,a_{5,1} + a_{8,5}\,a_{5,1}\,a_{1,8}\,a_{6,7}\,a_{7,6} \\ &\quad - a_{8,5}\,a_{5,1}\,a_{1,8}\,a_{4,2}\,a_{2,4} + a_{7,6}\,a_{6,8}\,a_{8,7}\,a_{1,5}\,a_{5,1} + a_{7,6}\,a_{6,8}\,a_{8,7}\,a_{4,2}\,a_{2,4} - a_{8,1}\,a_{1,8}\,a_{7,5}\,a_{5,3}\,a_{3,7} + a_{8,1}\,a_{1,8}\,a_{7,6}\,a_{6,3}\,a_{3,7} \\ &\quad + a_{7,5}\,a_{5,1}\,a_{1,2}\,a_{2,6}\,a_{6,7} + a_{7,5}\,a_{5,3}\,a_{3,7}\,a_{4,2}\,a_{2,4} - a_{7,6}\,a_{6,3}\,a_{3,7}\,a_{1,5}\,a_{5,1} - a_{7,6}\,a_{6,3}\,a_{3,7}\,a_{4,2}\,a_{2,4} \end{aligned}$$

F_4

$$\begin{aligned} &-a_{8,1}\,a_{1,2}\,a_{2,6}\,a_{6,8} + a_{8,7}\,a_{7,5}\,a_{5,1}\,a_{1,8} - a_{8,1}\,a_{1,8}\,a_{6,7}\,a_{7,6} + a_{8,1}\,a_{1,8}\,a_{4,2}\,a_{2,4} + a_{6,7}\,a_{7,6}\,a_{1,5}\,a_{5,1} + a_{6,7}\,a_{7,6}\,a_{4,2}\,a_{2,4} \\ &\quad - a_{1,5}\,a_{5,1}\,a_{4,2}\,a_{2,4} \end{aligned}$$

F_3

$$-a_{8,2}\,a_{2,6}\,a_{6,8} + a_{8,5}\,a_{5,1}\,a_{1,8} - a_{7,6}\,a_{6,8}\,a_{8,7} - a_{7,5}\,a_{5,3}\,a_{3,7} + a_{7,6}\,a_{6,3}\,a_{3,7}$$

F_2

$$-a_{8,1}\,a_{1,8} - a_{6,7}\,a_{7,6} + a_{1,5}\,a_{5,1} + a_{4,2}\,a_{2,4}$$

F_1

0

F_0

-1

"adjoint (-A)"

$$\begin{aligned} &[a_{4,2}\,a_{5,3}\,a_{2,4}\,a_{8,5}\,a_{7,6}\,a_{3,7}\,a_{6,8}, -a_{4,2}\,a_{5,3}\,a_{1,4}\,a_{8,5}\,a_{7,6}\,a_{3,7}\,a_{6,8}, \\ &\quad a_{4,2}\,a_{5,3}\,(-a_{1,4}\,a_{7,5}\,a_{2,6}\,a_{6,8}\,a_{8,7} - a_{2,4}\,a_{1,5}\,a_{7,6}\,a_{6,8}\,a_{8,7} - a_{2,4}\,a_{8,5}\,a_{7,6}\,a_{1,8}\,a_{6,7}), \\ &\quad -a_{1,2}\,a_{5,3}\,a_{2,4}\,a_{8,5}\,a_{7,6}\,a_{3,7}\,a_{6,8} - a_{8,2}\,a_{5,3}\,a_{1,4}\,a_{7,5}\,a_{2,6}\,a_{3,7}\,a_{6,8} - a_{8,2}\,a_{5,3}\,a_{2,4}\,a_{1,5}\,a_{7,6}\,a_{3,7}\,a_{6,8}, \\ &\quad -a_{4,2}\,a_{6,3}\,a_{2,4}\,a_{8,5}\,a_{7,6}\,a_{1,8}\,a_{3,7}, -a_{4,2}\,a_{5,3}\,a_{2,4}\,a_{8,5}\,a_{7,6}\,a_{1,8}\,a_{3,7}, a_{4,2}\,a_{5,3}\,a_{1,4}\,a_{8,5}\,a_{2,6}\,a_{3,7}\,a_{6,8}, \\ &\quad -a_{4,2}\,a_{5,3}\,(-a_{1,4}\,a_{7,5}\,a_{2,6}\,a_{3,7}\,a_{6,8} - a_{2,4}\,a_{1,5}\,a_{7,6}\,a_{3,7}\,a_{6,8})] \\ &\quad [0, 0, 0, -a_{5,1}\,a_{6,3}\,a_{2,4}\,a_{8,5}\,a_{7,6}\,a_{1,8}\,a_{3,7} - a_{8,1}\,a_{5,3}\,a_{1,4}\,a_{7,5}\,a_{2,6}\,a_{3,7}\,a_{6,8} - a_{8,1}\,a_{5,3}\,a_{2,4}\,a_{1,5}\,a_{7,6}\,a_{3,7}\,a_{6,8}, 0, 0, 0, 0] \\ &\quad [-a_{5,1}\,a_{4,2}\,a_{2,4}\,a_{8,5}\,a_{7,6}\,a_{3,7}\,a_{6,8}, a_{5,1}\,a_{4,2}\,a_{1,4}\,a_{8,5}\,a_{7,6}\,a_{3,7}\,a_{6,8}, \\ &\quad -a_{5,1}\,a_{4,2}\,(-a_{1,4}\,a_{7,5}\,a_{2,6}\,a_{6,8}\,a_{8,7} - a_{2,4}\,a_{1,5}\,a_{7,6}\,a_{6,8}\,a_{8,7} - a_{2,4}\,a_{8,5}\,a_{7,6}\,a_{1,8}\,a_{6,7}), \\ &\quad a_{5,1}\,(a_{1,2}\,a_{2,4}\,a_{8,5}\,a_{7,6}\,a_{3,7}\,a_{6,8} + a_{8,2}\,a_{1,4}\,a_{7,5}\,a_{2,6}\,a_{3,7}\,a_{6,8} + a_{8,2}\,a_{2,4}\,a_{1,5}\,a_{7,6}\,a_{3,7}\,a_{6,8}), \\ &\quad a_{8,1}\,a_{4,2}\,(-a_{1,4}\,a_{7,5}\,a_{2,6}\,a_{3,7}\,a_{6,8} - a_{2,4}\,a_{1,5}\,a_{7,6}\,a_{3,7}\,a_{6,8}), a_{5,1}\,a_{4,2}\,a_{2,4}\,a_{8,5}\,a_{7,6}\,a_{1,8}\,a_{3,7}, -a_{5,1}\,a_{4,2}\,a_{1,4}\,a_{8,5}\,a_{2,6}\,a_{3,7}\,a_{6,8}, \\ &\quad a_{5,1}\,a_{4,2}\,(-a_{1,4}\,a_{7,5}\,a_{2,6}\,a_{3,7}\,a_{6,8} - a_{2,4}\,a_{1,5}\,a_{7,6}\,a_{3,7}\,a_{6,8})] \end{aligned}$$

$$[-a_{8,1}\,a_{4,2}\,a_{5,3}\,a_{7,5}\,a_{2,6}\,a_{3,7}\,a_{6,8}\,,\,-a_{5,1}\,a_{4,2}\,a_{6,3}\,a_{8,5}\,a_{7,6}\,a_{1,8}\,a_{3,7}-a_{8,1}\,a_{4,2}\,a_{5,3}\,a_{1,5}\,a_{7,6}\,a_{3,7}\,a_{6,8}\,,$$
$$-a_{5,1}\,a_{4,2}\,a_{6,3}\,a_{7,5}\,a_{2,6}\,a_{1,8}\,a_{8,7}+a_{8,1}\,a_{4,2}\,a_{5,3}\,a_{7,5}\,a_{2,6}\,a_{1,8}\,a_{6,7}\,,$$
$$-a_{5,1}\,a_{8,2}\,a_{6,3}\,a_{7,5}\,a_{2,6}\,a_{1,8}\,a_{3,7}+a_{8,1}\,a_{1,2}\,a_{5,3}\,a_{7,5}\,a_{2,6}\,a_{3,7}\,a_{6,8}\,,\,a_{8,1}\,a_{4,2}\,a_{6,3}\,a_{7,5}\,a_{2,6}\,a_{1,8}\,a_{3,7}\,,$$
$$a_{8,1}\,a_{4,2}\,a_{5,3}\,a_{7,5}\,a_{2,6}\,a_{1,8}\,a_{3,7}\,,\,a_{5,1}\,a_{4,2}\,a_{6,3}\,a_{8,5}\,a_{2,6}\,a_{1,8}\,a_{3,7}+a_{8,1}\,a_{4,2}\,a_{5,3}\,a_{1,5}\,a_{2,6}\,a_{3,7}\,a_{6,8}\,,\,a_{5,1}\,a_{4,2}\,a_{6,3}\,a_{7,5}\,a_{2,6}\,a_{1,8}\,a_{3,7}$$
$$]$$
$$[-a_{8,1}\,a_{4,2}\,a_{5,3}\,a_{2,4}\,a_{7,6}\,a_{3,7}\,a_{6,8}\,,\,a_{8,1}\,a_{4,2}\,a_{5,3}\,a_{1,4}\,a_{7,6}\,a_{3,7}\,a_{6,8}\,,$$
$$-a_{5,1}\,a_{4,2}\,a_{6,3}\,a_{2,4}\,a_{7,6}\,a_{1,8}\,a_{8,7}+a_{8,1}\,a_{4,2}\,a_{5,3}\,a_{2,4}\,a_{7,6}\,a_{1,8}\,a_{6,7}\,,$$
$$-a_{5,1}\,a_{8,2}\,a_{6,3}\,a_{2,4}\,a_{7,6}\,a_{1,8}\,a_{3,7}+a_{8,1}\,a_{1,2}\,a_{5,3}\,a_{2,4}\,a_{7,6}\,a_{3,7}\,a_{6,8}\,,\,a_{8,1}\,a_{4,2}\,a_{6,3}\,a_{2,4}\,a_{7,6}\,a_{1,8}\,a_{3,7}\,,$$
$$a_{8,1}\,a_{4,2}\,a_{5,3}\,a_{2,4}\,a_{7,6}\,a_{1,8}\,a_{3,7}\,,\,-a_{8,1}\,a_{4,2}\,a_{5,3}\,a_{1,4}\,a_{2,6}\,a_{3,7}\,a_{6,8}\,,\,a_{5,1}\,a_{4,2}\,a_{6,3}\,a_{2,4}\,a_{7,6}\,a_{1,8}\,a_{3,7}\,]$$
$$[a_{8,1}\,a_{4,2}\,a_{5,3}\,a_{2,4}\,a_{7,5}\,a_{3,7}\,a_{6,8}\,,\,-a_{8,1}\,a_{4,2}\,a_{5,3}\,a_{1,4}\,a_{7,5}\,a_{3,7}\,a_{6,8}\,,$$
$$a_{5,1}\,a_{4,2}\,a_{6,3}\,a_{2,4}\,a_{7,5}\,a_{1,8}\,a_{8,7}-a_{8,1}\,a_{4,2}\,a_{5,3}\,a_{2,4}\,a_{7,5}\,a_{1,8}\,a_{6,7}\,,$$
$$a_{5,1}\,a_{8,2}\,a_{6,3}\,a_{2,4}\,a_{7,5}\,a_{1,8}\,a_{3,7}-a_{8,1}\,a_{1,2}\,a_{5,3}\,a_{2,4}\,a_{7,5}\,a_{3,7}\,a_{6,8}\,,\,-a_{8,1}\,a_{4,2}\,a_{6,3}\,a_{2,4}\,a_{7,5}\,a_{1,8}\,a_{3,7}\,,$$
$$-a_{8,1}\,a_{4,2}\,a_{5,3}\,a_{2,4}\,a_{7,5}\,a_{1,8}\,a_{3,7}\,,\,-a_{5,1}\,a_{4,2}\,a_{6,3}\,a_{2,4}\,a_{8,5}\,a_{1,8}\,a_{3,7}-a_{8,1}\,a_{4,2}\,a_{5,3}\,a_{2,4}\,a_{1,5}\,a_{3,7}\,a_{6,8}\,,$$
$$-a_{5,1}\,a_{4,2}\,a_{6,3}\,a_{2,4}\,a_{7,5}\,a_{1,8}\,a_{3,7}\,]$$
$$[0\,,\,0\,,\,-a_{5,1}\,a_{4,2}\,a_{6,3}\,a_{2,4}\,a_{8,5}\,a_{1,8}\,a_{7,6}-a_{8,1}\,a_{4,2}\,a_{5,3}\,a_{1,4}\,a_{7,5}\,a_{2,6}\,a_{6,8}-a_{8,1}\,a_{4,2}\,a_{5,3}\,a_{2,4}\,a_{1,5}\,a_{6,8}\,a_{7,6}\,,\,0\,,\,0\,,\,0\,,\,0\,,\,0\,]$$
$$[-a_{5,1}\,a_{4,2}\,a_{6,3}\,a_{2,4}\,a_{8,5}\,a_{3,7}\,a_{7,6}\,,\,a_{5,1}\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{8,5}\,a_{3,7}\,a_{7,6}\,,\,a_{5,1}\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{7,5}\,a_{2,6}\,a_{8,7}$$
$$+a_{5,1}\,a_{4,2}\,a_{6,3}\,a_{2,4}\,a_{1,5}\,a_{7,6}\,a_{8,7}-a_{8,1}\,a_{4,2}\,a_{5,3}\,a_{1,4}\,a_{7,5}\,a_{2,6}\,a_{6,7}-a_{8,1}\,a_{4,2}\,a_{5,3}\,a_{2,4}\,a_{1,5}\,a_{6,7}\,a_{7,6}\,,$$
$$-a_{5,1}\,(-a_{1,2}\,a_{6,3}\,a_{2,4}\,a_{8,5}\,a_{3,7}\,a_{7,6}-a_{8,2}\,a_{6,3}\,a_{1,4}\,a_{7,5}\,a_{2,6}\,a_{3,7}-a_{8,2}\,a_{6,3}\,a_{2,4}\,a_{1,5}\,a_{3,7}\,a_{7,6})\,,$$
$$-a_{8,1}\,a_{4,2}\,a_{6,3}\,(a_{1,4}\,a_{7,5}\,a_{2,6}\,a_{3,7}+a_{2,4}\,a_{1,5}\,a_{3,7}\,a_{7,6})\,,\,-a_{8,1}\,a_{4,2}\,a_{5,3}\,(a_{1,4}\,a_{7,5}\,a_{2,6}\,a_{3,7}+a_{2,4}\,a_{1,5}\,a_{3,7}\,a_{7,6})\,,$$
$$-a_{5,1}\,a_{4,2}\,a_{6,3}\,a_{1,4}\,a_{8,5}\,a_{2,6}\,a_{3,7}\,,\,-a_{5,1}\,a_{4,2}\,a_{6,3}\,(a_{1,4}\,a_{7,5}\,a_{2,6}\,a_{3,7}+a_{2,4}\,a_{1,5}\,a_{3,7}\,a_{7,6})]$$

"85"

$$A := \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & -1 & -1 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & -1 \\ -1 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Qualitative Stability Analysis

"Criterion i"

poly_coef_F0_to_Fn = [-1, -1, -1, 2, -2, -7, 3, -2, -1]

positive_feedback = [0, 0, 1, 5, 8, 10, 15, 7, 1]

negative_feedback = [-1, -1, -2, -3, -10, -17, -12, -9, -2]

absolute_feedback = [1, 1, 3, 8, 18, 27, 27, 16, 3]

wFn = [-1., -1., -0.33, 0.25, -0.11, -0.26, 0.11, -0.12, -0.33]

"Criterion ii"

$wD_7 = 0.000075$

ratio_to_model_C = 230.

"Class II Model"

Qualitative Press Perturbation Analysis

"Change in Abundance from Positive Input"

"From Increased Birth or Immigration or from Decreased Death or Emigration"

"adjoint (-A)"

$$\begin{bmatrix} 0 & -1 & 1 & -1 & -1 & 0 & 1 & 1 \\ -2 & 3 & -2 & 4 & 1 & -1 & -2 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 2 & -2 & 1 & -3 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & -2 & 2 & -3 & -1 & 1 & 1 & 1 \\ 2 & -2 & 1 & -2 & -1 & 1 & 1 & 1 \\ -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback (T)"

$$\begin{bmatrix} 6 & 3 & 1 & 5 & 1 & 2 & 1 & 1 \\ 6 & 3 & 2 & 6 & 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 4 & 2 & 1 & 5 & 2 & 1 & 1 & 1 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 2 & 5 & 1 & 1 & 1 & 1 \\ 4 & 2 & 1 & 4 & 1 & 1 & 1 & 1 \\ 3 & 3 & 0 & 3 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"weighted predictions (W)"

$$\begin{bmatrix} 0. & 0.33 & 1. & 0.20 & 1. & 0. & 1. & 1. \\ 0.33 & 1. & 1. & 0.67 & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 0.33 & 1. & 1. & 1. & 1. \\ 0.50 & 1. & 1. & 0.60 & 0. & 1. & 1. & 1. \\ 0.33 & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 0.50 & 1. & 1. & 0.60 & 1. & 1. & 1. & 1. \\ 0.50 & 1. & 1. & 0.50 & 1. & 1. & 1. & 1. \\ 0.33 & 0.33 & 1. & 0.33 & 1. & 1. & 1. & 1. \end{bmatrix}$$

"Change in Life Expectancy (E) from Positive Input"

"Increased Rate of Birth or Immigration"

"delta E"

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ -2 & 2 & -2 & 2 & 1 & -1 & -1 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -2 & 2 & -1 & 2 & 0 & -1 & -1 & -1 \\ -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 \\ 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 3 & 0 & 0 & 0 & 0 \\ 4 & 2 & 2 & 4 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 4 & 2 & 1 & 4 & 2 & 1 & 1 & 1 \\ 5 & 3 & 1 & 5 & 1 & 1 & 1 & 1 \\ 6 & 3 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 0.33 & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 0.33 & 1. & 0.33 & 1. & 1. & 1. & 1. \\ 0.50 & 1. & 1. & 0.50 & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 0.33 & 1. & 1. & 1. & 1. \\ 0.50 & 1. & 1. & 0.50 & 0. & 1. & 1. & 1. \\ 0.20 & 0.33 & 1. & 0.20 & 1. & 1. & 1. & 1. \\ 0. & 0.33 & 1. & 0.33 & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 0.33 \end{bmatrix}$$

"Decreased Rate of Death or Emigration"

"delta E"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -2 & 2 & -1 & 2 & 1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & 2 & -1 & 2 & 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & 1 & 1 & 0 & -1 & -1 \\ 0 & -1 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"absolute feedback"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 4 & 2 & 1 & 4 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 1 & 4 & 1 & 1 & 1 & 1 \\ 5 & 3 & 1 & 5 & 1 & 2 & 1 & 1 \\ 6 & 3 & 0 & 3 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"weighted delta E"

$$\begin{bmatrix} 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 0.33 & 1. & 1. & 1. & 1. \\ 0.50 & 1. & 1. & 0.50 & 1. & 1. & 1. & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 0.50 & 1. & 1. & 0.50 & 1. & 1. & 1. & 1. \\ 0.20 & 0.33 & 1. & 0.20 & 1. & 0. & 1. & 1. \\ 0. & 0.33 & 1. & 0.33 & 1. & 1. & 0.33 & 1. \\ 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

Symbolic Analyses

"Symbolically Specified Community Matrix"

$$\begin{bmatrix} 0 & 0 & 0 & 0 & -a_{1,5} & 0 & 0 & 0 \\ 0 & 0 & a_{2,3} & 0 & -a_{2,5} & 0 & 0 & -a_{2,8} \\ 0 & 0 & -a_{3,3} & 0 & 0 & -a_{3,6} & a_{3,7} & 0 \\ 0 & 0 & -a_{4,3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -a_{5,3} & -a_{5,4} & 0 & 0 & a_{5,7} & 0 \\ a_{6,1} & 0 & 0 & 0 & a_{6,5} & 0 & -a_{6,7} & -a_{6,8} \\ 0 & a_{7,2} & 0 & 0 & -a_{7,5} & a_{7,6} & 0 & -a_{7,8} \\ -a_{8,1} & -a_{8,2} & 0 & -a_{8,4} & 0 & 0 & 0 & 0 \end{bmatrix}$$

"system feedback"

$$F_8$$

$$-a_{6,7}\,a_{8,1}\,a_{7,2}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{2,8}\,a_{3,6} + a_{8,2}\,a_{6,1}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{7,6}\,a_{2,8}\,a_{3,7} - a_{8,4}\,a_{6,1}\,a_{7,2}\,a_{4,3}\,a_{1,5}\,a_{3,6}\,a_{2,8}\,a_{5,7}$$

$$F_7$$

$$\begin{aligned} &a_{8,1}\,a_{7,2}\,a_{5,3}\,a_{1,5}\,a_{3,6}\,a_{2,8}\,a_{6,7} + a_{8,1}\,a_{7,2}\,a_{2,3}\,a_{1,5}\,a_{3,6}\,a_{5,7}\,a_{6,8} - a_{6,1}\,a_{8,2}\,a_{5,3}\,a_{1,5}\,a_{7,6}\,a_{2,8}\,a_{3,7} \\ &\quad + a_{6,1}\,a_{8,2}\,a_{3,3}\,a_{1,5}\,a_{7,6}\,a_{2,8}\,a_{5,7} + a_{6,1}\,a_{8,2}\,a_{2,3}\,a_{1,5}\,a_{3,6}\,a_{5,7}\,a_{7,8} - a_{6,1}\,a_{8,2}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{2,8}\,a_{3,6} \\ &\quad - a_{8,1}\,a_{7,2}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{2,8}\,a_{3,7} + a_{7,2}\,a_{4,3}\,a_{8,4}\,a_{2,5}\,a_{3,6}\,a_{5,7}\,a_{6,8} + a_{7,2}\,a_{4,3}\,a_{8,4}\,a_{6,5}\,a_{3,6}\,a_{2,8}\,a_{5,7} \\ &\quad - a_{8,2}\,a_{4,3}\,a_{5,4}\,a_{2,5}\,a_{3,6}\,a_{6,7}\,a_{7,8} - a_{8,2}\,a_{4,3}\,a_{5,4}\,a_{2,5}\,a_{7,6}\,a_{3,7}\,a_{6,8} - a_{8,2}\,a_{4,3}\,a_{5,4}\,a_{6,5}\,a_{7,6}\,a_{2,8}\,a_{3,7} \\ &\quad + a_{8,2}\,a_{4,3}\,a_{5,4}\,a_{7,5}\,a_{3,6}\,a_{2,8}\,a_{6,7} - a_{6,1}\,a_{4,3}\,a_{8,4}\,a_{1,5}\,a_{3,6}\,a_{5,7}\,a_{7,8} - a_{8,1}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{3,6}\,a_{6,7}\,a_{7,8} \\ &\quad - a_{8,1}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{7,6}\,a_{3,7}\,a_{6,8} \end{aligned}$$

$$F_6$$

$$\begin{aligned} &a_{8,1}\,a_{5,3}\,a_{1,5}\,a_{7,6}\,a_{3,7}\,a_{6,8} - a_{7,6}\,a_{6,1}\,a_{1,5}\,a_{4,3}\,a_{5,4}\,a_{3,7} + a_{7,2}\,a_{2,3}\,a_{3,6}\,a_{6,1}\,a_{5,7}\,a_{1,5} - a_{8,2}\,a_{2,5}\,a_{4,3}\,a_{5,4}\,a_{3,7}\,a_{7,8} \\ &\quad + a_{8,4}\,a_{4,3}\,a_{3,6}\,a_{6,5}\,a_{5,7}\,a_{7,8} - a_{8,4}\,a_{4,3}\,a_{3,6}\,a_{6,7}\,a_{7,2}\,a_{2,8} - a_{8,2}\,a_{2,3}\,a_{3,6}\,a_{6,5}\,a_{5,7}\,a_{7,8} + a_{8,2}\,a_{2,5}\,a_{5,3}\,a_{3,6}\,a_{6,7}\,a_{7,8} \\ &\quad + a_{8,2}\,a_{2,5}\,a_{5,3}\,a_{7,6}\,a_{3,7}\,a_{6,8} - a_{8,1}\,a_{1,5}\,a_{4,3}\,a_{5,4}\,a_{3,7}\,a_{7,8} + a_{8,1}\,a_{1,5}\,a_{4,3}\,a_{5,4}\,a_{3,6}\,a_{6,8} + a_{8,2}\,a_{2,5}\,a_{4,3}\,a_{5,4}\,a_{3,6}\,a_{6,8} \\ &\quad + a_{8,1}\,a_{1,5}\,a_{5,3}\,a_{3,6}\,a_{6,7}\,a_{7,8} + a_{8,1}\,a_{1,5}\,a_{5,3}\,a_{3,7}\,a_{7,2}\,a_{2,8} - a_{3,3}\,a_{8,1}\,a_{1,5}\,a_{5,7}\,a_{7,2}\,a_{2,8} - a_{3,3}\,a_{8,1}\,a_{1,5}\,a_{5,7}\,a_{7,6}\,a_{6,8} \\ &\quad - a_{3,3}\,a_{8,2}\,a_{2,5}\,a_{5,7}\,a_{7,6}\,a_{6,8} + a_{8,2}\,a_{2,8}\,a_{7,6}\,a_{6,5}\,a_{5,3}\,a_{3,7} - a_{8,2}\,a_{2,8}\,a_{3,3}\,a_{7,6}\,a_{6,5}\,a_{5,7} + a_{8,2}\,a_{2,8}\,a_{6,1}\,a_{1,5}\,a_{5,3}\,a_{3,6} \\ &\quad + a_{8,2}\,a_{2,8}\,a_{6,5}\,a_{4,3}\,a_{5,4}\,a_{3,6} + a_{8,2}\,a_{2,8}\,a_{7,6}\,a_{6,1}\,a_{5,7}\,a_{1,5} - a_{8,2}\,a_{2,8}\,a_{7,5}\,a_{5,3}\,a_{3,6}\,a_{6,7} + a_{8,2}\,a_{2,8}\,a_{7,5}\,a_{4,3}\,a_{5,4}\,a_{3,7} \end{aligned}$$

$$- a_{8,2}\, a_{2,3}\, a_{3,6}\, a_{6,8}\, a_{7,5}\, a_{5,7} + a_{8,4}\, a_{4,3}\, a_{3,6}\, a_{6,8}\, a_{7,5}\, a_{5,7} - a_{7,2}\, a_{4,3}\, a_{5,4}\, a_{2,5}\, a_{3,6}\, a_{6,7}$$

$$F_5$$

$$a_{6,1}\, a_{1,5}\, a_{3,6}\, a_{4,3}\, a_{5,4} + a_{7,6}\, a_{6,5}\, a_{4,3}\, a_{5,4}\, a_{3,7} - a_{7,2}\, a_{2,3}\, a_{3,6}\, a_{6,5}\, a_{5,7} + a_{7,2}\, a_{2,5}\, a_{5,3}\, a_{3,6}\, a_{6,7} - a_{7,2}\, a_{2,5}\, a_{4,3}\, a_{5,4}\, a_{3,7}$$
$$- a_{7,5}\, a_{4,3}\, a_{5,4}\, a_{3,6}\, a_{6,7} + a_{7,6}\, a_{6,1}\, a_{1,5}\, a_{5,3}\, a_{3,7} - a_{3,3}\, a_{7,6}\, a_{6,1}\, a_{5,7}\, a_{1,5} - a_{8,2}\, a_{2,5}\, a_{5,7}\, a_{7,6}\, a_{6,8} - a_{8,4}\, a_{4,3}\, a_{3,6}\, a_{6,7}\, a_{7,8}$$
$$- a_{8,4}\, a_{4,3}\, a_{3,7}\, a_{7,2}\, a_{2,8} - a_{8,4}\, a_{4,3}\, a_{7,6}\, a_{3,7}\, a_{6,8} + a_{8,1}\, a_{1,5}\, a_{5,3}\, a_{3,7}\, a_{7,8} - a_{8,1}\, a_{1,5}\, a_{5,7}\, a_{7,2}\, a_{2,8} - a_{8,1}\, a_{1,5}\, a_{5,7}\, a_{7,6}\, a_{6,8}$$
$$+ a_{8,2}\, a_{2,3}\, a_{3,6}\, a_{6,7}\, a_{7,8} + a_{8,2}\, a_{2,3}\, a_{7,6}\, a_{3,7}\, a_{6,8} - a_{8,2}\, a_{2,5}\, a_{5,3}\, a_{3,6}\, a_{6,8} + a_{8,2}\, a_{2,5}\, a_{5,3}\, a_{3,7}\, a_{7,8} - a_{8,1}\, a_{1,5}\, a_{5,3}\, a_{3,6}\, a_{6,8}$$
$$- a_{3,3}\, a_{8,2}\, a_{2,5}\, a_{5,7}\, a_{7,8} - a_{8,2}\, a_{2,8}\, a_{7,5}\, a_{5,3}\, a_{3,7} - a_{8,2}\, a_{2,8}\, a_{7,6}\, a_{6,5}\, a_{5,7} + a_{8,2}\, a_{2,8}\, a_{3,3}\, a_{7,5}\, a_{5,7} + a_{8,2}\, a_{2,8}\, a_{3,3}\, a_{6,7}\, a_{7,6}$$
$$- a_{8,2}\, a_{2,8}\, a_{6,5}\, a_{5,3}\, a_{3,6} - a_{3,3}\, a_{8,1}\, a_{1,5}\, a_{5,7}\, a_{7,8}$$

$$F_4$$

$$-a_{8,1}\, a_{1,5}\, a_{5,7}\, a_{7,8} - a_{8,2}\, a_{2,3}\, a_{3,6}\, a_{6,8} + a_{8,2}\, a_{2,3}\, a_{3,7}\, a_{7,8} - a_{8,2}\, a_{2,5}\, a_{5,7}\, a_{7,8} + a_{8,4}\, a_{4,3}\, a_{3,6}\, a_{6,8} - a_{8,4}\, a_{4,3}\, a_{3,7}\, a_{7,8}$$
$$+ a_{8,2}\, a_{2,8}\, a_{7,5}\, a_{5,7} + a_{8,2}\, a_{2,8}\, a_{6,7}\, a_{7,6} + a_{7,2}\, a_{2,3}\, a_{3,6}\, a_{6,7} + a_{7,2}\, a_{2,5}\, a_{5,3}\, a_{3,7} + a_{7,5}\, a_{5,3}\, a_{3,6}\, a_{6,7} - a_{7,5}\, a_{4,3}\, a_{5,4}\, a_{3,7}$$
$$- a_{7,6}\, a_{6,1}\, a_{5,7}\, a_{1,5} - a_{7,6}\, a_{6,5}\, a_{5,3}\, a_{3,7} - a_{3,3}\, a_{7,2}\, a_{2,5}\, a_{5,7} + a_{3,3}\, a_{7,6}\, a_{6,5}\, a_{5,7} - a_{6,1}\, a_{1,5}\, a_{5,3}\, a_{3,6} - a_{6,5}\, a_{4,3}\, a_{5,4}\, a_{3,6}$$

$$F_3$$

$$a_{8,2}\, a_{2,8}\, a_{3,3} + a_{7,2}\, a_{2,3}\, a_{3,7} - a_{7,2}\, a_{2,5}\, a_{5,7} + a_{7,5}\, a_{5,3}\, a_{3,7} + a_{7,6}\, a_{6,5}\, a_{5,7} - a_{3,3}\, a_{7,5}\, a_{5,7} - a_{3,3}\, a_{6,7}\, a_{7,6} + a_{6,5}\, a_{5,3}\, a_{3,6}$$

$$F_2$$

$$a_{8,2}\, a_{2,8} - a_{7,5}\, a_{5,7} - a_{6,7}\, a_{7,6}$$

$$F_1$$

$$-a_{3,3}$$

$$F_0$$

$$-1$$

"adjoint (-A)"

$$[-a_{7,2}\, a_{4,3}\, a_{8,4}\, a_{2,5}\, a_{3,6}\, a_{5,7}\, a_{6,8} - a_{7,2}\, a_{4,3}\, a_{8,4}\, a_{6,5}\, a_{3,6}\, a_{2,8}\, a_{5,7} + a_{8,2}\, a_{4,3}\, a_{5,4}\, a_{2,5}\, a_{3,6}\, a_{6,7}\, a_{7,8}$$
$$+ a_{8,2}\, a_{4,3}\, a_{5,4}\, a_{2,5}\, a_{7,6}\, a_{3,7}\, a_{6,8} + a_{8,2}\, a_{4,3}\, a_{5,4}\, a_{6,5}\, a_{7,6}\, a_{2,8}\, a_{3,7} - a_{8,2}\, a_{4,3}\, a_{5,4}\, a_{7,5}\, a_{3,6}\, a_{2,8}\, a_{6,7},$$
$$a_{7,2}\, a_{4,3}\, a_{8,4}\, a_{1,5}\, a_{3,6}\, a_{5,7}\, a_{6,8} - a_{8,2}\, a_{4,3}\, a_{5,4}\, a_{1,5}\, a_{3,6}\, a_{6,7}\, a_{7,8} - a_{8,2}\, a_{4,3}\, a_{5,4}\, a_{1,5}\, a_{7,6}\, a_{3,7}\, a_{6,8},$$
$$a_{8,2}\, a_{4,3}\, a_{5,4}\, a_{1,5}\, a_{7,6}\, a_{2,8}\, a_{6,7}, a_{7,2}\, a_{2,3}\, a_{8,4}\, a_{1,5}\, a_{3,6}\, a_{5,7}\, a_{6,8} + a_{7,2}\, a_{5,3}\, a_{8,4}\, a_{1,5}\, a_{3,6}\, a_{2,8}\, a_{6,7}$$
$$- a_{8,2}\, a_{2,3}\, a_{5,4}\, a_{1,5}\, a_{3,6}\, a_{6,7}\, a_{7,8} - a_{8,2}\, a_{2,3}\, a_{5,4}\, a_{1,5}\, a_{7,6}\, a_{3,7}\, a_{6,8} - a_{8,2}\, a_{3,3}\, a_{5,4}\, a_{1,5}\, a_{7,6}\, a_{2,8}\, a_{6,7},$$
$$-a_{7,2}\, a_{4,3}\, a_{8,4}\, a_{1,5}\, a_{3,6}\, a_{2,8}\, a_{6,7}, -a_{7,2}\, a_{4,3}\, a_{8,4}\, a_{1,5}\, a_{3,6}\, a_{2,8}\, a_{5,7} + a_{8,2}\, a_{4,3}\, a_{5,4}\, a_{1,5}\, a_{7,6}\, a_{2,8}\, a_{3,7},$$
$$a_{8,2}\, a_{4,3}\, a_{5,4}\, a_{1,5}\, a_{3,6}\, a_{2,8}\, a_{6,7}, a_{7,2}\, a_{4,3}\, a_{5,4}\, a_{1,5}\, a_{3,6}\, a_{2,8}\, a_{6,7}]$$
$$[-a_{6,1}\, a_{4,3}\, a_{8,4}\, a_{2,5}\, a_{3,6}\, a_{5,7}\, a_{7,8} + a_{6,1}\, a_{4,3}\, a_{8,4}\, a_{7,5}\, a_{3,6}\, a_{2,8}\, a_{5,7} - a_{8,1}\, a_{4,3}\, a_{5,4}\, a_{2,5}\, a_{3,6}\, a_{6,7}\, a_{7,8}$$
$$- a_{8,1}\, a_{4,3}\, a_{5,4}\, a_{2,5}\, a_{7,6}\, a_{3,7}\, a_{6,8} - a_{8,1}\, a_{4,3}\, a_{5,4}\, a_{6,5}\, a_{7,6}\, a_{2,8}\, a_{3,7} + a_{8,1}\, a_{4,3}\, a_{5,4}\, a_{7,5}\, a_{3,6}\, a_{2,8}\, a_{6,7},$$
$$a_{6,1}\, a_{4,3}\, a_{8,4}\, a_{1,5}\, a_{3,6}\, a_{5,7}\, a_{7,8} + a_{8,1}\, a_{4,3}\, a_{5,4}\, a_{1,5}\, a_{3,6}\, a_{6,7}\, a_{7,8} + a_{8,1}\, a_{4,3}\, a_{5,4}\, a_{1,5}\, a_{7,6}\, a_{3,7}\, a_{6,8},$$
$$-a_{6,1}\, a_{4,3}\, a_{8,4}\, a_{1,5}\, a_{7,6}\, a_{2,8}\, a_{5,7} - a_{8,1}\, a_{4,3}\, a_{5,4}\, a_{1,5}\, a_{7,6}\, a_{2,8}\, a_{6,7}, a_{6,1}\, a_{2,3}\, a_{8,4}\, a_{1,5}\, a_{3,6}\, a_{5,7}\, a_{7,8}$$
$$+ a_{6,1}\, a_{3,3}\, a_{8,4}\, a_{1,5}\, a_{7,6}\, a_{2,8}\, a_{5,7} - a_{6,1}\, a_{5,3}\, a_{8,4}\, a_{1,5}\, a_{7,6}\, a_{2,8}\, a_{3,7} + a_{8,1}\, a_{2,3}\, a_{5,4}\, a_{1,5}\, a_{3,6}\, a_{6,7}\, a_{7,8}$$
$$+ a_{8,1}\, a_{2,3}\, a_{5,4}\, a_{1,5}\, a_{7,6}\, a_{3,7}\, a_{6,8} + a_{8,1}\, a_{3,3}\, a_{5,4}\, a_{1,5}\, a_{7,6}\, a_{2,8}\, a_{6,7}, a_{6,1}\, a_{4,3}\, a_{8,4}\, a_{1,5}\, a_{7,6}\, a_{2,8}\, a_{3,7},$$
$$-a_{8,1}\, a_{4,3}\, a_{5,4}\, a_{1,5}\, a_{7,6}\, a_{2,8}\, a_{3,7}, -a_{6,1}\, a_{4,3}\, a_{8,4}\, a_{1,5}\, a_{3,6}\, a_{2,8}\, a_{5,7} - a_{8,1}\, a_{4,3}\, a_{5,4}\, a_{1,5}\, a_{3,6}\, a_{2,8}\, a_{6,7},$$
$$-a_{6,1}\, a_{4,3}\, a_{5,4}\, a_{1,5}\, a_{7,6}\, a_{2,8}\, a_{3,7}]$$
$$[0, 0, 0, a_{6,1}\, a_{7,2}\, a_{8,4}\, a_{1,5}\, a_{3,6}\, a_{2,8}\, a_{5,7} - a_{6,1}\, a_{8,2}\, a_{5,4}\, a_{1,5}\, a_{7,6}\, a_{2,8}\, a_{3,7} + a_{8,1}\, a_{7,2}\, a_{5,4}\, a_{1,5}\, a_{3,6}\, a_{2,8}\, a_{6,7}, 0, 0, 0, 0]$$
$$[a_{6,1}\, a_{8,2}\, a_{4,3}\, a_{2,5}\, a_{3,6}\, a_{5,7}\, a_{7,8} - a_{6,1}\, a_{8,2}\, a_{4,3}\, a_{7,5}\, a_{3,6}\, a_{2,8}\, a_{5,7} + a_{8,1}\, a_{7,2}\, a_{4,3}\, a_{2,5}\, a_{3,6}\, a_{5,7}\, a_{6,8}$$
$$+ a_{8,1}\, a_{7,2}\, a_{4,3}\, a_{6,5}\, a_{3,6}\, a_{2,8}\, a_{5,7}, -a_{6,1}\, a_{8,2}\, a_{4,3}\, a_{1,5}\, a_{3,6}\, a_{5,7}\, a_{7,8} - a_{8,1}\, a_{7,2}\, a_{4,3}\, a_{1,5}\, a_{3,6}\, a_{5,7}\, a_{6,8},$$
$$a_{6,1}\, a_{8,2}\, a_{4,3}\, a_{1,5}\, a_{7,6}\, a_{2,8}\, a_{5,7}, -a_{6,1}\, a_{8,2}\, a_{2,3}\, a_{1,5}\, a_{3,6}\, a_{5,7}\, a_{7,8} - a_{6,1}\, a_{8,2}\, a_{3,3}\, a_{1,5}\, a_{7,6}\, a_{2,8}\, a_{5,7}$$
$$+ a_{6,1}\, a_{8,2}\, a_{5,3}\, a_{1,5}\, a_{7,6}\, a_{2,8}\, a_{3,7} - a_{8,1}\, a_{7,2}\, a_{2,3}\, a_{1,5}\, a_{3,6}\, a_{5,7}\, a_{6,8} - a_{8,1}\, a_{7,2}\, a_{5,3}\, a_{1,5}\, a_{3,6}\, a_{2,8}\, a_{6,7},$$
$$-a_{6,1}\, a_{8,2}\, a_{4,3}\, a_{1,5}\, a_{7,6}\, a_{2,8}\, a_{3,7} + a_{8,1}\, a_{7,2}\, a_{4,3}\, a_{1,5}\, a_{3,6}\, a_{2,8}\, a_{6,7}, a_{8,1}\, a_{7,2}\, a_{4,3}\, a_{1,5}\, a_{3,6}\, a_{2,8}\, a_{5,7},$$
$$a_{6,1}\, a_{8,2}\, a_{4,3}\, a_{1,5}\, a_{3,6}\, a_{2,8}\, a_{5,7}, a_{6,1}\, a_{7,2}\, a_{4,3}\, a_{1,5}\, a_{3,6}\, a_{2,8}\, a_{5,7}]$$
$$[a_{6,1}\, a_{7,2}\, a_{4,3}\, a_{8,4}\, a_{3,6}\, a_{2,8}\, a_{5,7} - a_{6,1}\, a_{8,2}\, a_{4,3}\, a_{5,4}\, a_{7,6}\, a_{2,8}\, a_{3,7} + a_{8,1}\, a_{7,2}\, a_{4,3}\, a_{5,4}\, a_{3,6}\, a_{2,8}\, a_{6,7}, 0, 0, 0, 0, 0, 0]$$
$$[a_{6,1}\, a_{8,2}\, a_{4,3}\, a_{5,4}\, a_{2,5}\, a_{3,7}\, a_{7,8} - a_{6,1}\, a_{8,2}\, a_{4,3}\, a_{5,4}\, a_{7,5}\, a_{2,8}\, a_{3,7} + a_{8,1}\, a_{7,2}\, a_{4,3}\, a_{5,4}\, a_{2,5}\, a_{3,7}\, a_{6,8}$$
$$+ a_{8,1}\, a_{7,2}\, a_{4,3}\, a_{5,4}\, a_{6,5}\, a_{2,8}\, a_{3,7}, -a_{6,1}\, a_{8,2}\, a_{4,3}\, a_{5,4}\, a_{1,5}\, a_{3,7}\, a_{7,8} - a_{8,1}\, a_{7,2}\, a_{4,3}\, a_{5,4}\, a_{1,5}\, a_{3,7}\, a_{6,8},$$

$$a_{6,1}\,a_{7,2}\,a_{4,3}\,a_{8,4}\,a_{1,5}\,a_{2,8}\,a_{5,7}+a_{8,1}\,a_{7,2}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{2,8}\,a_{6,7},\ -a_{6,1}\,a_{7,2}\,a_{3,3}\,a_{8,4}\,a_{1,5}\,a_{2,8}\,a_{5,7}$$
$$+a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{8,4}\,a_{1,5}\,a_{2,8}\,a_{3,7}-a_{6,1}\,a_{8,2}\,a_{2,3}\,a_{5,4}\,a_{1,5}\,a_{3,7}\,a_{7,8}-a_{8,1}\,a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{1,5}\,a_{3,7}\,a_{6,8}$$
$$-a_{8,1}\,a_{7,2}\,a_{3,3}\,a_{5,4}\,a_{1,5}\,a_{2,8}\,a_{6,7},\ -a_{6,1}\,a_{7,2}\,a_{4,3}\,a_{8,4}\,a_{1,5}\,a_{2,8}\,a_{3,7},\ a_{8,1}\,a_{7,2}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{2,8}\,a_{3,7},$$
$$a_{6,1}\,a_{8,2}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{2,8}\,a_{3,7},\ a_{6,1}\,a_{7,2}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{2,8}\,a_{3,7}]$$
$$[a_{6,1}\,a_{8,2}\,a_{4,3}\,a_{5,4}\,a_{2,5}\,a_{3,6}\,a_{7,8}-a_{6,1}\,a_{8,2}\,a_{4,3}\,a_{5,4}\,a_{7,5}\,a_{2,8}\,a_{3,6}+a_{8,1}\,a_{7,2}\,a_{4,3}\,a_{5,4}\,a_{2,5}\,a_{3,6}\,a_{6,8}$$
$$+a_{8,1}\,a_{7,2}\,a_{4,3}\,a_{5,4}\,a_{6,5}\,a_{2,8}\,a_{3,6},\ -a_{6,1}\,a_{8,2}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{3,6}\,a_{7,8}-a_{8,1}\,a_{7,2}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{3,6}\,a_{6,8},$$
$$a_{6,1}\,a_{8,2}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{2,8}\,a_{7,6},\ a_{6,1}\,a_{7,2}\,a_{5,3}\,a_{8,4}\,a_{1,5}\,a_{2,8}\,a_{3,6}-a_{6,1}\,a_{8,2}\,a_{2,3}\,a_{5,4}\,a_{1,5}\,a_{3,6}\,a_{7,8}$$
$$-a_{6,1}\,a_{8,2}\,a_{3,3}\,a_{5,4}\,a_{1,5}\,a_{2,8}\,a_{7,6}-a_{8,1}\,a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{1,5}\,a_{3,6}\,a_{6,8},\ -a_{6,1}\,a_{7,2}\,a_{4,3}\,a_{8,4}\,a_{1,5}\,a_{2,8}\,a_{3,6},$$
$$a_{8,1}\,a_{7,2}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{2,8}\,a_{3,6},\ a_{6,1}\,a_{8,2}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{2,8}\,a_{3,6},\ a_{6,1}\,a_{7,2}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{2,8}\,a_{3,6}]$$
$$[-a_{6,1}\,a_{7,2}\,a_{4,3}\,a_{8,4}\,a_{2,5}\,a_{3,6}\,a_{5,7}+a_{6,1}\,a_{8,2}\,a_{4,3}\,a_{5,4}\,a_{2,5}\,a_{3,7}\,a_{7,6}-a_{8,1}\,a_{7,2}\,a_{4,3}\,a_{5,4}\,a_{2,5}\,a_{3,6}\,a_{6,7},$$
$$a_{6,1}\,a_{7,2}\,a_{4,3}\,a_{8,4}\,a_{1,5}\,a_{3,6}\,a_{5,7}-a_{6,1}\,a_{8,2}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{3,7}\,a_{7,6}+a_{8,1}\,a_{7,2}\,a_{4,3}\,a_{5,4}\,a_{1,5}\,a_{3,6}\,a_{6,7},\ 0,$$
$$a_{6,1}\,a_{7,2}\,a_{2,3}\,a_{8,4}\,a_{1,5}\,a_{3,6}\,a_{5,7}-a_{6,1}\,a_{8,2}\,a_{2,3}\,a_{5,4}\,a_{1,5}\,a_{3,7}\,a_{7,6}+a_{8,1}\,a_{7,2}\,a_{2,3}\,a_{5,4}\,a_{1,5}\,a_{3,6}\,a_{6,7},\ 0,\ 0,\ 0,\ 0]$$